Strategic reasoning in conversations under imperfect information

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Conversations as a strategic activity

Bensten-Quayle 1988 Vice Presidential debate

Christie-Rubio 2016 Republican primary debate

Coleman's spokesman Sheehan's 2008 press conference

Conversations as a strategic activity

1. **Prosecutor**: *Do you have any bank accounts in Swiss banks, Mr. Bronston?*
2. **Bronston**: *No, sir.*
3. **Prosecutor**: *Have you ever?*
4. **Bronston**: *The company had an account there for about six months, in Zurich.*
5. **Prosecutor**: *Thank you Mr. Bronston.*

[Solan-Tiersma'05, *Speaking of crime: the language of criminal justice*]

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Conversations as a strategic activity

- **Justin**: *Have you been seeing Valentino this past week?*
- **Janet**: *Valentino has mononucleosis.*

Puzzling observations

- In situations where the agents' interests are broadly opposed (eg. political debates, marital disputes), people still act somewhat cooperatively.
- They answer more often than not their interlocutor's questions.
- You'd better attend to what your opponent says and gauge exactly what might be meant if you hope to win a debate.
- For eg. Bronston's overall interests are opposed to that of the Prosecutor. Yet he is somewhat cooperative, and exploits the implicatures of (4) to answer his questions.

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- Can be either symmetric or asymmetric.
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- The presense of a 'Jury'.
- Absence of a set-end or horizon.

Message Exchange games

$$\begin{array}{llllll} & & & & x_3 & y_3\ x_4 \cdots \\ x_1 & y_1 & x_2 & y_2 & & \\ & & & & x_3' & y_3'\ x_4' \cdots \end{array}$$

- $x_j \in V_0^+$, V_0: vocabulary of Player 0
- $y_j \in V_1^+$, V_1: vocabulary of Player 1

Message Exchange games

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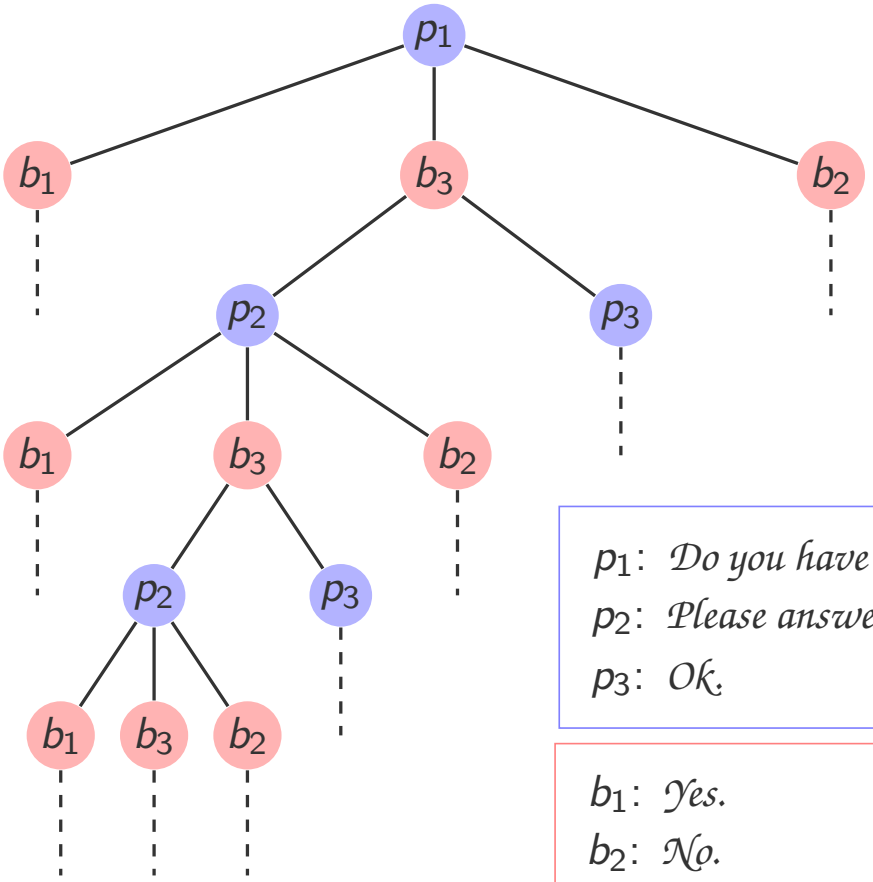
- Strategy $\sigma_0 : (V_0 \cup V_1)^* V_1^+ \to V_0^+$

Message Exchange games

The vocabulary is defined using the theory of SDRT [Asher,Lascarides '03].

- V is of the form $\pi : \phi$ where
  - $\pi \in$ DU: the set of discourse constituent labels.
  - ϕ is a formula from some fixed language eg. some higher order logic for describing elementary discourse move contents.
- There is also a set of relations $\mathcal{R} \subset \text{DU} \times \text{DU}$, eg. question-answer-pair (qap).
- These relations specify when a discourse move can be played 'coherently' in a context.
- Can be described as a graph structure.

Example of an ME game



p_1: *Do you have any bank accounts in Swiss Banks, Mr. Bronston?*
p_2: *Please answer my question directly.*
p_3: *Ok.*

b_1: *Yes.*
b_2: *No.*
b_3: *The company had an account...*

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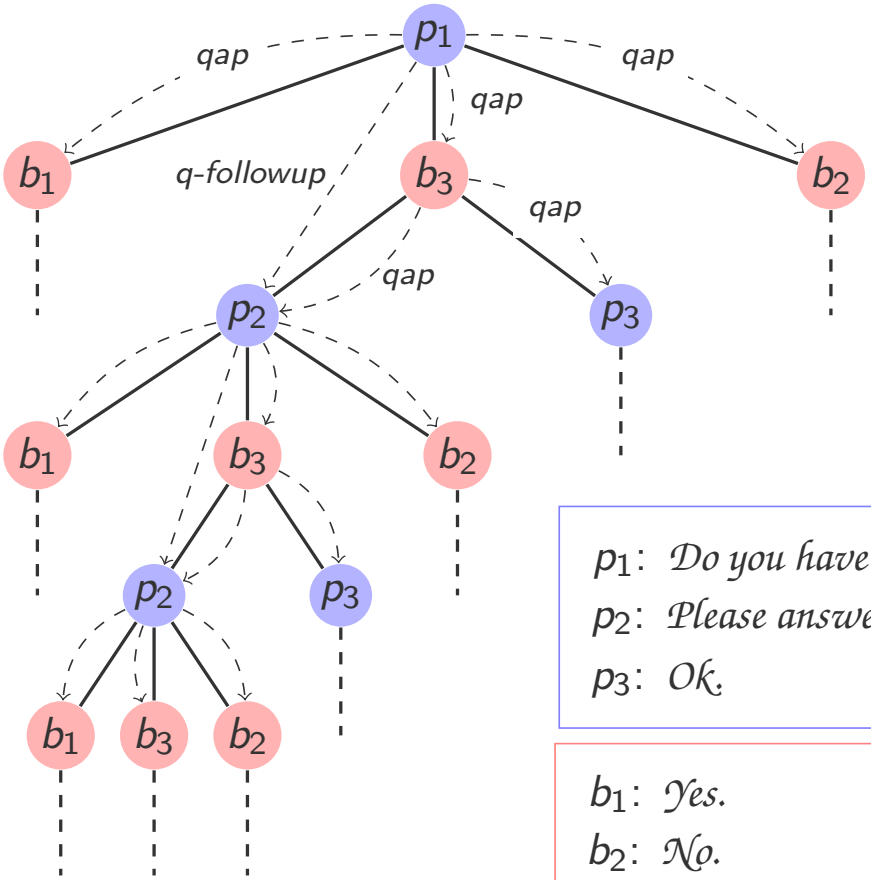

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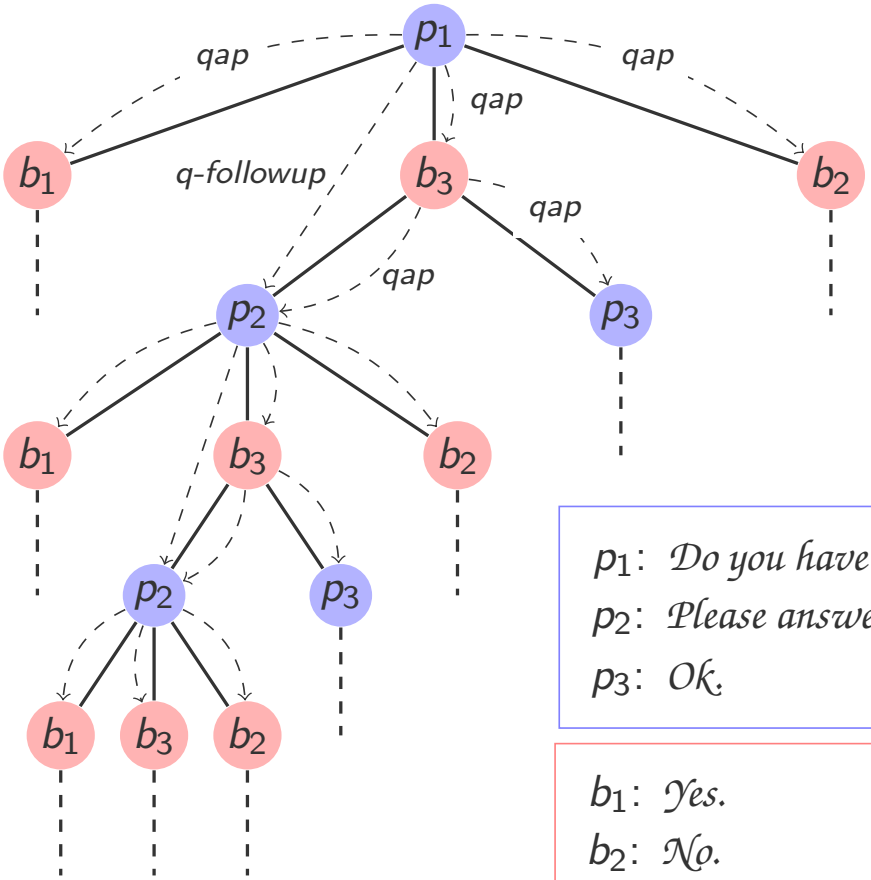

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Prosecutor has a winning strategy!

Properties of ME games I

- Potentially unbounded, no set end.
- Messages come with a conventionally associated meaning due to the constraints enforced by the Jury: an agent who asserts ϕ commits to its content.
- We can add a language of simple propositional modal logic over the language of SDRT [Asher, Venant'15]:

$$\neg\phi \mid \phi_1 \vee \phi_2 \mid C_i\phi,\ i \in \{0,1\} \mid C^*\phi$$

- Commitment is modeled as a Kripke modal operator via an alternativeness relation in a pointed model with a distinguished (actual) world w_0.

Properties of ME games II

- Each discourse move defines an action, that updates the model's commitment structure in the style of PAL.
- Entailment relation $\models$ that ensures $\phi \models C^* C_i \phi$.
- Usual axioms.
- Allows movement from sequence of discourse moves to sequence of updates on the model.

Past approaches

- Signaling games and its cousins
  - Signaling Games [Spence'73].
  - Persuasion Games [Rubinstein-Glazer'04].
  - Cheap Talk [Crawford-Sobel'82, Farrell'87].
  - Long Cheap Talk [Aumann-Hart'03, Aumann-Maschler'95, Forges'90].
- Limitations of the signaling framework
  - Does not take into account the linguistic structure of messages.
  - Works well when the interests are aligned (Gricean players) or partially aligned.
  - Non-cooperative setting: problem of interpretation.
  - With costs: no exchange in equilibrium; without costs: 'babbling equilibrium' [Crawford-Sobel'82].
  - Inherently asymmetric.

Advantages of our approach

- Exploits the rich linguistic content of messages.
- The interpretation problem is solved extrinsically
  Prosecutor: *I want to know if* **you** *ever had an account in Swiss banks. Answer my question directly.*
- Avoids undesirable backward induction arguments.
- The set of plays has a rich topological structure.
- The player goals based on linguistic evaluations by the Jury can be characterized in terms of their structural complexity
  - Consistency, Coherence, Responsiveness etc. are all FO($<$) definable.
  - CNEC is not FO($<$) definable.
- Indicative of the strategic complexity of the player goals.

Deciding the winner finitely

Questions

- How does the Jury determine the winning sets?
- When and how does the Jury decide to stop the game?

Enter Weighted ME games

- The Jury assigns a score or weight to every move of each player.
- The weights are history dependent.
- The final score is the 'discounted sum' of the individual weights.
- Discounting reflects the fact: "play your best cards first".
- Permits the Jury to stop the game after a fixed finite number of turns with the assurance that neither player can gain or lose more than a certain amount.

Imperfect information

- It is crucial that the players are uncertain about the Jury conditions.
- They are also uncertain about the strategies of the other players.
- What constitutes rational play? equilibrium concepts?
- Type theory [Harsanyi '68].

Types and beliefs I

- Each player i has a (possibly infinite) set of types T_i.
- Each type t_i of Player i has a (first-order) belief function $\beta_i(t_i)$ which assigns to t_i a probability distribution over the types of the other players and the Jury.
- The higher-order beliefs can be defined in a standard way.
- Each type t_i starts the game with an initial belief, called the 'prior belief'.

Types and beliefs II

- After each move, all the players dynamically update their beliefs through Bayesian updates.
- 'optimal strategies', 'best-response', 'rationality', 'common belief in rationality' etc. can then be defined.
- Equilibrium concepts: NE, iterated removal of dominated strategies, correlated equilibrium, rationalizability etc. can be explored in terms of the beliefs.

Back to Bronston-Prosecutor

(I) The company had an account in the Swiss banks **but** Bronston himself did not.

Types of the Jury:

tj_1	accepts (I) and clears him of all charges
tj_2	does not accept (I) and he is charged with contempt of court
tj_3	does not accept (I) but does not charge him yet either

Beliefs of the players:

	tp_1	tp_2	tp_3	tj_1	tj_2	tj_3
tb_1	0	1	0	1	0	0
tb_2	0	0	1	0	1	0
tb_2	0	1	0	0	0	1

	tb_1	tb_2	tb_3	tj_1	tj_2	tj_3
tp_1	1	0	0	1	0	0
tp_2	1	0	0	0	1	0
tp_2	0	0	1	0	0	1

σ_1	*yes*
σ_2	*no*
σ_3	indirect

τ_1	followup
τ_2	acknowledgement

Rational type-strategies:
Bronston: $(\cdot, \sigma_3)$
Prosecutor: $(tp_1, \tau_2), (tp_2, \tau_1), (tp_2, \tau_2), (tp_3, \tau_2)$

Inexistence results

Theorem (Hellman, Levy '03,'12,'13)

A Bayesian game with a non-separable type space may not have a (measurable) equilibrium.

- Guaranteeing equilibria:
  - Restricting types to subsets definable in a countable language (FO definable, natural language).
  - Restricting types to subsets generated by resource bounded agents.
    - Limited memory.
    - Limited computation power.
  - Debates usually have a 'moderator' to assign turns.