Hierarchies of probability logics

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Abstract. Our aim is to present what we call the lower and the upper hierarchies of the real valued probability logics with probability operators of the form $P_{\geqslant s}$ and Q_F, where $s \in [0,1]_{\mathbb{Q}} = [0,1] \cap \mathbb{Q}$ and F is a recursive subset of $[0,1]_{\mathbb{Q}}$. The intended meaning of $P_{\geqslant s}\alpha$ is that the probability of α is at least s, while the intended meaning of $Q_F\alpha$ is that the probability of α is in F.

Introduction

The modern probability logics arose from the work of Jerome Keisler on generalized quantifiers and hyperfinite model theory in the mid seventies of the twentieth century [8]. Another branch of research that was involved with automatization of reasoning under uncertainty have led to development of numerous Hilbert style formal systems with modal like probability operators, see for instance [5, 2, 11, 13, 14, 17, 18, 20, 23, 24]. The simplest form of such representation of uncertainty does not allow iteration of probability operators, so formulas are Boolean combinations of the basic probability formulas, i.e. formulas of the form

$$\texttt{ProbOp}(\alpha_1, \dots, \alpha_n),$$

where $\alpha_1, \dots, \alpha_n$ are classical (propositional or predicate) formulas and `ProbOp` is an n-ary probability operator. Weighted probability formulas used by Fagin, Halpern and Megiddo in [2] can be treated as n-ary probability operators. For instance,

$$w(\alpha) + 3w(\beta) - 5w(\gamma) \geqslant 1$$

is example of a ternary probability operator.

The vast majority of those formal systems have unary or binary probability operators. The unary operators are used for statements about probability of classical formulas: for example we use

$$P_{\geqslant 3/4}(p \vee q)$$

to express “the probability of $p \vee q$ is at least 3/4”, while

$$Q_{\{\frac{n}{n+1} \mid n \in \mathbb{N}\}}(p \vee q)$$

in our notation reads “the probability of $p \vee q$ is an element of the set $\{\frac{n}{n+1} \mid n \in \mathbb{N}\}$”. The binary operators are usually used for the expression of conditional probability: for instance, we use

$$CP_{\geqslant 1/3}(p, q)$$

to express that the conditional probability of p given q is at least $1/3$.

Over the course of two decades we have developed various probability logics with the mentioned types of probability operators - an extensive survey including a uniform notation for logics is presented in [17]. The aim of this paper is to put the certain class of probability logics into the wider context of mathematical phenomenology - to compare mathematical concepts according to some natural criterion (expressive power, class of models, consistency strength and so on). Here we will focus on the classification of two sorts of probability logics: $LPP_{2,P,Q,O}$ logics introduced in [12] and $LPP_2^{\mathrm{Fr}(n)}$ logics introduced in [3, 13, 17, 20, 24] (L for logic, the first P for propositional, and the second P for probability). Independently, several authors in [4, 6] have developed the fuzzy logics $FP(Ł_n)$ that extend Łukasiewicz logic. The $LPP_2^{\mathrm{Fr}(n)}$ logics can be embedded into those logics. For the $LPP_{2,P,Q,O}$ logics we introduce the comparison criterion with respect to the classes of models, while the $LPP_2^{\mathrm{Fr}(n)}$ logics we compare in terms of the interpretation method. We show that both criteria can be joined in a single one. Thus we have obtained the hierarchy of probability logics where the lattice of $LPP_{2,P,Q,O}$ logics is the end extension of the lattice of $LPP_2^{\mathrm{Fr}(n)}$ logics.

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