ON PROBABLE CONDITIONALS

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Conditional statements are not statements of causality.

The statements of causality require the cause to precede the effect in time, whereas conditional statements do not require temporal order.

Confusion is strengthened in natural languages.

They use the same “if … then…” form for causality and conditional.

We are interested in probable conditionals:

$pr(A\rightarrow B)$ „probability of , B if A"

$pr(A|B)$ „probability of B , if A"

LEWIS:

If $pr(A \rightarrow B)$ is the same as $pr(B|A)$ then

$$pr(B|A) = pr(A \rightarrow B) = pr(A \rightarrow B|B)\, pr(B) + pr(A \rightarrow B|-B)\, pr(-B) =$$

$$pr(B \rightarrow (A \rightarrow B))\, pr(B) + pr(-B \rightarrow (A \rightarrow B))\, pr(-B) =$$

$$pr(AB \rightarrow B)\, pr(B) + pr(A(-B) \rightarrow B)\, pr(-B) =$$

$$1pr(B) + 0pr(-B) = pr(B)$$

Standard probability:

$pr(A \rightarrow B) = pr(-A \vee B) = pr(-A \vee AB) = pr(-A) + pr(AB) = pr(-A) + pr(A)\ pr(B|A)$

$pr(A \rightarrow B) = pr(-A) + pr(A)\ pr(B|A) = 1 - x + xc \geq c = pr(B|A)$

It is equal only if:

$pr(A) = x = 1$ i.e. $pr(A \rightarrow B) = pr(B|A) = pr(B)$ or

$pr(B|A) = c = 1$ i.e. If $pr(A \rightarrow B) = pr(B|A) = 1$

More elementary:

S = If “it is even on the dye” then “it is two on the dye” = $E \rightarrow T$

$pr(-(E \rightarrow T)) = pr(\text{even \& not two}) = 1/3$

i.e. $pr(E \rightarrow T) = 2/3$ but $pr(T|E) = 1/3$

Define $A{\uparrow}B$, which means "A makes B more probable"

i.e. "A supports B", as $pr(B/A)>pr(B)$.

Define $A{\downarrow}B$, which means "A makes B less probable",

i.e. "A subverts B", as $pr(B/A)<pr(B)$.

The independence relation $A \perp B$, is defined as $pr(B|A)=pr(B)$.

For lot of people it is tempting to transfer the properties of conditionals to properties of supports.

This is quite a common error.

(Even if support $\uparrow$ is a kind of probable conditional it does not have properties of conditional $\rightarrow$.)

People think:

“if A supports B and B supports C, then A supports C”.

(They think of “supports” as transitive.)

Confronted with a concrete counterexample:

A=“having white hair”,
B=“being over 50”,
C=“being completely bald”,

they change their minds.

People think:

“If A supports C and B supports C,
then their conjunction does so even more”.

Confronted with a concrete counterexample:

A crime is committed by two men,
one in a red jacket another in a black coat.

A=“first witness recognized the suspect as the man in the red jacket“,
B=“second witness recognized him as the man in the black coat”
C=“the suspect is guilty“,

they change their minds.

There are other concrete counterexamples in Carnap's *Logical Foundations of Probability* (chapter 6.).

It seems that people do not err in concrete settings and do err in abstract settings.

This is the reason that an abstract analysis of supports $\uparrow$ (compared to conditionals $\rightarrow$) could be of some interest.

The basic properties of conditionals are:

(1) $A \to B, B \to C \Rightarrow A \to C$	is valid	(transitivity)
(2) $A \to B \Rightarrow B \to A$	is not valid	(converse fallacy)
(3) $A \to B \Rightarrow -B \to -A$	is valid	(contraposition)
(4) $A \to B \Rightarrow -A \to -B$	is not valid	(inverse fallacy)

(5) $C \rightarrow A,\ C \rightarrow B \Rightarrow C \rightarrow A \& B$ is valid (conjunction intr)

(6) $C \rightarrow A,\ C \rightarrow B \Rightarrow C \rightarrow A \vee B$ is valid (disjunction intr)

(7) $A \rightarrow C,\ (B \rightarrow C) \Rightarrow A \& B \rightarrow C$ is valid (conjunction elim)

(8) $A \rightarrow C,\ (B \rightarrow C) \Rightarrow A \vee B \rightarrow C$ is valid (disjunction elim)

The corresponding properties of $A \uparrow B$, except (3), are exactly the opposite.

(i) Property (2) is valid for $\perp$, $\uparrow$ and $\downarrow$ (these relations are symmetrical).

(ii) Property (4) is valid for $\perp$, $\uparrow$ and $\downarrow$.

(iii) Property (1) is not valid for $\perp$, $\uparrow$ and $\downarrow$ (these relations are not transitive).

(iv) Properties (5), (6), (7) and (8) are not valid for $\perp$, $\uparrow$ and $\downarrow$.

(i) The symmetry of $\uparrow$ and $\downarrow$ follows from:

$$pr(A|B)pr(B)=pr(B|A)pr(A).$$

If $pr(A|B)>pr(A)$ and $pr(B|A)\leq pr(B)$ then

$pr(A|B)pr(B)>pr(A)pr(B)\geq pr(B|A)pr(A)$, contradiction.

(ii) It is easy to prove that:

$$A{\uparrow}B \quad \text{iff} \quad A{\downarrow}\text{-}B$$

$$A{\downarrow}B \quad \text{iff} \quad A{\uparrow}\text{-}B,$$

and (4) then follows by symmetry of $\uparrow$ and $\downarrow$.

For $(\perp)\uparrow$ we have:

$A(\perp)\uparrow B \quad \Leftrightarrow \quad pr(A|B)(=)>pr(A)$

$\Leftrightarrow \quad pr(-A|B)=1–pr(A|B)(=)<1–pr(A)=pr(-A)$

$\Leftrightarrow \quad -A(\perp)\downarrow B.$

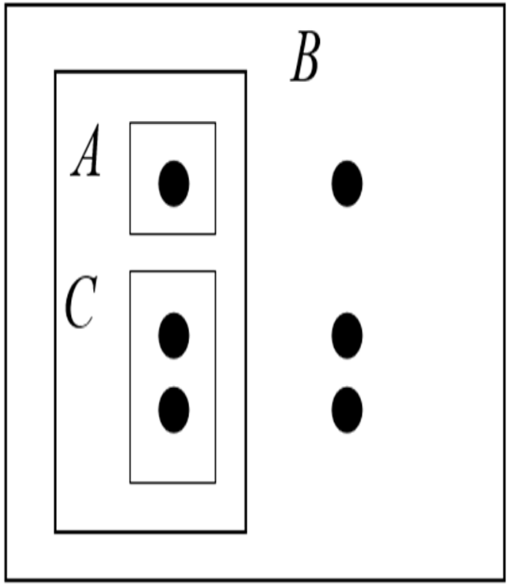
(iii) If $\perp$ and $\downarrow$ were transitive then:

$$A{\downarrow}B \text{ iff } B{\downarrow}A \text{ which implies } A{\downarrow}A,$$

$$A{\perp}B \text{ iff } B{\perp}A \text{ which implies } A{\perp}A,$$

but $A{\downarrow}A$ and $A{\perp}A$ are both false.

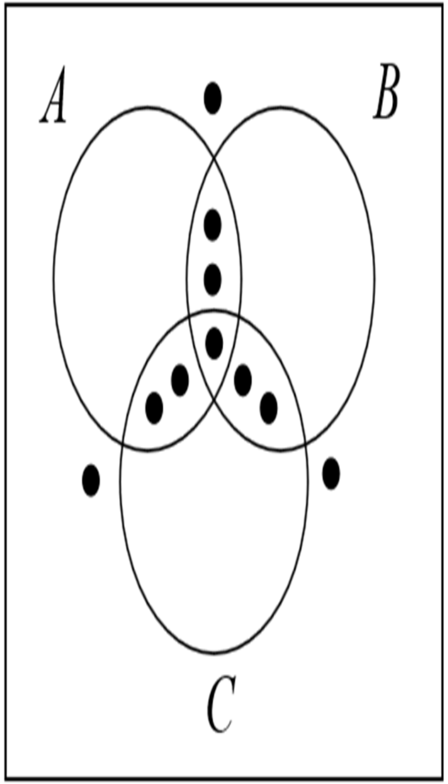
This argument does not work for $\uparrow$, because $A{\uparrow}A$ is true.



pr(B|A)=1 1/2=pr(B) i.e. A↑B

pr(C|B)=2/3 1/3=pr(C) i.e. B↑C

pr(C|A)=0 1/3=pr(C) i.e. A↓C

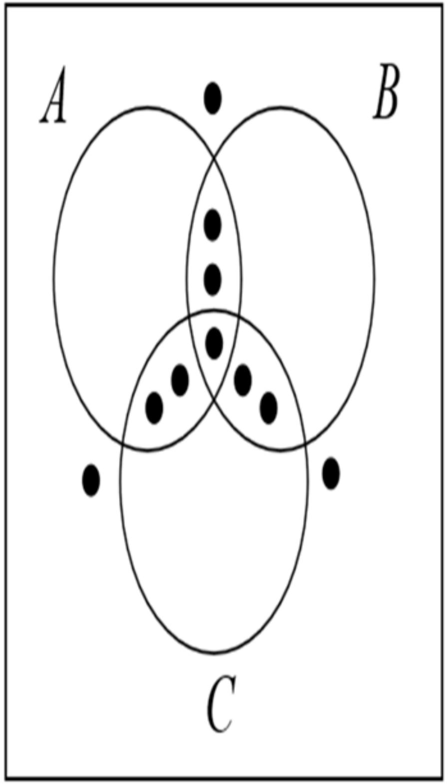


pr(A|C)=3/5 1/2=pr(A) i.e. C↑A

pr(B|C)=3/5 1/2=pr(B) i.e. C↑B

pr(A&B|C)=1/5 3/10=pr(A&B) i.e. C↓(A&B)

This proves (5↑).



$pr(C|A)=3/5$ $1/2=pr(C)$ i.e. $A\uparrow C$

$pr(C|B)=3/5$ $1/2=pr(C)$ i.e. $B\uparrow C$

$pr(C|A\&B)=1/3$ $1/2=pr(A\&B)$ i.e. $(A\&B)\downarrow C$

This proves $(7\downarrow)$.

From (5$\uparrow$), it follows that:

$-C\uparrow -A, -C\uparrow -B \Rightarrow -C\uparrow(-A\&-B)$ is not valid.

Then it follows from(ii) that:

$C\uparrow A, C\uparrow B \Rightarrow C\uparrow -(-A\&-B)$ is not valid.

Hence: $C\uparrow A, C\uparrow B \Rightarrow C\uparrow(A\vee B)$ is not valid.

This proves (6$\uparrow$).

(8) follows (analogously) from (7).