MULTIPLE CONCLUSION DEDUCTIONS IN CLASSICAL LOGIC

MARCEL MARETIĆ
UNIVERSITY OF ZAGREB

Abstract. Natural deduction systems are, unlike Gentzen's sequent calculus, not related to semantic trees. Natural deductions arise from syntactic approach to logic – from proof search and inference rules. In that sense they are adequate for minimal and intuitionistic logic. Addition of *tertium non datur* (TND) or *reduction ad absurdum* (RAA) yields deduction calculus for classical logic.

Kneale in [2] proposes multiple conclusion deductions as an elegant and symmetrical version of deduction calculus that provides a good fit for classical logic. Kneale's inference rules are local – hypotheses are never discharged. Proofs, which Kneale calls *developments*, are formula trees branching downward *and upward*.

Kneale's calculus of developments is not complete. Shoesmith and Smiley in [1] propose adjustments for completion of the calculus. Our approach to multiple conclusion calculus is simple and better motivated. Unlike Shoesmith and Smiley, who in [1] motivate multiple conlcusion deductions syntactically, we relate multiple conclusion deductions to semantic trees. We present an elegant and analytic proof search for multiple conclusion deductions.

Essential steps of the algorithm are:

(1) analysis: construction of analytic deductions;
(2) synthesis: matching of analytic deductions that completes the proof search.

Thus, multiple conclusion deductions are analytic in the sense that they yield a simple analytic proof search (as in [3]).

Steps of the proof search algorithm can be motivated semantically. Analysis (step 1) corresponds to semantic analysis and branching of a clausal semantic tree, whereas synthesys (step 2) corresponds to branch closing on the clausal semantic tree. Therefore, proof search for multiple conclusion deductions is algorithmically equivalent to Beth's semantic trees.

References

[1] Shoesmith D.J., Smiley T.J., *Multiple Conclusion Logic*, Cambridge University Press, 1978.
[2] Kneale W., Kneale M., *Development of Logic*, Clarendon Press, 1956, pp. 538–548.

[3] Smullyan R. M., *First Order Logic*, Courier Dover Publications, 1995.