VJEROJATNOST KAO LOGIKA

(TEORIJA VJEROJATNOSTI JEST LOGIKA)

ZVONIMIR ŠIKIĆ

VERITAS NA $L(\wedge, \vee, -)$:

$$\boxed{\begin{aligned} &(V1)\ V(-A) = 1 - V(A) \\ &(V2)\ V(A \vee B) = \max\{V(A), V(B)\} \\ &(V3)\ V(AB) = \min\{V(A), V(B)\} \end{aligned}}$$

$$V : L \to \{0, 1\}$$

PROBABILITAS MORA BITI PROŠIRENE!!

PROBABILITAS NA $\mathcal{L}$:

$$\boxed{\begin{array}{l}(P1)\ \models A \Rightarrow P(A)=1 \\ (P2)\ A,B\models\ \ \rightarrow P(A\vee B)=P(A)+P(B)\end{array}}$$

$$P:\mathcal{L}\to[0,1]$$

$(P1')$ $P(A)+P(-A)=1$

Dz: $1 \overset{(1)}{=} P(A\vee\bar{A})$
$\overset{(2)}{=} P(A)+P(\bar{A})$

$(P3)$ $A\models B \Rightarrow P(A)\leq P(B)$

Dz: $A\models B \Rightarrow A,\bar{B}\models \longrightarrow$
$\cancel{1}\geq P(A\vee\bar{B}) \underset{(1')}{\overset{(2)}{=}} P(A)+\cancel{1}-P(B)$

$(P3')$ $A\equiv B \Rightarrow P(A)=P(B)$

OČITO ✓

$(P1)$ & $(P2)$ $\Longleftrightarrow$

$$\boxed{\begin{array}{l} (P1')\ P(-A) = 1 - P(A) \\ (P2)\ P(A \vee B) = P(A) + P(B) \\ \qquad \text{ZA } A, B \models \end{array}}$$

__Dz:__ $(P1)\&(P2) \Rightarrow (P1')$ ✓

$(P1')\&(P2) \Rightarrow (P1)$:

$$\models A \implies A \equiv A \vee \bar{A}$$

$$\implies P(A) \overset{(3')}{=} P(A \vee \bar{A}) \overset{(2)}{=} P(A) + P(\bar{A}) \overset{(1')}{=} 1 \ ✓$$

GDJE JE KONJUNKCIJA $\wedge$

???

$R(A) \equiv P(\bar{A})$

"RIZIK OD A"

$$\boxed{(P2') \quad R(AB) = R(A) + R(B) \quad \text{ZA} \models A, B}$$

Dz: $R(AB) = P(\bar{A} \vee \bar{B})$

$= P(\bar{A}) + P(\bar{B}) = R(A) + R(B)$

$$\boxed{A \models B \implies R(A) \geq R(B)}$$ P3

ILI OPĆENITO:

$$\boxed{A_1 \ldots A_m \models B \implies \sum R(A_i) \geq R(B)}$$

Dz: INDUKCIJOM

$$A_1 \dots A_{n-1} \models A \vee \bar{A}_n \overset{p.i.}{\Rightarrow}$$

$$\sum_{1}^{n-1} R(A_i) \geq R(A \vee \bar{A}_n) =$$

$$= 1 - P(A \vee \bar{A}_n) \overset{(2'')}{\geq} 1 - (P(A) + P(\bar{A}_n))$$

$$= (1 - P(A)) - (1 - P(A_n))$$

$$= R(A) - R(A_n) \Rightarrow$$

$$\sum_{1}^{n} R(A_i) \geq R(A)$$

$$\boxed{(P2'')\ P(AB)+P(A\vee B)=P(A)+P(B)}$$

Dz:

$$\boxed{P(A\vee B)}=P(A\vee \bar{A}B)=$$

$$P(A)+P(\bar{A}B)\overset{*}{=}$$

$$\boxed{P(A)+P(B)-P(AB)}$$

$$*\ \boxed{P(B)}=P(AB\vee \bar{A}B)=$$

$$=\boxed{P(AB)+P(\bar{A}B)}$$

$$\Longrightarrow P(\bar{A}B)=P(B)-P(AB)$$

$\sqrt{a}$

P JE PROŠIRENJE OD V:

(P1') $V(A) + V(\bar{A}) = 1$

(P2'') $V(A \vee B) + V(AB) =$

$= \max\{V(A), V(B)\} +$

$+ \min\{V(A), V(B)\} =$

$= V(A) + V(B)$

SA $\{0, 1\}$ NA $[0, 1]$

SVAKI $P: L \to \{0,1\}$ JE V:

(V1) $P(-A) \underset{(1')}{=} 1 - P(A)$

(V2) $P(A \vee B) \underset{(3)}{\geq} \mathrm{MAX}\{P(A), P(B)\}$

$$P(A \vee B) \underset{(2'')}{\leq} P(A) + P(B)$$

P(A)	P(B)	≥ MAX	≤ +
1	1	1	0,1
1	0	1	0,1
0	1	1	0,1
0	0	0,1	0

(V3) $P(AB) = 1 - P(\bar{A} \vee \bar{B}) \underset{(V2)}{=}$

$1 - \mathrm{MAX}\{P(\bar{A}), P(\bar{B})\} =$

$1 - \mathrm{MAX}\{1 - P(A), 1 - P(B)\} =$

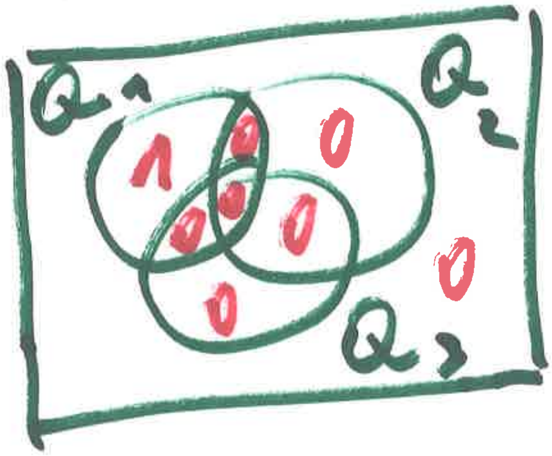
$1 - (1 - \mathrm{MIN}\{P(A), P(B)\}) =$

$\mathrm{MIN}\{P(A), P(B)\}$

(MOŽE I KAO V2)

V-MODEL

$$M:\{\underbrace{Q_1,\dots,Q_n}_{\text{At(omi)}}\}\to\{0,1\}$$



$\wedge$ JE ODABIR ČELIJE

NPR: $Q_1\,\overline{Q}_2\,\overline{Q}_3 = C_i$

$$\underbrace{1 \quad 0 \quad 0}_{\text{REDAK U IST. T.}}$$

ALTERNANT = ČELIJA = REDAK U IST. T.
(U POTP. ALT. NORM. FORMI)

ILI:

$$M:\{C_1, C_2, \dots, C_{2^n}\}\to\{0,1\}$$

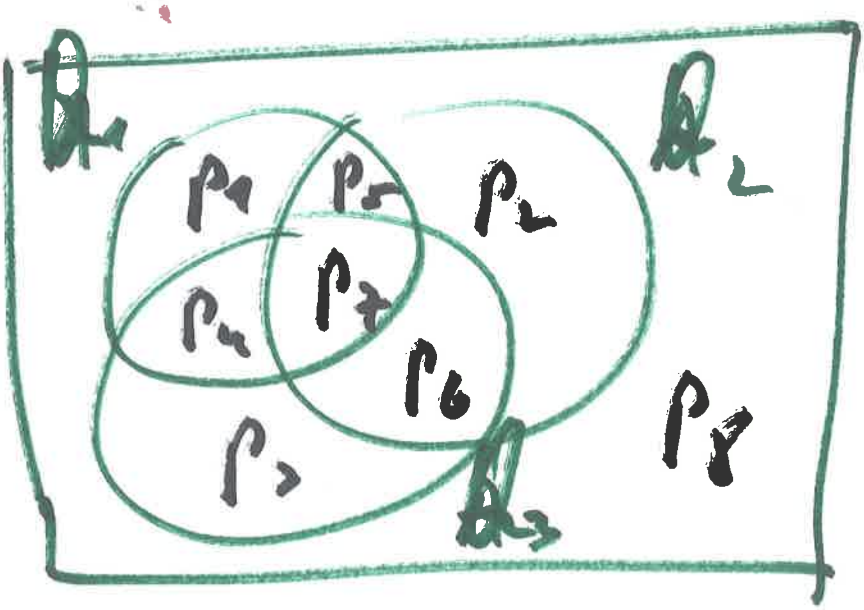
TAKVO DA JE $\sum_i M(C_i)=\sum_i c_i = 1$

(VIDI SLIKU)

P-MODEL

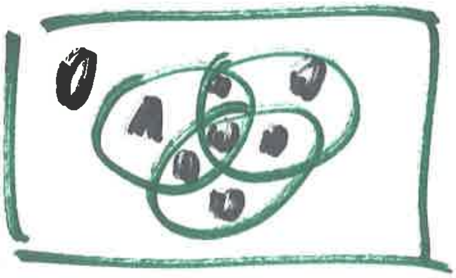
$$M : \{C_1, \dots, C_{2^n}\} \to [0,1]$$

takav da $\sum_i M(C_i) = \sum_i p_i = 1$



$$\sum p_i = 1$$

OČITO JE V-MODEL SPECIJALNI P-MODEL



$$(\forall A \in L_m)(A \equiv \bigvee_A C_i \lor A \equiv 0)$$

M SE PROŠIRUJE NA CIJELI JEZIK L_m NA SLJ. NAČIN:

$$P_M(A) := \sum_A M(C_i) = \sum_A C_i$$

(ILI 0 ZA $A \equiv 0$)

AKO JE M V-MODEL ONDA JE $P_M = V_M : L_m \to \{0,1\}$

INAČE $P_M : L_m \to [0,1]$

TO SU ISTINA I VJEROJATNOST U MODELU M !!!

"KONAČNI" KOLMOGOROV:

$$(\Omega, P : \Omega \to [0,1])$$

$$\begin{array}{c|c} \omega_1 & p_1 \\ \vdots & \vdots \\ \omega_m & p_m \end{array} \quad \text{t.d. } \sum_i p_i = 1$$

$$\Downarrow$$

$Q_i =$ "DESIO SE ω_i" tj.

$$\omega_i = \overline{Q_1} \cdots Q_i \cdots \overline{Q_m}$$

$M(\omega_i) = p_i$ INAČE $M = 0$

"KONAČNI" LOGIČKI

$$\Downarrow$$

$\omega_i = C_i$ t.d. $M(C_i) > 0$

$$p_i = M(C_i)$$

"KONAČNI" KOLMOGOROV

VERISTIČKA LOG. POSLJEDICA

$$A_1, \ldots, A_m \models B$$

DEFINIRANO SA

$$(\forall n)\left[(\forall i)\, V_n(A_i) = 1 \Rightarrow V_n(B) = 1\right]$$

$$(\forall n)\left[(\forall i)\, V_n(A_i) \in \alpha_i \Rightarrow V_n(B) \in \beta\right]$$

POOPĆENJE ZA $\alpha_i, \beta \subseteq \{0, 1\}$

($\alpha_i = \{0,1\}$ NEPOTR. PRETPOSTAVKA
$\beta = \{0,1\}$ VALJANA IMPL. (TRIV.) Ξ
$\alpha_i, \beta = 0 \Rightarrow -A_i, -B$ $\alpha_i \beta = 1$)

ZBOG $M \Leftrightarrow V_M$

$$(\forall V)\left[(\forall i)\, V(A_i) \in \alpha_i \rightarrow V(B) \in \beta\right]$$

PROBABIL. LOGIČKA POSLJEDICA

$$P(A_1) \in \alpha_1, \ldots, P(A_m) \in \alpha_m \models P(B) \in \beta$$

$$\emptyset \neq \alpha_i, \beta \subseteq [0, 1]$$

DEFINIRANO SA:

$$(\forall M)\Big((\forall i) P_M(A_i) \in \alpha_i \rightarrow P_M(B) \in \beta\Big)$$

ili zbog $M \leftrightarrow P_M$

$$(\forall P)\Big((\forall i) P(A_i) \in \alpha_i \rightarrow P(B) \in \beta\Big)$$

VERISTIČKA L. POSLJ. JE SPEC. SLUČAJ U KOJEM JE $M: L_m \rightarrow \{0, 1\}$ I $\alpha_i, \beta \in \{0, 1\}$ I TADA JE $P_M = V_M$ t. v.

NPR

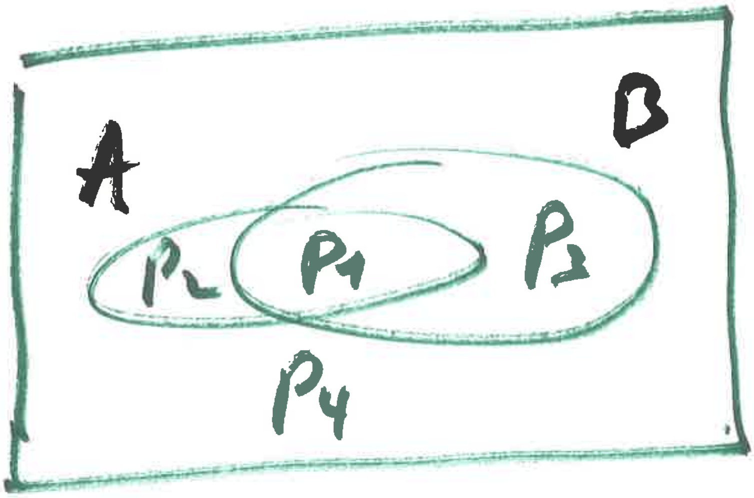
$$P(A)=p,\ P(A\to B)=q \models P(B)\in\beta$$

SIGURNO VRIJEDI ZA $\beta=[0,1]$

PROBLEM JE NAĆI OPTIMALNI β.

$(\forall M)$ tj.

$(\forall p_1 p_2 p_3 p_4)$



$$p_1+p_2=p$$
$$p_1+p_3+p_4=q$$
$$p_1+p_2+p_3+p_4=1$$
$$p_1+p_3=\ ?$$

$$p_1=p+q-1\geq 0$$
$$p_2=1-q\geq 0$$
$$p_3=1-p-p_4\geq 0$$
$$\boxed{0\leq p_4\leq 1-p}$$
$$\boxed{p_1+p_3=q-p_4}$$

$$\Longrightarrow \boxed{p+q-1\leq p_1+p_3\leq q}$$

OPTIMALNI $\beta=[p-q+1,\ q]$

TO SU ZADACI LIN. PROGRAMIRANJA

$$P(A)=p,\ P(A\to B)=q \models P(B)\in[p+q-1,\ q]$$

$$P(A)=p,\ P(B)=q \models P(AB)\in[p+q-1,\ \min\{p,q\}]$$

$$P(A)=p,\ P(B)=q \models P(A\vee B)\in[\max\{p,q\},\ p+q]$$

$$P(A\to B)=p,\ P(B\to C)=q \models P(A\to C)\in[p+q-1,\ 1]$$

$$P(A \to B) = p,\ P(\bar{A} \to B) = q \models P(B) = p + q - 1$$

$$P(A \to B) = p \models P(B \to A) \in [1-p, 1]$$

$$P(A) \in [p_1, p_2],\ P(A \to B) \in [q_1, q_2] \models P(B) \in [p_1 + q_1 - 1, q_2]$$

$$P(A \to B) \in [p_1, p_2];\ P(B \to C) \in [q_1, q_2] \models P(A \to C) \in [p_1 + q_1 - 1, 1]$$

NE OVISI O GORNJ. MEĐAMA

ALI !!! $P(A \to B)$ NIJE $P(B|A)$

"S NESIGURNOŠĆU ε"

$$P(A) \in [1-\varepsilon, 1],\ P(A \to B) \in [1-\varepsilon, 1] \models P(B) \in [1-2\varepsilon, 1]$$

$$P(A \to B) \in [1-\varepsilon, 1],\ P(B \to C) \in [1-\varepsilon, 1] \models P(A \to C) \in [1-2\varepsilon, 1]$$

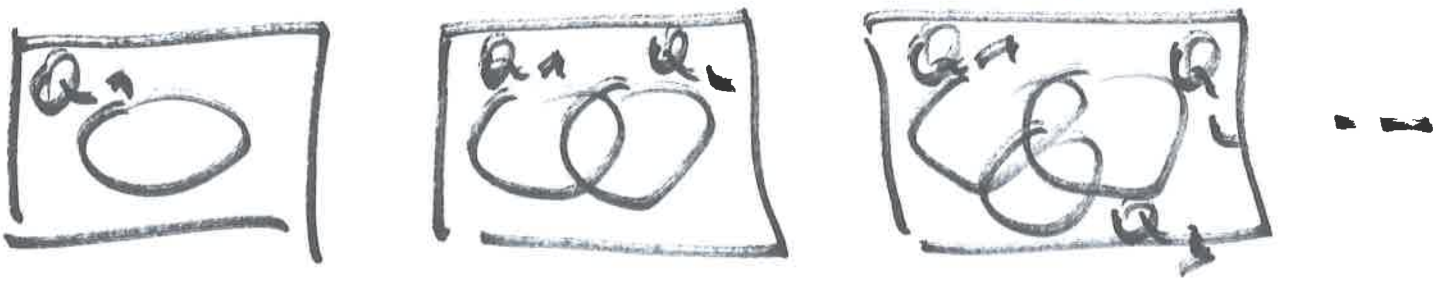
TEOREM (LIN. PROGRAMIRANJA):

RELACIJA $\models$ JE ODLUČIVA ZA RACIONALNE INTERVALE α_i (UKLJ. I OPTIMALNOST) I OPTIMALNI β JE RAC. INT.

BESKONAČNI SKUP $At = \{Q_1, \ldots\}$

GENERIRA L_∞ I IMA ĆELIJE:

$$1 \;\begin{cases} C_1 = Q_1 & \begin{cases} C_{11} = Q_1 Q_2 & \cdots \\ C_{10} = Q_1 \overline{Q}_2 & \cdots \end{cases} \\ C_0 = \overline{Q}_1 & \begin{cases} C_{01} = \overline{Q}_1 Q_2 & \cdots \\ C_{00} = \overline{Q}_1 \overline{Q}_2 & \cdots \end{cases} \end{cases}$$



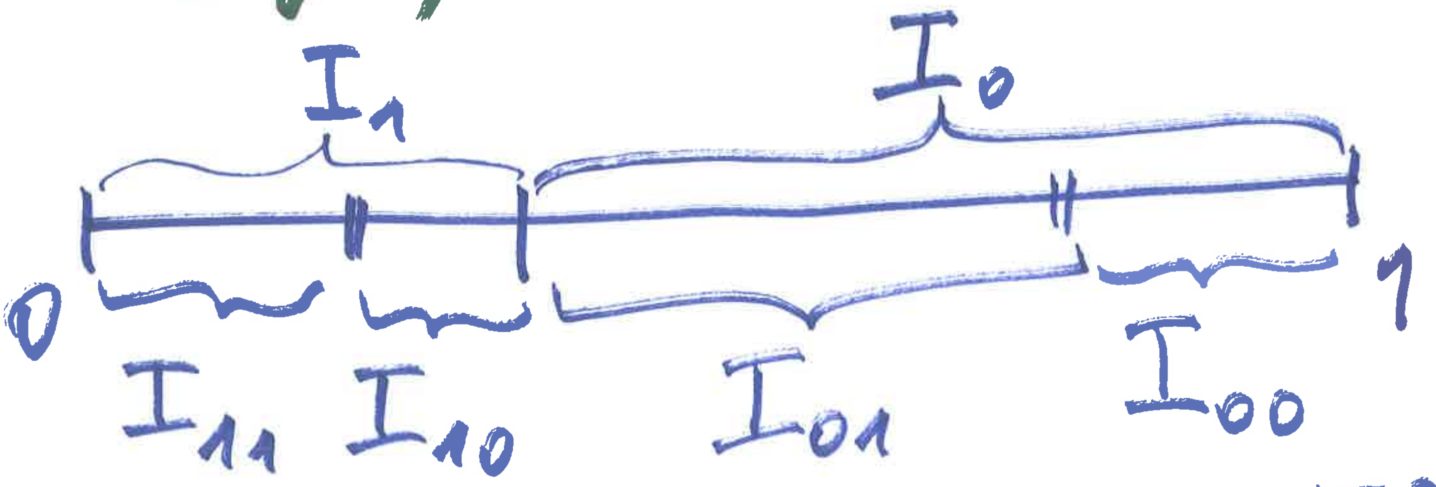
PROBABILIST. MODEL ZA L_∞:

$$M : \{C_0, C_1, C_{00}, C_{01}, \ldots\} \to [0,1]$$

$$r_1 + r_0 = 1 \quad \& \quad r_{\sigma 1} + r_{\sigma 0} = r_\sigma$$

$M \to P_M$ ZA A IZGRAĐEN OD $Q_1, \ldots, Q_n$ KAO U L_n

$M=\{I_\sigma\}$; ∞ BISEKC. OD $[0,1]$:



ITP.

$$C_\sigma \Leftrightarrow I_\sigma \Leftrightarrow |I_\sigma| = M(C_\sigma) = r_\sigma$$

"SVAKI ČVOR STABLA, C_σ, JE INTERVAL"
(ČVOROVA IMA $\aleph_0$)

"PROPOZ IZ L_∞ JE KON. PODSKUP ČVOROVA C_σ (MOGU SVI S RAZINE n)"

$$A = \bigvee_A C_\sigma \;\&\; P_M(A) = \sum_A |I_\sigma|$$

(I NJIH IMA $\aleph_0$)

KAKO UKLOPITI ∞ KONJUNKCIJE I ALTERNACIJE ?

(KONJUNKCIJE NPR. $\overline{R}_1 \overline{R}_2 \overline{R}_3 \ldots$ SU "REALNE TOČKE", ČIJA JE VJEROJATNOST 0 AKO SVI $I_\sigma \to 0$)

$$\boxed{\begin{array}{l} (P_\infty)\ P\left(\bigwedge_1^\infty A_i\right) := \lim_{n\to\infty} P\left(\bigwedge_1^n A_i\right) \\ \Updownarrow \\ (P_{\infty'})\ P\left(\bigvee_1^\infty A_i\right) := \lim_{n\to\infty} P\left(\bigvee_1^n A_i\right) \end{array}}$$

σ-ADITIVNOST JE POSLJEDICA:

$$\boxed{P\left(\dot{\bigvee_1^\infty} A_i\right) := \lim_{n\to\infty} P\left(\dot{\bigvee_1^n} A_i\right)} =$$

$$\lim_{n\to\infty}\left(P(A_1)+\cdots+P(A_n)\right) := \boxed{\sum_1^\infty P(A_i)}$$

DZ. OD (VI):

$$\boxed{P\left(\bigwedge_{1}^{\infty} A_i\right)} = P\left(--\bigwedge_{1}^{\infty} A_i\right) =$$

$$P\left(-\bigvee_{1}^{\infty} \overline{A}_i\right) = 1 - P\left(\bigvee_{1}^{\infty} \overline{A_i}\right) =$$

$$1 - \lim_{n\to\infty} P\left(\bigvee_{1}^{n} \overline{A}_i\right) =$$

$$1 - \lim_{n\to\infty} P\left(--\bigvee_{1}^{n} \overline{A}_i\right) =$$

$$1 - \lim_{n\to\infty} P\left(-\bigwedge_{1}^{n} A_i\right) =$$

$$1 - \lim_{n\to\infty} \left(1 - P\left(\bigwedge_{1}^{n} A_i\right)\right) =$$

$$\boxed{\lim_{n\to\infty} P\left(\bigwedge_{1}^{n} A_i\right)}$$

BOREL-CANTELLI

$P(A_i) = P(\text{"i-ti POKUŠ. USPJ."}) = p_i \Rightarrow$

$P(B) = P(\text{"U } \infty \text{ NIZU } \infty \text{ MNOGO USPJ."})$

$$= \begin{cases} 0 & \text{ZA } \sum p_i < \infty \\ 1 & \text{ZA } \sum p_i = \infty \end{cases}$$

Dz:

$\neg B \equiv (\exists k) \neg (A_k \vee A_{k+1} \vee \cdots)$

$B \equiv (\forall k)(A_k \vee A_{k+1} \vee \cdots)$

tj. $B \equiv (A_1 \vee A_2 \vee A_3 \vee \cdots) \wedge$

$(A_2 \vee A_3 \vee \cdots) \wedge$

$(A_3 \vee \cdots) \wedge$

$\vdots$

$$\boxed{\sum p_i < \infty} \Rightarrow \left(B \models A_k \vee A_{k+1} \vee \cdots \right)$$

$$\Rightarrow P(B) \leq P(A_k \vee A_{k+1} \vee \cdots)$$

$$\leq p_k + p_{k+1} + \cdots \xrightarrow[k\to\infty]{} 0 \,]$$

$$\boxed{\sum p_i = \infty} \Rightarrow$$

$$P(B) = P\big((A_1 \vee A_2 \vee A_3 \vee \cdots) \wedge (A_2 \vee A_3 \vee \cdots) \wedge \cdots \big] =$$

$$1 - P(\bar{A}_1 \bar{A}_2 \bar{A}_3 \cdots \vee \bar{A}_2 \bar{A}_3 \cdots \vee \cdots) \geq$$

$$1 - [\underbrace{P(\bar{A}_1 \bar{A}_2 \bar{A}_3 \cdots)}_{\substack{= (1-p_1)(1-p_2)(1-p_3)\cdots \\ = 0}} + \underbrace{P(\bar{A}_2 \bar{A}_3 \cdots)}_{\substack{= (1-p_2)(1-p_3)\cdots \\ = 0}} + \cdots] = 1$$

$$\left(\sum p_i = \infty\right) \qquad 0 \qquad 0 \qquad \cdots$$

(SERIJALNA NEZ. A_i)

SLABI ZAKON VELIKIH BROJEVA

$$(\forall \varepsilon)\left[P_n\left|\frac{U_n}{n}-p\right|>\varepsilon\right] \xrightarrow[n\to\infty]{} 0$$

GDJE SE $B_n = (\Omega_n, P_n)$ MIJ-
EMJA S n (PA DOSEGNUTA δ-
BLISKOST U_n/n I p NIJE GARAN-
TIRANA ZA SVE $N > n$)

"KOJI P TEŽI NULI?"

Dz: $P_n(C) = p^k q^{n-k} \xrightarrow{\text{BAN. RAČUN}}$

$$P_n\left[\bigcup\left\{C : \left|\frac{k}{n}-p\right|>\varepsilon\right\}\right] =$$

$$\sum_{\{C : |\frac{k}{n}-p|>\varepsilon\}} P_n(C) < \frac{pq}{n\varepsilon^2} < \frac{1}{4n\varepsilon^2}$$

POTREBAN JE (Ω, P) KOJI UKLJUČUJE SVE (Ω_n, P_n)!

Ω = SKUP SVIH, KONAČ. I BRSKON. $\pm A_1 \pm A_2 \pm A_3 \pm A_4 \pm \cdots$

KAKO NA NJEMU DEFINIRATI P?

KOLMOGOROV:

$$\Omega_{KON} = \{[a,b] : [a,b] \subseteq [0,1]\}$$

$$P([a,b]) = b - a$$

$$\Omega = \sigma(\Omega_{KON})$$

SIGMA ALG. GENER. S Ω_{KON}

$$2^{\aleph_0}$$

TEOREM

$(\exists!)$ PROŠIRENJE OD P SA Ω_{KON} NA Ω KOJE JE σ-ADITIVNO

U LOGIČKOM PRISTUPU SA STR. 17... SVI 'P_m' SU UKY-UČENI U P DEFINIRAN NA ∞ MNOGO ĆELIJA C_σ

DAKLE DOKAZ SA STR. 23 JE DOKAZ ZA JEDAN $\underline{\underline{P}}$

U LOGIČKOM PRISTUPU "SLABI" ZAKON JE "JAKI".

JEFTINO???

TO JE BORELOVA x_0 VJEROJ. (1909)

PRIJELAZ IZ KLAS. U MODERN. TEOR.
(BARON-NOVIKOFF AHES 1978)

PROBLEM 1

P(NIJEDAN USPJEH U ∞ NIZU) =

$$P(\bar{A}_1 \bar{A}_2 \bar{A}_3 \cdots) = \lim_{n\to\infty} P(\bar{A}_1 \cdots \bar{A}_n) =$$

$$\lim_{n\to\infty} (1-p_1)\cdots(1-p_n) = \prod_1^\infty (1-p_i) < \infty$$

AKKO $\sum_1^\infty p_i < \infty$

PROBLEM 2

P(k USPJEHA U ∞ NIZU)

PROBLEM 3

P(∞ USPJEHA U ∞ NIZU)

"BOREL-CANTELLI" st. 21-22

TO JE BORELOVA $\aleph_0$ VJEROJAT. (1909)

B-N (AHES 78): PRIJELAZ IZ KLAS. U MODERNU TEORIJU <u>ALI</u>

AUTOR MOD. TEORIJE MJERE JE NIJE KORISTIO U SVOJOJ T. VJEROJ.!
(IAKO JE U GEOMETR. VJ. IZ 1905)

MISLIO JE DA JE $|\Omega| = \aleph_0$,
ŠTO JE POTPUNO ZBUNJUJUĆE,
JER JE $|\Omega| = 2^{\aleph_0}$!

B-N OČITO NE UVIĐAJU MOGUĆNOST PROBAB. VJEROJ. KOJA OPRAVDAVA BORELOVE ARGUMENTE (ZA NJIH POSTOJI SAMO KOLMOGOROV)

DEF. LOG. PROB. POSL. OSTAJE ISTA

TEOREM:

$\models P(A \vee \neg A) = 1$

$\models P(A) = P(AB) + P(A\overline{B})$

$(P(AB) = p,\ P(A\overline{B}) = q \models P(A) = p+q)$

$\models P(A \vee B) + P(AB) = P(A) + P(B)$

$\models A \to B \iff \models P(A) \leq P(B)$

$\iff \models P(A \vee B) = P(A) + P(B)$

$A, B \models$

$\models A \iff P(A) = 1$

$A \models \iff P(A) = 0$

UVJETNA VJEROJATNOST

$$P(B|A) := \frac{P(AB)}{P(A)} = \frac{\sum_{AB} c_i}{\sum_{A} c_i}$$

ZA $P(A) \neq 0$, INAČE NEDEFIN.

$(c = 2^X (X \in [0,1])$

$$\boxed{P(AB) = P(A)\,P(B|A)}$$

(COX DOKAZIVO)

ZA BILO KOJI IF-VEZNIK $\circ$

$$P(B|A) \neq P(A \circ B)$$

LIN. RAZL. FORMA OD c_i S KOEF. IZ $\{0,1\}$

LIN. FORMA OD c_i S KOEF. IZ $\{0,1\}$

OPRAVDANJE DEF. UV. VJER:

(1) $P(A|U)$ JE VJER. FUN. OD A

(2) $P(U|U) = 1$

(3) $A, B \subseteq U \Rightarrow \dfrac{P(A)}{P(B)} = \dfrac{P(A|U)}{P(B|U)}$

$\Longrightarrow$

$A \subseteq U \xRightarrow{2,3} P(A|U) = \dfrac{P(A)}{P(U)}$ ✓

$A \not\subseteq U \Rightarrow A = AU \vee A\overline{U}$

$\xRightarrow{1,2} P(A|U) = P(AU|U) + \cancel{P(A\overline{U}|U)}$

$$\boxed{AU \subseteq U \Longrightarrow P(A|U) = \frac{P(AU)}{P(U)}}$$

21a

$$\boxed{A, B \text{ STOH. NEZAV.} :\equiv P(AB) = P(A)P(B)}$$

tj. $P(A|B) = P(A)$ $\quad P(B|A) = P(B)$

$$\boxed{\text{LOGIČ. NEZAV} \underset{\not\Leftarrow}{\Rightarrow} \text{STOH. NEZAV.}}$$

A_i ESENC. (BITAN) U A $:\equiv$

$$A(1/A_i) \not\equiv A(0/A_i)$$

A, B LOG. NEZ. $:\equiv$

A^* i B^* NEMAJU ZAJ. BITNOG ATOMA

- ATOMI SU NEZAVISNI
- A, B NEMAJU ZAJ. ATOMA ONDA SU NEZAVISNI

TEOREM

ZA SVAKI A POSTOJI EKVIVALENTNI A' BEZ NEBITNIH ATOMA

DZ

$$A \underset{IF}{\equiv} A(1/A_i)A_i \vee A(0/A_i)\bar{A}_i$$

$$(A_i \text{ NEB}) \equiv A(1/A_i)(A_i \vee -A_i)$$

$$\underset{IF}{\equiv} A(1/A_i)$$

TJ. NEBITNI A_i JE ELIM. ITD.

KOROLAR

A KONTINGETNA ($\not\models A$; $A \not\models$)

AKKO SADRŽI BAR JEDAN BITNI ATOM.

TEOREM ZA LOGIČKI NEZAVISNE I KONTINGENTNE A, B:

$F(A,B)$ JE TAUTOLOGIJA AKKO $F(A_1, A_2)$ JE TAUTOLOGIJA.

DZ: $(\Leftarrow)$ SUPSTITUCIJA

$(\Rightarrow)$

$$1 \equiv F(A,B) \underset{IF}{\equiv} F(1,1)AB \vee F(1,0)A\bar{B} \vee F(0,1)\bar{A}B \vee F(0,0)\bar{A}\bar{B}$$

A, B KONTING. $\Rightarrow$ PRIM. OBJE VRIJ. tj.
A, B NEZAV. $\Rightarrow$ PRIM. IH NEZAV

$$M_1(A,B) = (1,1) \quad \cdots \quad M_4(A,B) = (0,0)$$

$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow$$

$$F(1,1) = 1 \quad \cdots \quad F(0,0) = 1$$

$$\Rightarrow F(A_1, A_2) \underset{IF}{\equiv} F(1,1)A_1A_2 \vee \cdots \vee F(0,0)\bar{A}_1\bar{A}_2$$

$$\equiv A_1A_2 \vee A_1\bar{A}_2 \vee \bar{A}_1A_2 \vee \bar{A}_1\bar{A}_2 \equiv 1$$

$B_1, \dots, B_m$ STOH. NEZAVISNE $\overset{\text{def}}{\equiv}$

SVE KOMB (NJIH $2^m - m - 1$) STOH. NEZ.

TEOREM (FELLER)

$B_1, \dots, B_m$ STOH. NEZAVISNE AKKO

$$(\forall \pm)\, P(\underbrace{\pm B_1 \wedge \dots \wedge \pm B_m}_{\text{ĆELIJE NAD } B_1, \dots, B_m}) = P(\pm B_1) \cdot \ldots \cdot P(\pm B_m)$$

KOROLAR $B_1 \dots B_m$ ST. NEZAV.

$\Longrightarrow \pm B_1, \dots, \pm B_m$ ST. NEZAV.

DZ: ĆELIJE NAD $\pm B_1 \dots \pm B_m$ SU ISTO

ŠTO I ĆELIJE NAD $B_1, \dots, B_m$.

KOROLAR $A \equiv \bigvee_A C_i \Longrightarrow$

$$P(A) = \sum_A P(C_i) = \sum_A P(\pm A_1) \cdots P(\pm A_m)$$

TEOREM A, B LOG. NEŽAVISNI

$\Longrightarrow A, B$ STOHAST. NEŽAVISNI

Dž: $A \equiv \bigvee_A C_i, \quad B \equiv \bigvee_B C_j$

(LOG. NEŽ.) $\{C_i\} \cap \{C_j\} = \phi$

$$\boxed{P(AB)} = P\left(\left(\bigvee_A C_i\right)\left(\bigvee_B C_j\right)\right) =$$

$$P\left(\bigvee_A \bigvee_B C_i C_j\right) =$$

$$\sum_A \sum_B P(C_i C_j) =$$

$$\sum_A \sum_B P(C_i) P(C_j) =$$

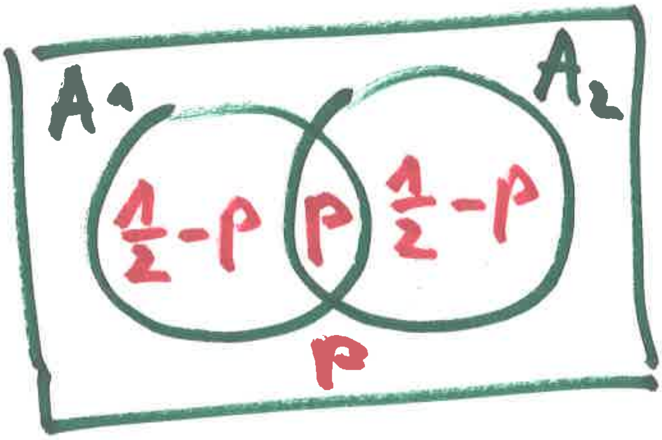
$$\left(\sum_A P(C_i)\right)\left(\sum_B P(C_j)\right) =$$

$$\boxed{P(A)P(B)}$$

TEOREM A, B STOH. NEZAV.

$\not\Rightarrow$ A, B LOGIČKI NEZAVISNI

Dz:



$$P[(\underbrace{A_1A_2 \vee \bar{A}_1\bar{A}_2}_{A})(\underbrace{\bar{A}_1\bar{A}_2 \vee \bar{A}_1A_2}_{B})] = p$$

$$P(\overbrace{A_1A_2 \vee \bar{A}_1\bar{A}_2}^{A})P(\overbrace{\bar{A}_1\bar{A}_2 \vee \bar{A}_1A_2}^{B}) =$$

$$= 2p\left(p + \frac{1}{2} - p\right) = p$$

DAKLE; A, B STOH. NEZAVISNI

A_1 BITAN U A i B

DAKLE; A, B LOGIČKI ZAVISNI

DEF. POSLJEDICE ISTA:

$$\boxed{P(A_1|B_1)\in\alpha_1 \;\cdots\cdots \models P(A|B)\in\beta}$$

ZNAČI:

$$(\forall M)\left[(\forall i)P_M(A_i|B_i)\in\alpha_i \rightarrow P_M(A|B)\in\beta\right]$$

(STANDARDNI) TEOREMI:

$$\models P(ABC)=P(A)P(B|A)P(C|AB)$$

$$\models P(A|B)=P(AB|B)$$

$$\models P(A)P(A|B)=P(B)P(B|A)$$

$$\models P(C\underbrace{(C\rightarrow A)}_{\equiv_{IP} CA})=P(C)P(A|C)$$

KOROLAR (ZADNJEG T.)

$$\boxed{P(C \to A) = P(A|C)}$$

AKKO$_1$ $\boxed{P(C) = 1 \lor P(C \to A) = 1}$

AKKO$_2$ $\boxed{C, C \to A \text{ (ST. NEZAVIS.}}$

Dz: $P(C \to A) = P(A|C)$

$$\iff 1 - P(C\bar{A}) = P(AC)/P(C)$$

$$= \frac{P(C) - P(C\bar{A})}{P(C)} = 1 - \frac{P(C\bar{A})}{P(C)}$$

$$\iff P(C\bar{A}) = P(C\bar{A})/P(C)$$

$$\iff_1 P(C) = 1 \lor \underbrace{P(C\bar{A}) = 0}_{P(C \to A) = 1}$$

$\iff_2$ TRIV. IZ ZADNJ. TEOR. ST 26

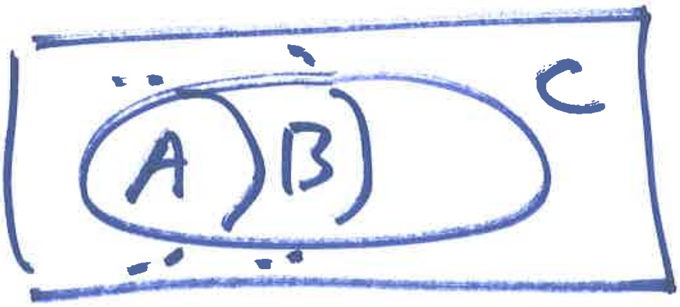
<u>TEOREM</u> $P_C(A) :\equiv P(A|C)$

JE PROBAB. F. ZA SVAKI $P(C)\neq 0$.

<u>Dz:</u> (P1) $P(C\to A)=1 \models P(A|C)=1$

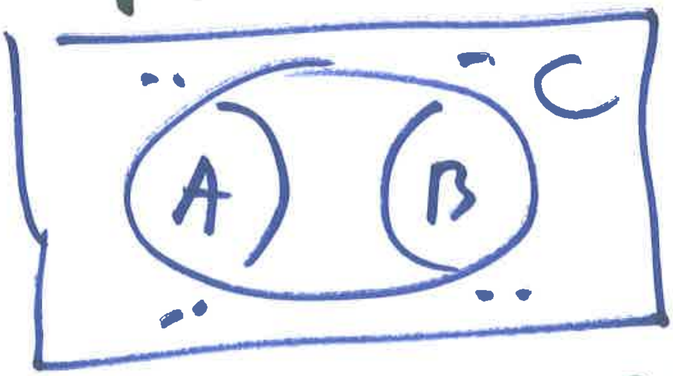
(PRETHODNI KOROLAR)

(P2) $P(C\to(A\to B))=1 \models P(A|C)\leq P(B|C)$



(P3) $P(C\to(A\to \bar{B}))=1 \models$

$P(A\vee B|C)=P(A|C)+P(B|C)$



(UVJ) $\models P(AB|C)=P(A|C)P(B|AC)$

"UVJETNA VJ. ZA P_C"

TEOREM

$$P(C \to A) = 1 \equiv P(A|C) = 1$$

$$P(A|B) = 1 \equiv P(\bar{B}|\bar{A}) = 1$$

Dz: PRVA $\equiv$ JE "KOR. ZADNJEG"

$$P(A|B) = 1 \Rightarrow$$

$$P(AB) = P(B)P(A|B) = P(B) =$$

$$= P(AB) + P(\bar{A}B)$$

$$\Rightarrow P(\bar{A}B) = 0$$

$$\Rightarrow P(\bar{A}\bar{B}) + \cancel{P(\bar{A}B)} = P(\bar{A})$$

$$\Rightarrow \frac{P(\bar{A}\bar{B})}{P(\bar{A})} = 1 \text{ tj. } P(\bar{B}|\bar{A}) = 1$$

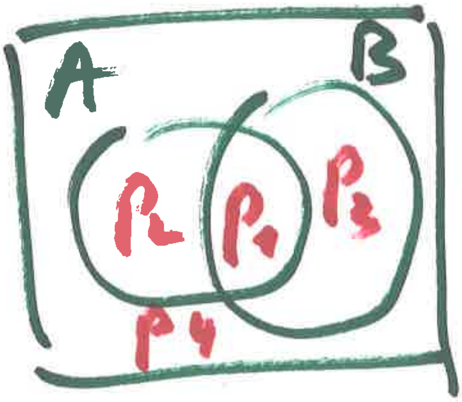
$$(\text{uz } P(\bar{A}) \neq 0)$$

TEOREM

OPTIMALNO

$$P(A|B)=p \models P(\bar B|\bar A)\in[0,1]$$

Dz:



OPTIMIZIRAJ

$$P(\bar B|\bar A)=\frac{p_4}{p_3+p_4}$$

uz $\frac{p_1}{p_1+p_3}=p$

tj. $(1-p)p_1+p\,p_3=0$

$$p_1+p_2+p_3+p_4=1$$

$$p_1,p_2,p_3,p_4\geq 0$$

RAZLOMLJENO LINEARNO PROGRAM.

DAJE OPTIMUM:

$$0\leq\frac{p_4}{p_3+p_4}\leq 1$$

$$P(A|B)=p \models P(B|A)\in[0,1]$$

TEOREM

$P(B|A)=p,\ P(B|\bar{A})=q \vDash P(B)\in[p,q]$

(IZ JEDNE URNE S p IZ DRUGE S q
ONDA IZ NEKE OD NJIH S $r\in[p,q]$)

Dz:

$$P(AB)=P(A)\cdot p$$

$$P(\bar{A}B)=P(\bar{A})q$$

$$+ \qquad\qquad = q-qP(A)$$

$$P(B)=\underbrace{(p-q)\underbrace{P(A)}_{[0,1]}+q}_{[p,q]}$$

MODUS PONENS

$$P(B|A)=p,\ P(A)=q \models P(B)\in[pq,\ 1-\bar{p}q]$$

$$P(B|A)=[1-\varepsilon,1],\ P(A)=[1-\varepsilon,1]$$
$$\models P(B)\in[(1-\varepsilon)^2,1]$$

"POJAČANA" IMPLIKACIJA

$$P(B|A_1)=p,\ P(B|A_2)=q \models P(B|A_1A_2)\in[0,1]$$

DOKAZIVANJE PO SLUČAJEVIMA

$$P(B|A_1)=1-\varepsilon,\ P(B|A_2)=1-\varepsilon \models$$
$$P(B|A_1\vee A_2)\in\left[1-\frac{2\sigma}{1+\sigma},\ 1-\frac{\sigma}{2-\sigma}\right]$$

JEDAN BOOLEOV PROBLEM

$$P(A_1)=\theta_1,\ P(A_2)=\theta_2,$$

$$P(D|A_1)=w_1,\ P(D|A_2)=w_2$$

$$\models P(D|A_1 \vee A_2) \in [D, G]$$

$$D = \mathrm{MAX}\left(\frac{\theta_2 w_2}{\theta_2+\theta_1\overline{w_1}},\ \frac{\theta_1 w_1}{\theta_1+\theta_2\overline{w_2}},\ \theta_1 w_1,\ \theta_2 w_2\right)$$

$$G = \mathrm{MIN}\left(\frac{\theta_1 w_1+\theta_2 w_2}{\theta_1 w_1+\theta_2},\ \frac{\theta_1 w_1+\theta_2 w_2}{\theta_1+\theta_2 w_2},\ \overline{\theta_1}+\theta_1 w_1,\ \overline{\theta_2}+\theta_2 w_2\right)$$

BOOLEOVA LOGIKA JE BILA PROBABIL. LOGIKA !!