

# The Rationality of Escalation an unexpected use of Coinduction in Economics

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ENS de Lyon

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## 1 Escalation

- In 2014
- In 1720
- In 1971

## 2 Sequential games

- Sequential games (intuitive presentation)
- Infinite games
- Finite sequential games in COQ
- Infinite sequential games in COQ

## 3 Examples of games with escalation

- The 0, 1 game
- “Illogic” conflict of escalation revisited

## 4 Escalation and cognitive psychology

## 5 Conclusion

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# High-frequency trading

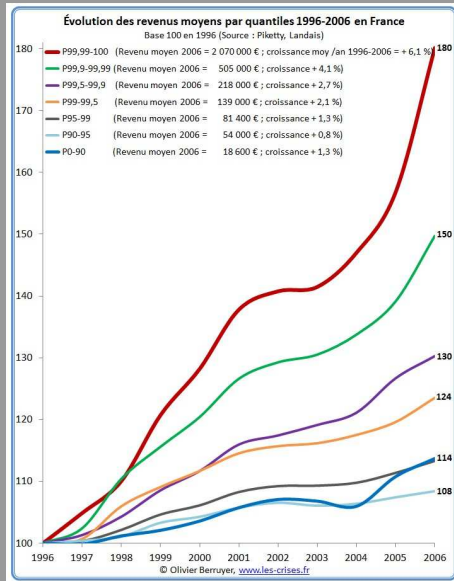
*High-frequency trading (HFT) is a type of algorithmic trading, specifically the use of sophisticated technological tools and computer algorithms to rapidly trade securities.*

*HFT uses proprietary trading strategies carried out by computers to move in and out of positions in seconds or fractions of a second.*

Wikipedia (2014)



# Evolution of inequalities



# South Sea Bubble

*I can calculate the movement of the stars,  
but not the madness of men.*

claimed to be Newton's view  
on the outcome of  
the South Sea Bubble (1720).

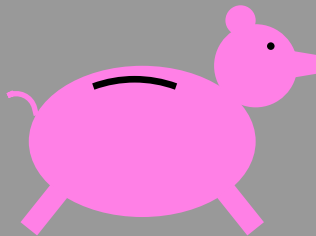
# The Dollar Auction

In 1971, in a paper called  
The Dollar Auction game:  
A paradox in noncooperative behavior and escalation  
Martin Shubik described an infinite game.



# The Dollar Auction (*the story revisited*)

For charity, an object is sold on an auction made a special way.  
There is a piggy bank (or a hat).



To bid, each person puts one euro in the piggy bank which is never returned to him.

# The Dollar Auction: a game with costs

Assume that there are two bidders (*Alice* and *Bob*) and that the value of the object is  $v$  €.

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- the bidder who has the object has a cost of  $n$  and
- the bidder who does not get the object has a cost of  $v + n$ .

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- It should be studied using tools designed for infiniteness.  
namely **coinduction**.

# Is escalation in the Dollar Auction rational?

- Escalation is irrational.

*Once two bids have been obtained from the crowd, the **paradox** of escalation is real [...] A total of payments between three and five dollars is not uncommon* Shubik (1971), p .110.



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**Theorem:** *The dollar auction game has an escalation.*

There is no paradox.

# Why this discrepancy?

- For Osborne et al. the resources are finite.

*Each person's wealth is  $w$ , which exceeds  $v$ ; neither player may bid more than her wealth.*

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# A problem with Shubik's analysis

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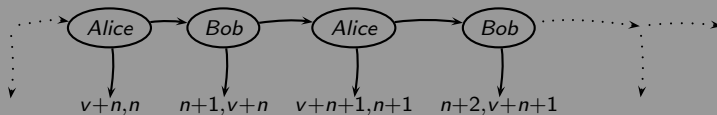
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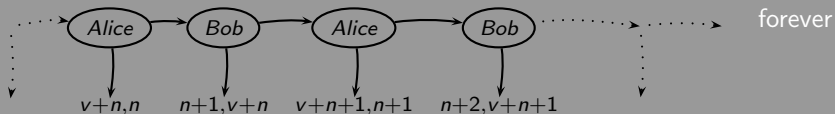
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# The Dollar Auction pictured



The dollar auction

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The dollar auction

We will focus on a simpler game (1 and 0, are costs)



The 0, 1 game



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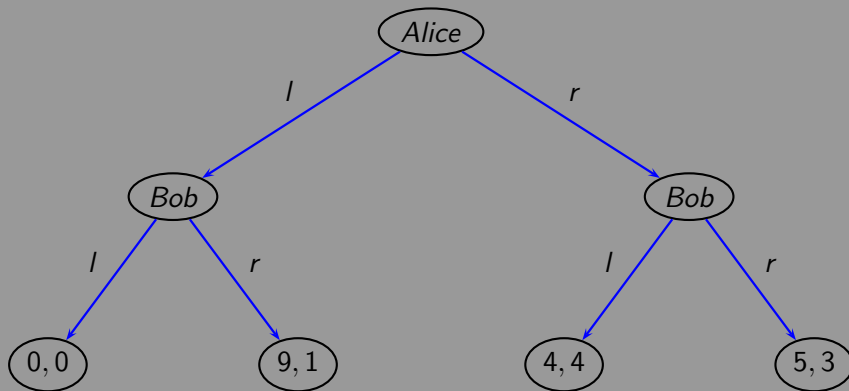
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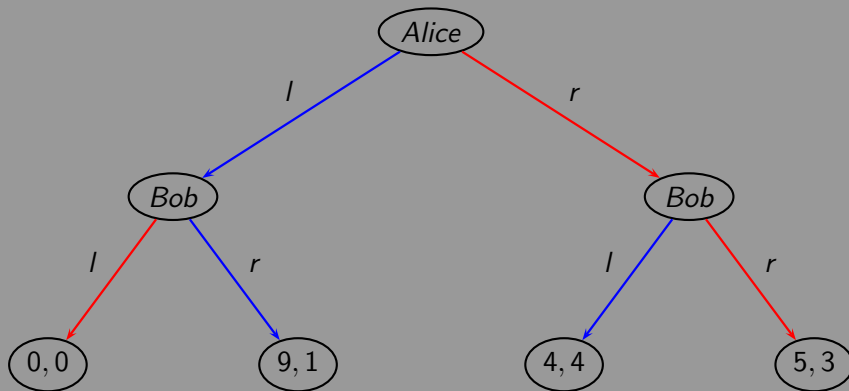
# What is a *sequential* game?

A **sequential game** is described by a labeled tree



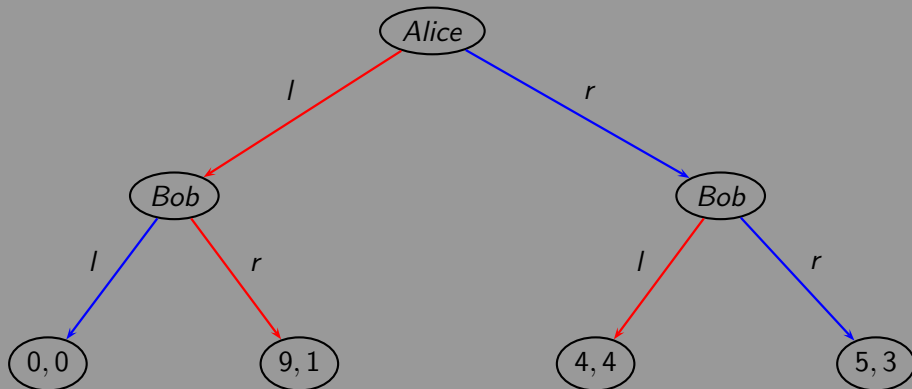
# What is a *strategy profile* ?

A **strategy profile** is described by a labeled tree plus choices



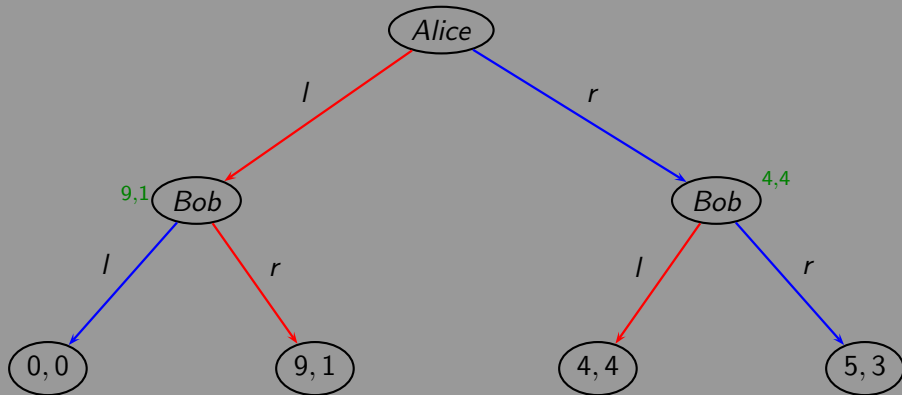
# What is a *Nash equilibrium*?

A **Nash equilibrium** is a strategy profile where if an agent changes alone his action he will get a utility which is not better.



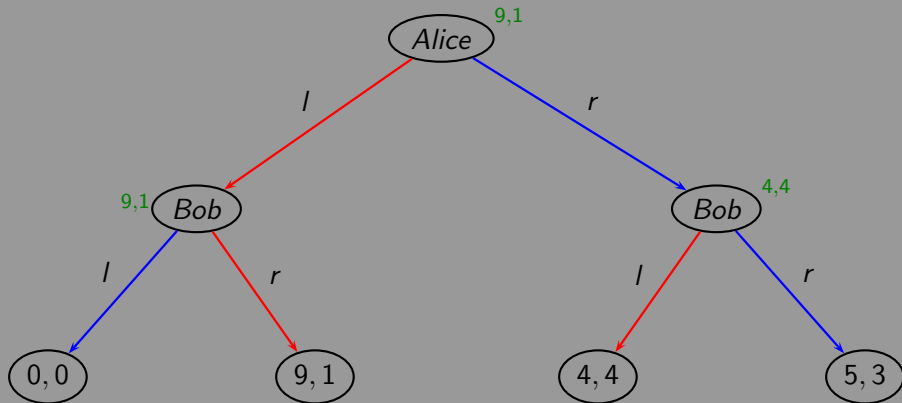
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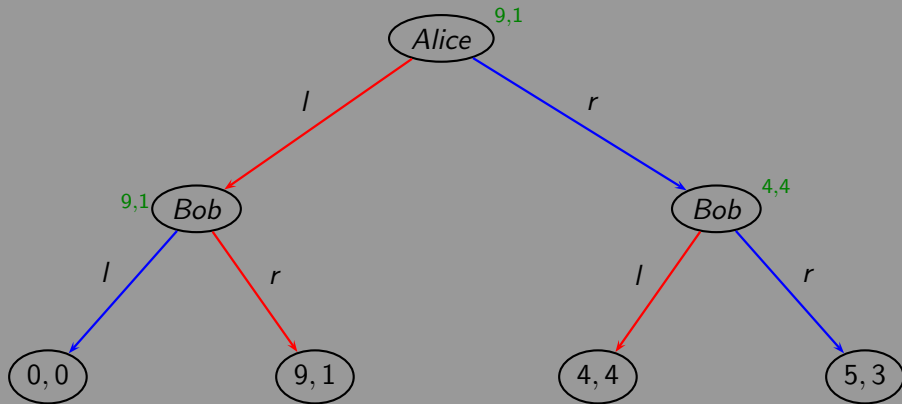
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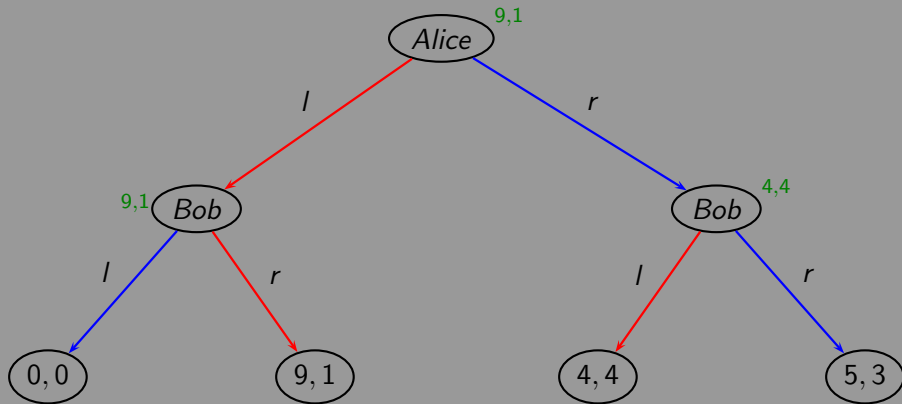
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This way of computing this Nash equilibrium is called **backward induction**.

# What is a *Nash equilibrium*?

A **Nash equilibrium** is a strategy profile where if an agent changes alone his action he will get a utility which is not better.



This Nash equilibrium is also called a **backward induction equilibrium**.



We are interested in infinite games,

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and to the extension of backward induction to infinite games.

# Finite sequential games as inductive objects

A *finite sequential games* is described *by induction* from its subgames.

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Without loss of generality, we restrict to **binary sequential games**.

A **binary finite sequential game** is

- either a **node**, assigned to a **player**, with **two subgames**,
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```
Inductive FinGame : Set :=  
| gLeaf : Utility_fun → FinGame  
| gNode : Agent → FinGame → FinGame → FinGame.
```

# Utility and utility functions

*Utility\_fun* is a function which associates a utility (a cost or a payoff) with an agent:

**Definition** `Utility_fun` := Agent  $\rightarrow$  Utility.

# A finite strategy profile is also an inductive

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Inductive FinStratProf : Set :=  
| sLeaf : Utility_fun → FinStratProf  
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A strategy profil is written  $\ll f \gg$  and  $\ll a, c, sl, sr \gg$ .



# From finite strategy profile to utility function

```
Fixpoint f2u (s:FinStratProf) : Utility_fun :=  
match s with  
| <<uf>> => uf  
| <<a, l, sl, sr>> => (f2u sl)  
| <<a, r, sl, sr>> => (f2u sr)  
end.
```

# Backward induction

On finite strategy profiles.

**Inductive** BI: FinStratProf  $\rightarrow$  Prop :=

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BI sl  $\rightarrow$  BI sr  $\rightarrow$  (f2u sr a  $\preceq$  f2u sl a)  $\rightarrow$  BI ( $\ll$ a left sl sr  $\gg$ )

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# Coinductive *Games*

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The concept of infinite strategy profile is also defined as a coinductive:

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# From infinite strategy profiles to utility function

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It returns a value only on strategies which go eventually to a leaf.

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Roughly speaking,

*following the path given by the strategy profile one is lead to a leaf.*

# Existence and uniqueness

On strategies that leads to a leaf, one gets  
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**Lemma** *Existence\_i2u*:  $\forall (a:Agent) (s:StratProf),$   
 $LeadsToLeaf\ s \rightarrow \exists u:Utility, i2u\ a\ u\ s.$

**Lemma** *Uniqueness\_i2u*:  $\forall (a:Agent) (u\ v:Utility) (s:StratProf),$   
 $LeadsToLeaf\ s \rightarrow i2u\ a\ u\ s \rightarrow i2u\ a\ v\ s \rightarrow u=v.$

# The predicate *Always leads to leaf*

A strategy profile always leads to leaf,  
if all strategy sub profiles lead to a leaf.

Also called *strongly convergent* and written  $\Downarrow$

Proposition

$$\Downarrow s = \Box(\downarrow s)$$

# SubGame Perfect Equilibria

**CoInductive** *SGPE*:  $\text{StratProf} \rightarrow \text{Prop} :=$

| *SGPE\_leaf*:  $\forall f:\text{Utility\_fun}, \text{SGPE } (\ll f \gg)$



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 $s2u\ sl\ a\ u \rightarrow s2u\ sr\ a\ v \rightarrow (v \preceq u) \rightarrow$   
 $\text{SGPE } (\ll a,l,sl,sr \gg)$

| *SGPE\_right*:  $\forall (a:\text{Agent}) (u\ v:\text{Utility}) (sl\ sr:\text{StratProf}),$   
 $\text{AlwLeadsToLeaf } (\ll a,r,sl,sr \gg) \rightarrow$   
 $\text{SGPE } sl \rightarrow \text{SGPE } sr \rightarrow$   
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## 1 Escalation

- In 2014
- In 1720
- In 1971

## 2 Sequential games

- Sequential games (intuitive presentation)
- Infinite games
- Finite sequential games in COQ
- Infinite sequential games in COQ

## 3 Examples of games with escalation

- The 0, 1 game
- “Illogic” conflict of escalation revisited

## 4 Escalation and cognitive psychology

## 5 Conclusion

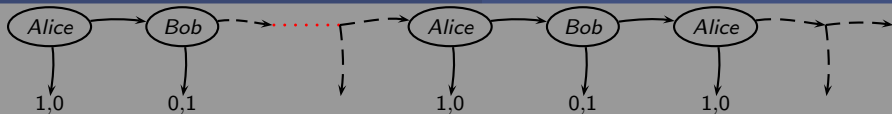


The 0, 1 game



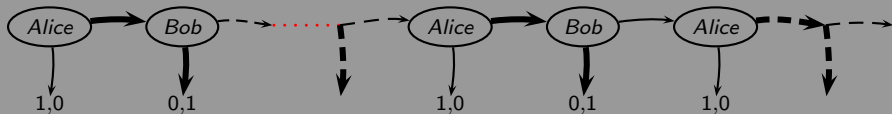
The 0, 1 game

0 and 1 are costs



The 0, 1 game

0 and 1 are costs



The strategy profile **Alice continues, Bob stops**



The 0, 1 game

0 and 1 are costs

The strategy profile **Alice continues, Bob stops**The strategy profile **Alice stops, Bob continues**

# Two SGPE's

## Theorem

*The strategy profile **Alice continues, Bob stops** is a SGPE.*

# Two SGPE's

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# The escalation

At each step Alice is rational if she continues.



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As a sequence of rational decisions, escalation is rational in 0, 1 game.

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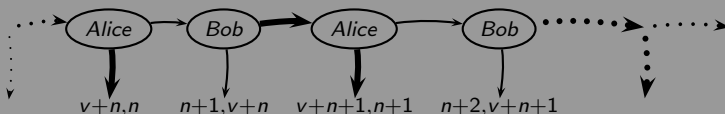
## 5 Conclusion

# The *Dollar Auction* revisited



# Alice abandons

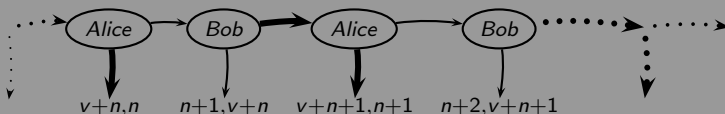
In Shubik's game, we can prove that the strategy  
Alice abandons and Bob continues



is a **SubGame Perfect equilibrium**.

# Alice abandons

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Alice abandons and Bob continues

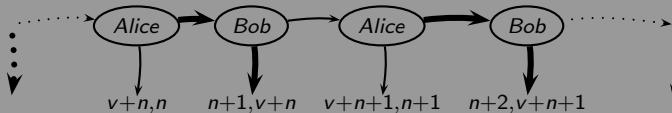


is a **SubGame Perfect equilibrium**.

Alice takes Bob's threat as credible and considers it is better to give up.

# Bob gives up

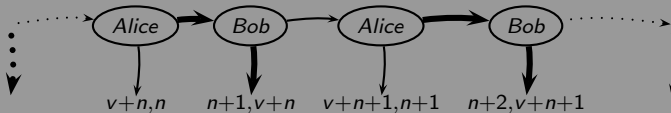
The strategy Alice continues and Bob gives up



is a **SubGame Perfect Equilibrium**.

# Bob gives up

The strategy Alice continues and Bob gives up



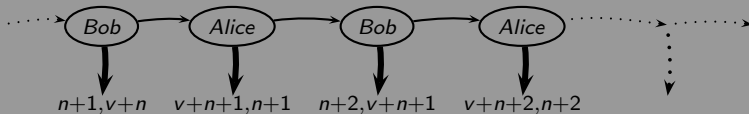
is a **SubGame Perfect Equilibrium**.

Bob takes Alice's threat as credible.



# Always give up

The strategy always give up



is a **not a SubGame Perfect Equilibrium** and therefore **not a Nash equilibrium**.

# Escalation in the Dollar Auction

At each turn if the agent continues she (he) is rational.

# Escalation in the Dollar Auction

At each turn if the agent continues she (he) is rational.

Escalation is rational in the Dollar Auction game.

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# A book

A recent book addresses the new trends on rational thought.

K.E. Stanovich.

*What Intelligence Tests Miss: The Psychology of Rational Thought.*

Yale University Press, 2010.

# Two levels of rationality

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# Conclusion

- Reasoning on infinite sequential games is **subtle**, however necessary.
- **Escalation is rational** if the agents **believe** in a world of **infinite resources**.
- Coalgebras and coinduction are the right tools for **rethinking of economics**.