Display-type calculi

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joint work with
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Informal presentation of Display Calculi

Display calculi: variation of sequent calculi

Sequent calculi:

- **One structural symbol ;**

$$\frac{X \vdash A \quad Y \vdash B}{X \,;\, Y \vdash A \wedge B} \qquad \frac{A \vdash X \quad B \vdash Y}{A \vee B \vdash A \,;\, B}$$

- **Cut Rule:**

$$\frac{X \vdash A \qquad A \vdash Y}{X \vdash Y}$$

Display calculi: One structural symbol for each logical connective.

Informal presentation of Display Calculi

Why: modularity and cut elimination meta theorem

How do they work? 1 property = 1 rule

Some rules

$$\frac{X \vdash A \quad Y \vdash B}{X ; Y \vdash A \wedge B} \qquad \frac{X \vdash A}{\circ X \vdash \Diamond A} \qquad \frac{\circ X \vdash Y}{X \vdash \bullet Y}\text{display}$$

; structural symbols $\longrightarrow$ to manipulate structures
$\wedge$ operational symbols $\longrightarrow$ formulas such as $\Diamond A$ are "frozen".

Main feature: display property

display property $\Longrightarrow$ cut elimination meta theorem

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Display postulate

Structural symbols	$>$		;	
Operational symbols	$\succ\!\!-$	$\rightarrow$	$\wedge$	$\vee$

Display Postulates

$$(\,;<)\ \frac{X\,;Y \vdash Z}{\overline{\quad X \vdash Z < Y\quad}} \qquad (\,;>)\ \frac{X\,;Y \vdash Z}{\overline{\quad Y \vdash X > Z\quad}}$$

Adjunction

$$A \wedge B \leq C \quad \text{iff} \quad B \leq A \rightarrow C \quad \text{iff} \quad A \leq C \leftarrow B$$

Modularity: Structural rules for propositional base

Intuitionistic base

$$Id\ \frac{}{p \vdash p} \qquad \frac{X \vdash A \quad A \vdash Y}{X \vdash Y}\ Cut$$

$$\mathrm{I}_L^1\ \frac{X \vdash Y}{\mathrm{I} \vdash Y < X} \qquad \frac{X \vdash Y}{X < Y \vdash \mathrm{I}}\ \mathrm{I}_R^1 \qquad \mathrm{I}_L^2\ \frac{X \vdash Y}{\mathrm{I} \vdash X > Y} \qquad \frac{X \vdash Y}{Y > X \vdash \mathrm{I}}\ \mathrm{I}_R^2$$

$$\mathrm{I}W_L\ \frac{\mathrm{I} \vdash X}{Y \vdash X} \qquad \frac{X \vdash \mathrm{I}}{X \vdash Y}\ \mathrm{I}W_R \qquad C_L\ \frac{X ; X \vdash Y}{X \vdash Y} \qquad \frac{Y \vdash X ; X}{Y \vdash X}\ C_R$$

$$W_L^1\ \frac{X \vdash Z}{Y \vdash Z < X} \qquad \frac{X \vdash Z}{X < Z \vdash Y}\ W_R^1 \qquad W_L^2\ \frac{X \vdash Z}{Y \vdash X > Z} \qquad \frac{X \vdash Z}{Z > X \vdash Y}\ W_R^2$$

$$E_L\ \frac{Y ; X \vdash Z}{X ; Y \vdash Z} \qquad \frac{Z \vdash X ; Y}{Z \vdash Y ; X}\ E_R \qquad A_L\ \frac{X ; (Y ; Z) \vdash W}{(X ; Y) ; Z \vdash W} \qquad \frac{W \vdash (Z ; Y) ; X}{W \vdash Z ; (Y ; X)}\ A_R$$

Classical base

$$Gri_L\ \frac{X > (Y ; Z) \vdash W}{(X > Y) ; Z \vdash W} \qquad \frac{W \vdash X > (Y ; Z)}{W \vdash (X > Y) ; Z}\ Gri_R$$

Cut elimination: Proof by induction

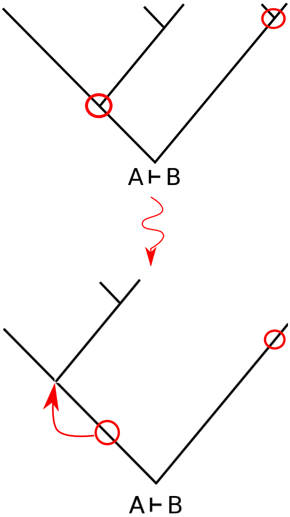
Complexity of the cut formula

$$\dfrac{\dfrac{\begin{array}{c}\vdots\ \pi_1\\ Z \vdash \circ A\end{array}}{Z \vdash \Box A} \qquad \dfrac{\begin{array}{c}\vdots\ \pi_2\\ A \vdash Y\end{array}}{\Box A \vdash \circ Y}}{Z \vdash \circ Y}\ Cut$$

$$\Downarrow$$

$$\text{Display}\ \dfrac{\text{Display}\ \dfrac{\begin{array}{c}\vdots\ \pi_1\\ Z \vdash \circ A\end{array}}{\bullet Z \vdash A} \qquad \begin{array}{c}\vdots\ \pi_2\\ A \vdash Y\end{array}}{\dfrac{\bullet Z \vdash Y}{Z \vdash \circ Y}}\ Cut$$

Height of the cut



Applied logics

PDL, DEL, Monotone Modal Logic, Game Logic, ...

'bad' logics for proof theory:

- not closed under uniform substitution,
- dynamic interactions: difficult to handle,
- non-normal modal logics: no adjunction,
- iteration operators call for ω-rules,

$\Longrightarrow$ **some display rules might be NOT sound**

Question: Can we guarantee full display property then?

Conjecture: YES, under some conditions. (work in progress)

But what if conjecture is false?

Let's discuss alternative strategies.

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EAK: Multi-type

EAK syntax: $p \in \mathsf{Prop}$, $a \in \mathsf{Agent}$, $\alpha \in \mathsf{Action}$,

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \underbrace{\Diamond_a \varphi \mid \Diamond_\alpha \varphi}_{\textbf{unary}}$$

multi-type syntax:

$$\text{Formulas} \quad \varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \underbrace{\langle a \rangle \varphi \mid \langle \alpha \rangle \varphi}_{\textbf{binary}}$$

$$\text{Actions} \quad \alpha ::= \alpha \mid \langle a, \alpha \rangle$$

$$\text{Agents} \quad a ::= a$$

$\langle a, \alpha \rangle$: new modality, induistinguishability between actions
$\Longrightarrow$ Increase expressivity

EAK: Multi-type

Interaction Axiom:

$$\langle\alpha\rangle\langle\mathtt{a}\rangle A \rightarrow \mathtt{Pre}(\alpha) \wedge \bigvee\{\langle\mathtt{a}\rangle\langle\beta\rangle A \mid \alpha\mathtt{a}\beta\}$$

Rule without multi-type:

$$\frac{\Big(\{\mathtt{a}\}\{\beta\}\, X \vdash Y \mid \alpha\mathtt{a}\beta\Big)}{\{\alpha\}\{\mathtt{a}\}X \vdash \mathbf{;}\Big(Y \mid \alpha\mathtt{a}\beta\Big)}$$

Rule without multi-type:

$$\frac{X \vdash (\mathtt{a}\blacktriangle\alpha) \blacktriangleright (\mathtt{a} \blacktriangleright Y)}{X \vdash \mathtt{a} \blacktriangleright (\alpha \blacktriangleright Y)}$$

EAK: Multi-type

Formulas $\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \langle a \rangle \varphi \mid \langle \alpha \rangle \varphi$

Actions $\alpha ::= \alpha \mid \langle a, \alpha \rangle$

Agents $a ::= a$

Display property:

- dynamic modalities:

$$\mathbb{M}, w \Vdash \langle \alpha \rangle \varphi \quad \text{iff} \quad \mathbb{M}, w \Vdash Pre(\alpha) \quad \text{and} \quad \mathbb{M}_\alpha, w_\alpha \Vdash \varphi$$

$\Longrightarrow$ final coalgebra semantics

- Fact: virtual adjoints for $\langle a, \alpha \rangle$ are safe
  $\rightsquigarrow$ Very ad hoc proof
  $\rightsquigarrow$ **no** known **uniform method** to guarantee safeness

Monotone Modal Logic: Visibility

MML syntax: $\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \langle\exists\forall\rangle\varphi$

Problem: $\langle\exists\forall\rangle$ does not distribute over $\vee$ or $\wedge$
$\Longrightarrow$ NO adjunction $\Longrightarrow$ NO display property.

Solution: Surgical cut + Visibility

Visibility. If A is active in a rule, then A is in isolation.

$$Cut_L \frac{Z \vdash A \qquad (X \vdash Y)[A]^{pre}}{(X \vdash Y)[Z]^{pre}} \qquad \frac{(X \vdash Y)[A]^{suc} \qquad A \vdash Z}{(X \vdash Y)[Z]^{suc}} Cut_R$$

Conclusions & Future works

To summarise

- ✓ Multi-type
- ✓ Weak display + strong visibility
- ✓ Virtual adjoints with adhoc proof

Further research:

- Linear Logic: avoiding *closed-enough rules*
- PDL: avoiding *omega-rule*
- PDL + MML $\longrightarrow$ Game Logic

$\rightarrow$ Find uniform proof of safeness of virtual adjoints.

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