A Logic for Temporal Beliefs and Intentions - Completeness and belief revision

Marc van Zee and Dragan Doder

University of Luxembourg
University of Belgrade

Logics and Applications (LAP'16)
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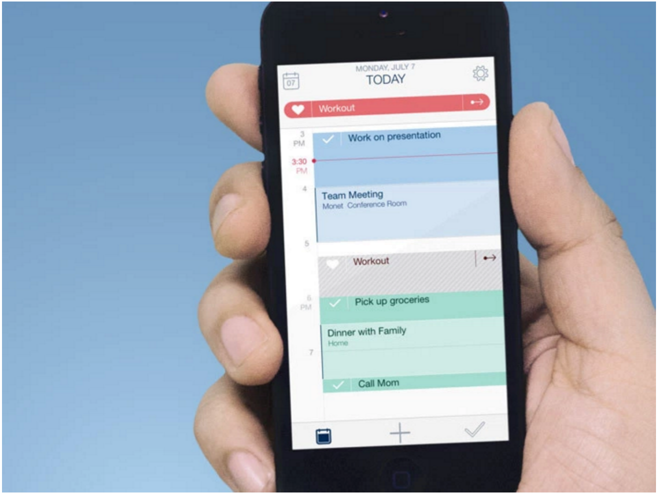
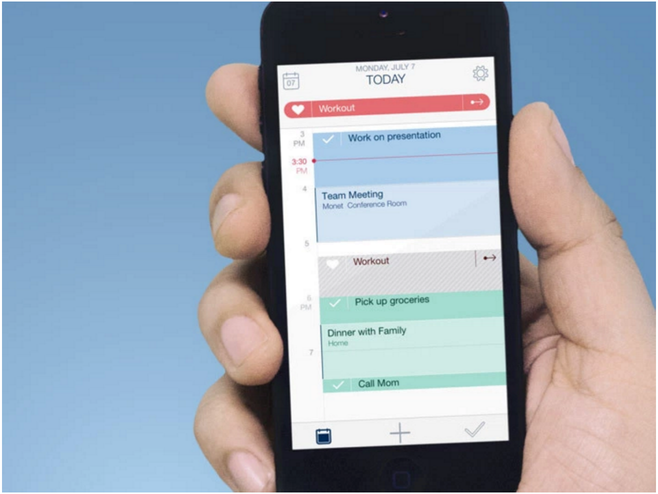
Timeful App

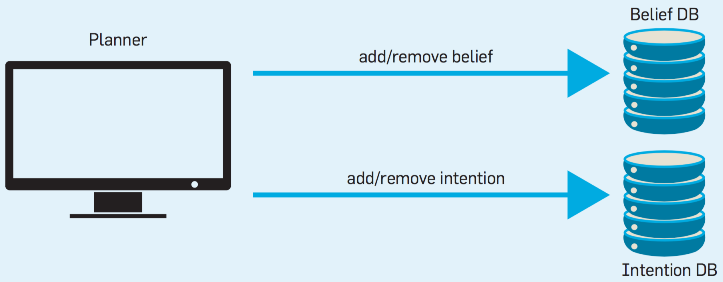

MONDAY, JULY 7
TODAY
07
Workout
3 PM
Work on presentation
3:30 PM
4
Team Meeting
Monet Conference Room
5
Workout
6 PM
Pick up groceries
Dinner with Family
Home
7
Call Mom

Timeful App



Yoav Shoham, ‘Why knowledge representation matters’, *Commununications of the ACM*, **59**(1), 47–49, (January 2016).

Timeful starting point: The Database Perspective



Yoav Shoham, ‘Logical theories of intention and the database perspective’, *Journal of Phil. Logic*, **38**(6), 633–647, (2009).

(. . .) a generalization of the AGM scheme for belief revision, (...). In the AGM framework, the intelligent database is responsible for storing the planner's beliefs and ensuring their consistency. In the enriched framework, there are two databases, one for beliefs and one for intentions, which are responsible for maintaining not only their individual consistency but also their mutual consistency."

On AGM belief revision

- Belief revision – the process of changing beliefs to take into account a new piece of information.
- The AGM postulates – properties that should be satisfied by any (rational) revision operators defined on deductively closed sets of propositional formulas.
- Katsuno and Mendelzon (KM) represent a belief set B as a propositional formula ψ such that $B = \{\varphi \mid \psi \vdash \varphi\}$. They define the following six postulates for revision on ψ and show that these are equivalent to the eight AGM postulates.

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(R1) $\psi \circ \varphi$ implies φ
(R2) If $\psi \wedge \varphi$ is satisfiable, then $\psi \circ \varphi \equiv \psi \wedge \varphi$
(R3) If φ is satisfiable, then $\psi \circ \varphi$ is also satisfiable
(R4) If $\psi \equiv \psi'$ and $\varphi \equiv \varphi'$, then $\psi \circ \varphi \equiv \psi' \circ \varphi'$
(R5) $(\psi \circ \varphi) \wedge \varphi'$ implies $\psi \circ (\varphi \wedge \varphi')$
(R6) If $(\psi \circ \varphi) \wedge \varphi'$ is satisfiable, then $\psi \circ (\varphi \wedge \varphi')$ implies $(\psi \circ \varphi) \wedge \varphi'$

Katsuno - Mendelzon representation theorem

$\mathbb{I}$ – the set of all interpretations over some propositional language, faithful assignment – a function that assigns each ψ to a pre-order $\leq_\psi$ on models satisfying the following three conditions:

1. If $I, I' \in Mod(\psi)$, then $I <_\psi I'$ does not hold.
2. If $I \in Mod(\psi)$ and $I' \notin Mod(\psi)$, then $I <_\psi I'$ holds.
3. If $\psi \equiv \phi$, then $\leq_\psi = \leq_\phi$.

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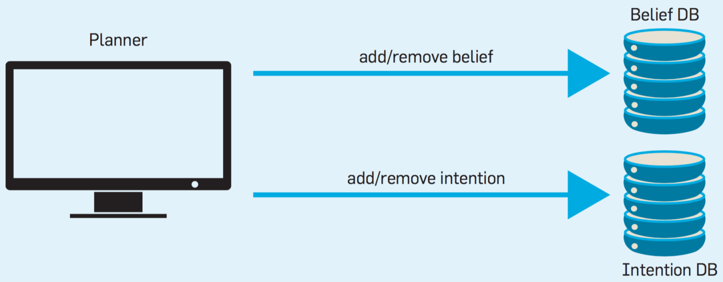
Theorem

A revision operator $\circ$ satisfies postulates (R1)-(R6) iff there exists a faithful assignment that maps each formula ψ to a total preorder $\leq_\psi$ such that

$$Mod(\psi \circ \varphi) = \min(Mod(\varphi), \leq_\psi).$$

Shoham's Database Perspective



Shoham's Database Perspective

Shoham's Coherence Conditions

1. Beliefs must be internally consistent.
2. Intentions must be internally consistent.
   - 2.1 At most one action is intended for any given moment.
   - 2.2 If two intended actions immediately follow one another, the earlier cannot have postconditions that are inconsistent with the preconditions of the latter.
3. Intentions must be consistent with beliefs.
   - 3.1 If you intend to take an action you cannot believe that its preconditions do not hold.
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Icard, Pacuit and Shoham (KR'10) proposed a formalization (IPS), but we discuss flaws in our papers (omitted here)

Objective and Contributions

Objective:

- Develop a logic for Shoham's database perspective including coherence conditions and study belief and intention revision.

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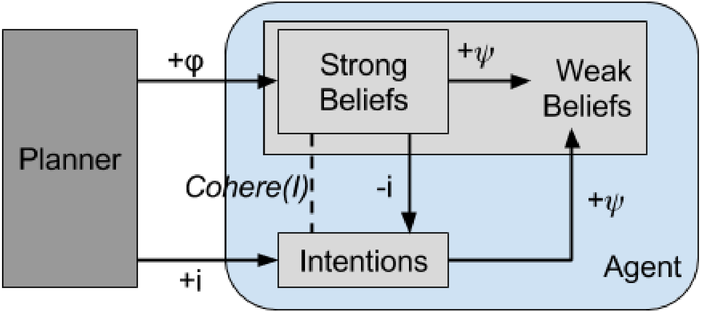
Results from:

- International Joint Conference on Artificial Intelligence – IJCAI 2015
- AAAI Spring Symposia – Logical Formalizations of Commonsense Reasoning 2015
- European Conference on Artificial Intelligence – ECAI 2016

Preview of our formalization



- Planner can add strong beliefs and intentions.
- Strong belief revision may trigger intention revision.
- Intention revision may only trigger revision of weak beliefs.
- The agent should be coherent after revision.

Table of Contents

Parameterized-time Action Logic

PAL: beliefs about action and time

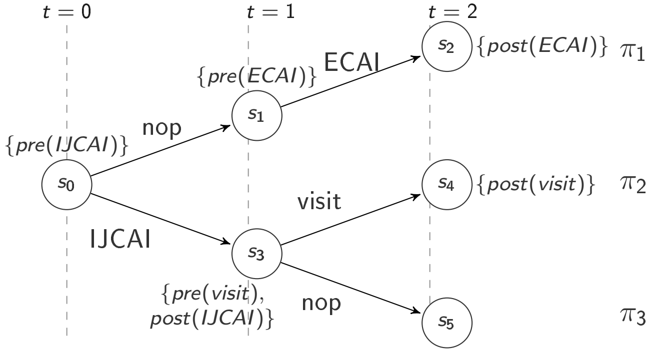
- Language:
  $\varphi ::= p_t \mid pre(a)_t \mid post(a)_t \mid do(a)_t \mid \Box_t\varphi \mid \varphi \wedge \varphi \mid \neg\varphi$
- CTL*-like tree semantics
- Model = a tree and a path: $m = (T, \pi)$
- Equivalence relation $\sim_t$ on paths:
  $\pi' \sim_t \pi$ iff π and π' are same up to time t
- Truth Definition of $\Box_t$:
  $T, \pi \models \Box_t\varphi$ iff for all π' in T: if $\pi' \sim_t \pi$, then $T, \pi' \models \varphi$

Running Example

Traveling to Conferences

- An agent located in Luxembourg
- Considers to attend IJCAI (USA) in July 2016 and ECAI (NL) in August 2016.
- Insufficient budget available for traveling to both conferences.
- The agent believes:
  - it is possible to attend IJCAI at time 0 and that it is possible to attend ECAI at time 1.
  - it is impossible to attend both conferences.
- If the agent decides to attend IJCAI, then it would like to visit to a colleague at time 1 (August 2016).

Example PAL Model



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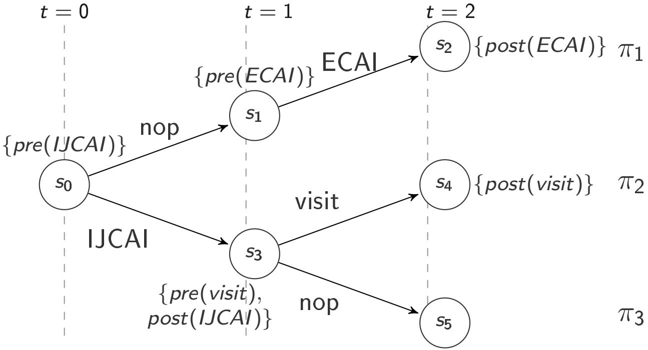

$$T, \pi_3 \models \neg do(visit)_1$$

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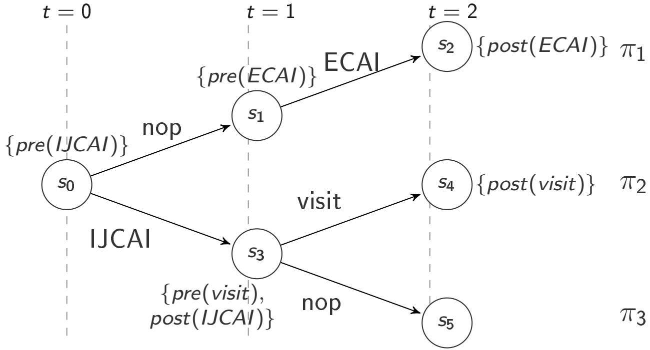

$$T, \pi_3 \models \neg do(visit)_1$$
$$T, \pi_1 \models \Diamond_0(do(IJCAI)_0 \wedge \neg do(visit)_1)$$

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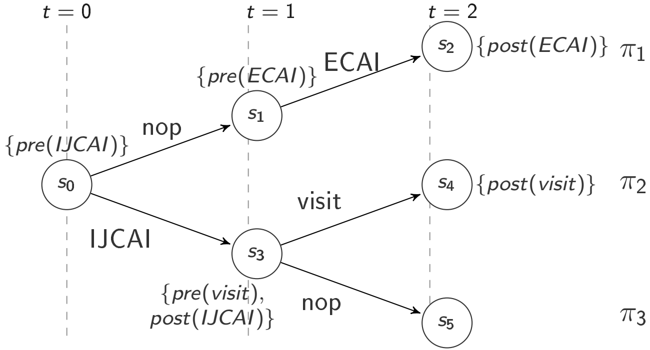

$$T, \pi_3 \models \neg do(visit)_1$$
$$T, \pi_1 \models \Diamond_0(do(IJCAI)_0 \wedge \neg do(visit)_1)$$
$$T, \pi_1 \models \neg\Diamond_0(do(IJCAI)_0 \wedge do(ECAI)_1)$$

Axiomatization:

Propositional tautologies (PROP)

$\Box_t(\varphi \rightarrow \varphi') \rightarrow (\Box_t\varphi \rightarrow \Box_t\varphi')$ (K)

$\Box_t\varphi \rightarrow \varphi$ (T)

$\Diamond_t\varphi \rightarrow \Box_t\Diamond_t\varphi$ (5)

$\chi_t \rightarrow \Box_t\chi_t$, where $\chi \in \mathit{Prop}$ (A1)

$\Diamond_t\chi_t \rightarrow \chi_t$, where $\chi \in \mathit{Prop}$ (A2)

$\Box_t\varphi \rightarrow \Box_{t+1}\varphi$ (A3)

$\mathit{do}(a)_t \rightarrow \Box_{t+1}\mathit{do}(a)_t$ (A4)

$\Diamond_{t+1}\mathit{do}(a)_t \rightarrow \mathit{do}(a)_t$ (A5)

$\bigvee_{a\in \mathit{Act}} \mathit{do}(a)_t$ (A6)

$\mathit{do}(a)_t \rightarrow \bigwedge_{b\neq a} \neg \mathit{do}(b)_t$ (A7)

$\mathit{pre}(a)_t \rightarrow \Diamond_t\mathit{do}(a)_t$ (A8)

$\mathit{do}(a)_t \rightarrow \mathit{post}(a)_{t+1}$ (A9)

$\Diamond_t(\mathit{do}(a)_t \wedge \alpha) \rightarrow \Box_t(\mathit{do}(a)_t \rightarrow \alpha)$ (A10)
where $\alpha \in \mathit{Past}(t+1)$

From φ, infer $\Box_t\varphi$ (NEC)

From $\varphi, \varphi \rightarrow \varphi'$, infer φ' (MP)

Strong and Weak Beliefs

Definition (Strong Beliefs)

The set of all *strong beliefs* $\mathbb{B}_t$ in time t:

$$\varphi ::= \Box_t \psi \mid \varphi \wedge \varphi \mid \neg\varphi,$$

where $\psi \in \mathscr{L}_{PAL}$. A *strong belief set B* is a set of strong beliefs closed under consequence.

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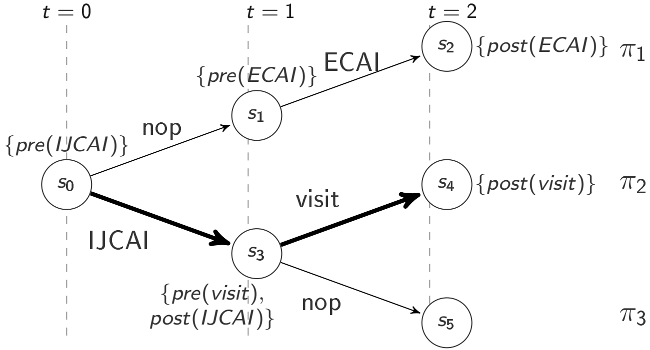
Intention database: $I = \{(a_1, t_1), \ldots, (a_n, t_n)\}$.

Definition (Weak Beliefs)

Given a pair (B, I), the weak beliefs are defined as:

$$WB(B, I) = Cn(B \cup \{do(a)_t \mid (a, t) \in I\}).$$

Examples of Strong/Weak Beliefs. $I = \{(IJCAI, 0), (visit, 1)\}$



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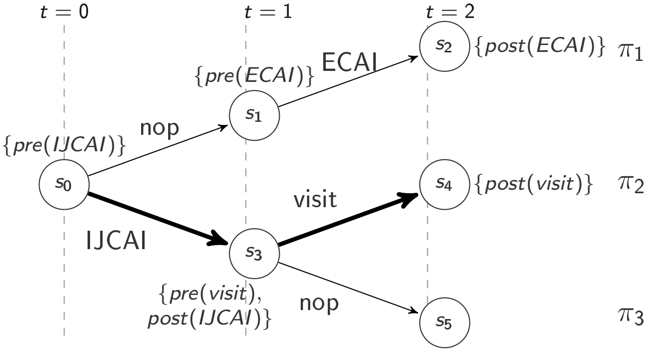


Strong beliefs:

$\Diamond_0 do(IJCAI)_0$

$\Diamond_0 do(ECAI)_1$

$\neg\Diamond_0(post(ECAI)_2 \wedge post(IJCAI)_1)$

Weak beliefs:

$do(IJCAI)_0$

$post(IJCAI)_1$

$\neg post(ECAI)_2$

Formalizing Shoham's Coherence Conditions

Icard *et al.* (IPS) Coherence Condition

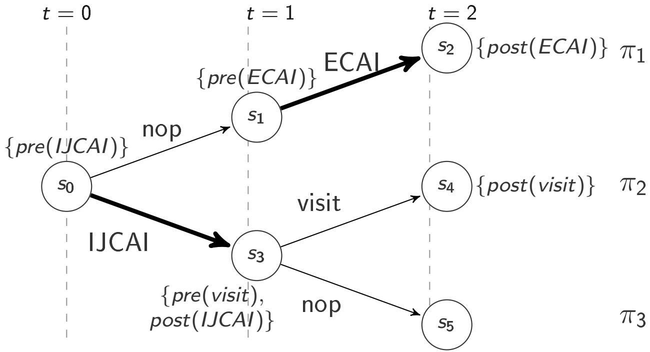
Let M be a set of PAL models. The pair (M, I) is coherent iff there exists some $m \in M$ s.t.

$$m \models \Diamond_0 \bigwedge_{(a,t) \in I} pre(a)_t.$$

This condition is too weak.

Analysis of IPS Coherence Condition

Let $I = \{(IJCAI, 0), (ECAI, 1)\}$



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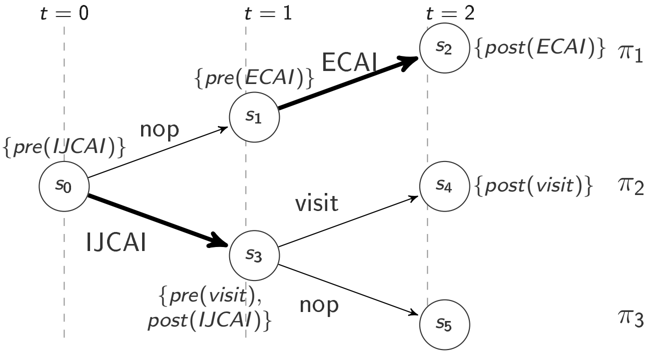

(M, I) is coherent, because
$T, \pi_1 \models \Diamond_0(pre(IJCAI)_0 \wedge pre(ECAI)_1)$.
However, the agent does not have the precondition to execute *both actions on the same path*.

PAL-P: Extension of PAL

Definition (PAL-P Language)

The language $\mathscr{L}$ is obtained from $\mathscr{L}_{PAL}$ by adding $\{pre(a, b, \ldots)_t \mid \{a, b, \ldots\} \subseteq \text{Act}, t \in \mathbb{N}\}$ to the set of propositions.

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- We extend definition of model
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$$pre(\ldots, a, b)_t \rightarrow pre(\ldots, a)_t \qquad \text{(A11)}$$
$$(pre(a, b, \ldots)_t \wedge do(a)_t) \rightarrow pre(b, \ldots)_{t+1} \qquad \text{(A12)}$$

Shoham's Coherence Conditions

Definition (Coherence)

Given an intention database $I = \{(b_{t_1}, t_1), \ldots, (b_{t_n}, t_n)\}$ with $t_1 < \ldots < t_n$, let

$$Cohere(I) = \Diamond_0 \bigvee_{\substack{a_k \in Act : k \notin \{t_1, \ldots, t_n\} \\ a_k = b_k : k \in \{t_1, \ldots, t_n\}}} pre(a_{t_1}, a_{t_1+1}, \ldots, a_{t_n})_{t_1}.$$

For a given set of models M, we say that (M, I) is *coherent* iff there exists some $m \in M$ with $m \models Cohere(I)$. For a given agent $A = (B, I)$, we say that the A is *coherent* iff B is consistent with $Cohere(I)$, i.e., $B \not\vdash \neg Cohere(I)$.

Theorem

The Shoham's Coherence conditions hold.

Belief and Intention Revision

Overview:

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- We prove a representation theorem relating the postulates for revision to an ordering among models and a selection function that accommodates new intentions while restoring coherence.
- Difficulty: when revising a belief database that is bounded up to some time t with a strong belief, we have to ensure that the resulting belief database is also bounded up to t, and that it remains a strong belief.

Belief and Intention Revision

Definition (Agent Revision Function)

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$(P9)$ If $(\psi', I \cup \{i\})$ is coherent, then $I \cup \{i\} \subseteq I'$.

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$(P11)$ If $I = I_2$, $i = i_2$, and $\psi' \equiv \psi'_2$, then $I' = I'_2$.
$(P12)$ For all I'' with $I' \subset I'' \subseteq I \cup \{i\}$:$(\psi', I'')$ is not coherent.

Selection Function

Definition (Selection Function)

Given an intention database I, a *selection function* $\gamma_I^t : \mathbb{M}_{SB} \times \mathbb{I} \rightarrow \mathbb{D}$ maps a set of models of a strong belief and an intention to an updated intention database—all bounded up to t— such that if $\gamma_I^t(M^{|t}, \{i\}) = I'$, then:

1. $(M^{|t}, I')$ is coherent.
2. If $(M^{|t}, \{i\})$ is coherent, then $i \in I'$.
3. If $(M^{|t}, I \cup \{i\})$ is coherent, then $I \cup \{i\} \subseteq I'$.
4. $I' \subseteq I \cup \{i\}$.
5. For all I'' with $I' \subset I'' \subseteq I \cup \{i\}$:$(M^{|t}, I'')$ is not coherent.

Faithful assignment

Definition (Faithful assignment)

A *faithful assignment* is a function that assigns to each strong belief formula $\psi \in \mathbb{B}^{|t}$ a total pre-order $\leq^t_\psi$ over $\mathbb{M}$ and to each intention database $I \in \mathbb{D}^{|t}$ a selection function γ^t_I and satisfies the following conditions:

1. If $m_1, m_2 \in Mod(\psi)$, then $m_1 \leq^t_\psi m_2$ and $m_2 \leq^t_\psi m_1$.
2. If $m_1 \in Mod(\psi)$ and $m_2 \notin Mod(\psi)$, then $m_1 < m_2$.
3. If $\psi \equiv \phi$, then $\leq^t_\psi = \leq^t_\phi$.
4. If $T^{|t} = T_2^{|t}$, then $(T, \pi) \leq^t_\psi (T_2, \pi_2)$ and $(T_2, \pi_2) \leq^t_\psi (T, \pi)$.

Representation Theorem

Theorem (Representation Theorem)

An agent revision operator $*_t$ *satisfies postulates (P1)-(P12) iff there exists a faithful assignment that maps each* ψ *to a total pre-order* $\leq^t_\psi$ *and each* I *to a selection function* γ^t_I *such that if* $(\psi, I) *_t (\varphi, i) = (\psi', I')$*, then:*

1. $Mod(\psi') = \min(Mod(\varphi), \leq^t_\psi)$
2. $I' = \gamma^t_I(Mod(\psi'), i)$

Conclusion

- We formalize Shoham's database perspective using PAL
- In order to formalize the coherence condition, we extend PAL with preconditions for action sequences
- We formalize a coherence condition and show that it formalizes Shoham's conditions.
- We provide postulates for the joint revision of beliefs and intentions.
- We prove a representation theorem in Katsuno & Mendelzon style.