

Learning equilibria in monoidal computer

(5 dubious statements about outsmarting)

Dusko Pavlovic
University of Hawaii

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Outline

Dubious Statement 1

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Dubious

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Is there a logic of social behaviors?

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Is there a logic of social behaviors?

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And the herd of swine rushed towards the steep bank and threw themselves in the sea.

Matthew 8:30-37

Mark 5:1-20

Luke 8:27-38

Behaviors are counted, predicted and sold

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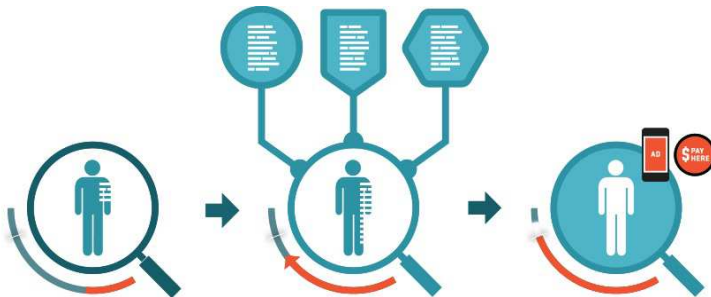
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Artificial Intelligence has been achieved

Not just computers behaving like people
but also people behaving like computers

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Working hypotheses

- ▶ math of social interactions
= game theory

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Working hypotheses

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- ▶ math of social interactions
= game theory
- ▶ $\text{people} \subseteq \text{computers}$
 $\implies \text{social behaviors} \subseteq \text{computations}$

Working hypotheses

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- ▶ math of social interactions
= game theory
- ▶ people \subseteq computers
 \implies social behaviors \subseteq computations
- ▶ **logic of social behaviors**
= game theory + computation

Trouble: game theory + computation =



- ▶ simple games have **complex strategies**
 - ▶ Rabin (1957): BR undecidable
 - ▶ Blass (1972): BR at any hyperarithmetic level

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Trouble: game theory + computation =



- ▶ simple games and regular (FSM implemented) strategies have **complex strategy search space**
 - ▶ Gilboa, Ben-Porath (1980s): BR search unfeasible
 - ▶ Nachbar (1990s): equilibrium undecidable

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Every problem is a game

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Every problem is a game

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Problem (Landau)

Are there infinitely many n such that $n^2 + 1$ is prime?

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Every problem is a game

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Game

- ▶ Alice picks a_1 .
- ▶ Bob picks b_1
- ▶ Alice picks a_2 .
- ▶ Bob picks b_2
- ▶ Alice picks a_3 .
- ▶ Alice wins if $(a_1 + b_1)^2 + 1 = (a_2 + 2)(a_3 + 2)$

Every problem is a game

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Strategies

- ▶ If n is the greatest number with $n^2 + 1$ prime, then Alice picks $a_1 > p$, — and wins.

Every problem is a game

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Strategies

- ▶ If n is the greatest number with $n^2 + 1$ prime, then Alice picks $a_1 > p$, — and wins.
- ▶ If there are infinitely many primes $p = n^2 + 1$, then Bob picks b_1 so that $(a_1 + b_1)^2 + 1$ is prime, — and wins.

Every problem is a game

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Strategies

- ▶ If n is the greatest number with $n^2 + 1$ prime, then Alice picks $a_1 > p$, — and wins.
- ▶ If there are infinitely many primes $p = n^2 + 1$, then Bob picks b_1 so that $(a_1 + b_1)^2 + 1$ is prime, — and wins.
- ▶ Since Landau's Problem is open, **we don't know who wins, or how hard it is to win.**

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Every problem is a game.

Corollary

- ▶ Finding simple games with hard strategies is like finding simple statements with hard proofs.
 - ▶ It shouldn't stop us.

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Corollary

- ▶ Finding simple games with hard strategies is like finding simple statements with hard proofs.
 - ▶ It shouldn't stop us.
- ▶ The fact that game theory + computation =  is the usual proof search challenge.
 - ▶ It's fun!

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- ### Dubious Statement 5

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Game theory in one slide

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$$\begin{array}{c}
 \frac{\prod_{i=1}^n A_i \xrightarrow{u_k} \mathbb{R}}{\prod_{k \neq i=1}^n A_i \xrightarrow{BR_k} A_k} \\
 \hline
 \prod_{i=1}^n A_i \xrightarrow{BR = \langle BR_k \circ \pi_k \rangle_{i=1}^n} \prod_{i=1}^n A_i \\
 \hline
 \begin{array}{ccc}
 1 & \xrightarrow{NE} & \prod_{i=1}^n A_i \\
 \searrow NE & & \nearrow BR \\
 & \prod_{i=1}^n A_i &
 \end{array}
 \end{array}$$

Position games

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$$\begin{array}{c}
 A \times X \xrightarrow{\langle u_k, s_k \rangle} \mathbb{R} \times X \\
 \hline
 A_{-k} \times X \xrightarrow{BR_k} A_k \\
 \hline
 A \times X \xrightarrow{BR = \langle BR_k \circ \pi_k \rangle_{i=1}^n} A \\
 \hline
 \begin{array}{ccc}
 X & \xrightarrow{NE} & A \\
 \searrow \langle NE, id \rangle & & \nearrow BR \\
 & A \times X &
 \end{array}
 \end{array}$$

where

$$A = \prod_{i \in n} A_i$$

$$X = \prod_{i \in n} X_i$$

$$A_{-i} = \prod_{\substack{k \in n \\ k \neq i}} A_k$$

Dubious Statement 2

**Utility is a red herring in game theory:
only used to derive strategies.**

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**Utility is a red herring in game theory:
only used to derive strategies.**

**Game theory is a theory of fixed points
of strategies.**

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Categories help with strategies.

Category of strategies

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Definition

Let \mathcal{C} be a cartesian category, and $\Delta : \mathcal{C} \longrightarrow \mathcal{C}$ a commutative monad over it.

The category $\mathcal{S} = \mathcal{S}_{\Delta\mathcal{C}}$ of Δ -strategies over \mathcal{C} consists of

- ▶ players $A = \langle M_A, S_A \rangle \in \mathcal{C}^2$
- ▶ strategies $(A \xrightarrow{\Phi} B) \in \mathcal{C}(M_A \times S_B, \Delta(M_B \times S_B))$

Category of strategies

Composition

$$\frac{A \xrightarrow{\Phi} B \quad B \xrightarrow{\Psi} C}{A \xrightarrow{\Phi; \Psi} C}$$

is given by

$$(\Phi; \Psi)_{a\gamma c\gamma'} = \sum_{\beta b} \Phi_{a\beta b\beta} \cdot \Psi_{b\gamma c\gamma'}$$

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Category of strategies

Composition

$$\frac{A \xrightarrow{\Phi} B \quad B \xrightarrow{\Psi} C}{A \xrightarrow{\Phi; \Psi} C}$$

is given by

$$\begin{aligned}(\Phi; \Psi)_{a\gamma c\gamma'} &= \sum_{\beta b} \Phi_{a\beta b\beta} \cdot \Psi_{b\gamma c\gamma'} \\ &= \sum_{\beta} \sum_b \Phi_{a\beta b\beta} \cdot \Psi_{b\gamma c\gamma'}\end{aligned}$$

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Category of strategies

Composition

$$\frac{M_A \otimes S_B \xrightarrow{\Phi} M_B \otimes S_B \quad M_B \otimes S_C \xrightarrow{\Psi} M_C \times S_C}{M_B \otimes S_C \xrightarrow{\Phi; \Psi} M_C \times S_C}$$

=

$$\begin{aligned} \text{Tr}_{S_B} \Big(S_B \otimes M_A \otimes S_C \cong M_A \otimes S_B \otimes S_C \xrightarrow{\Phi \otimes S_C} M_B \otimes S_B \otimes S_C \cong \\ \cong S_B \otimes M_B \otimes S_C \xrightarrow{S_B \otimes \Psi} S_B \otimes M_C \times S_C \Big) \end{aligned}$$

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Games of perfect and complete information

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$$S_A = S_B = (\mathbb{R} \times \mathbb{R})^{M_A \times M_B}$$

Best Response strategies

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$$\begin{aligned}\langle b, \sigma^A \rangle &\xrightarrow{\Sigma_A} \langle a, \sigma^A \rangle && \iff \forall x \in M_A. \sigma_{xb}^A \leq \sigma_{ab}^A \\ \langle a, \sigma^B \rangle &\xrightarrow{\Sigma_B} \langle b, \sigma^B \rangle && \iff \forall y \in M_B. \sigma_{ay}^B \leq \sigma_{ab}^B\end{aligned}$$

Nash equilibrium

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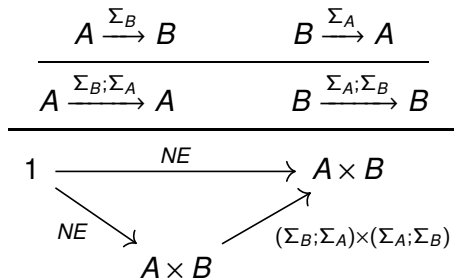
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Games of imperfect and complete information

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$$\begin{aligned}S_A &= P_A \times (\mathbb{R} \times \mathbb{R})^{M_A \times M_B} \\S_B &= P_B \times (\mathbb{R} \times \mathbb{R})^{M_A \times M_B}\end{aligned}$$

Games of perfect and incomplete information

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$$\begin{aligned} S_A &= \mathbb{R}^{M_A \times M_B} \times \Delta S_B \\ S_B &= \mathbb{R}^{M_A \times M_B} \times \Delta S_A \end{aligned}$$

Games of perfect and incomplete information

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$$S_A = S_B = \prod_{i=0}^{\infty} \Delta^i(\mathbb{R}^{M_A \times M_B})$$

Outsmarting

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$$[A, B] = [M_A \times \Delta[B, A], M_B \times \Delta[B, A]]$$

$$[B, A] = [M_B \times \Delta[A, B], M_A \times \Delta[A, B]]$$

where $[X, Y]$ are Y 's strategies against X

What is the math of outsmarting?

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String diagrams: Types

$$X \mid \quad A \mid \quad \mid B \quad D \mid$$
$$X \otimes A \otimes B \otimes D$$

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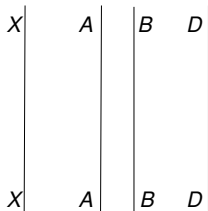
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String diagrams: Identities



$X \otimes A \otimes B \otimes D$

↑
id

$X \otimes A \otimes B \otimes D$

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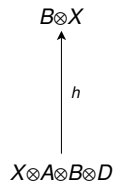
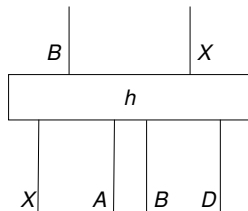
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String diagrams: Operations



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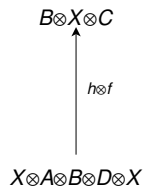
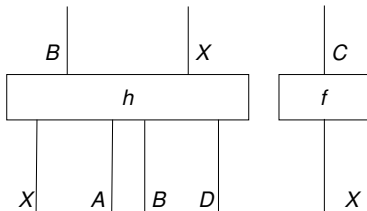
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String diagrams: Parallel composition



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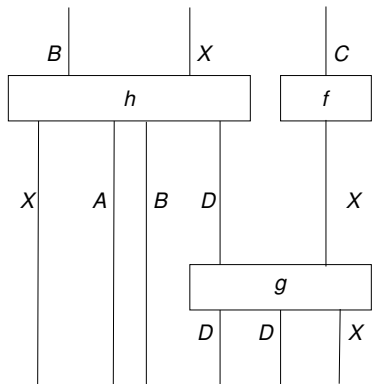
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String diagrams: Sequential composition

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$$\begin{array}{c}
 B \otimes X \otimes C \\
 \uparrow h \otimes f \\
 X \otimes A \otimes B \otimes D \otimes X \\
 \uparrow X \otimes A \otimes B \otimes g \\
 X \otimes A \otimes B \otimes D \otimes D \otimes X
 \end{array}$$

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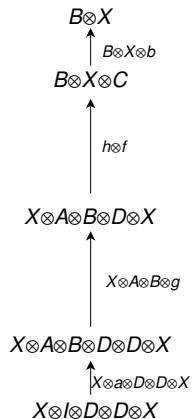
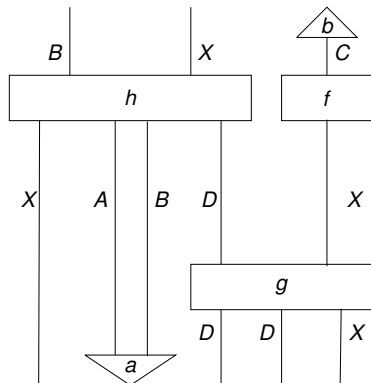
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String diagrams: Values and deletion

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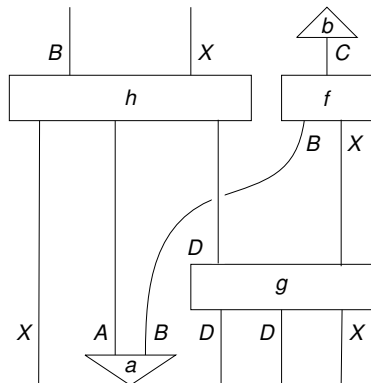
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String diagrams: Symmetry



$$\begin{array}{c}
 B \otimes X \\
 \uparrow B \otimes X \otimes b \\
 B \otimes X \otimes C \\
 \uparrow h \otimes f \\
 X \otimes A \otimes D \otimes B \otimes X \\
 \uparrow X \otimes A \otimes c \otimes X \\
 X \otimes A \otimes B \otimes D \otimes X \\
 \uparrow X \otimes A \otimes B \otimes g \\
 X \otimes A \otimes B \otimes D \otimes D \otimes X \\
 \uparrow X \otimes a \otimes D \otimes D \otimes X \\
 X \otimes I \otimes D \otimes D \otimes X
 \end{array}$$

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Data services: Copying and deletion

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- ▶ the *copying* operation $A \xrightarrow{\delta} A \otimes A$, and
- ▶ the *deleting* operation $A \xrightarrow{\top} I$,

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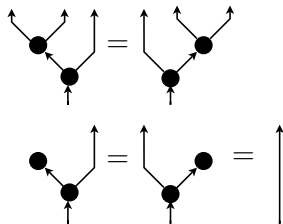
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which together form a *comonoid*, i.e. satisfy the equations

$$\delta; (\delta \otimes A) = \delta; (A \otimes \delta)$$

$$\delta; (\top \otimes A) = \delta; (A \otimes \top) = \text{id}_A$$



Data services: Comparison

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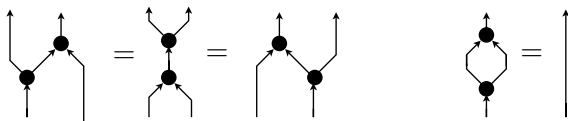
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- ▶ the *comparison* operation $A \otimes A \xrightarrow{\varrho} A$

which is required to be associative and thus makes A into a *semigroup*.

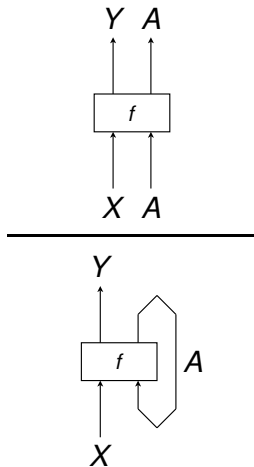
The copying and the comparison operations are further required to satisfy the *data distribution* conditions

$$(\delta \otimes A); (A \otimes \varrho) = \varrho; \delta = (A \otimes \delta); (\varrho \otimes A) \quad \delta; \varrho = \text{id}$$



Trace

$$\frac{X \otimes A \xrightarrow{f} Y \otimes A}{X \xrightarrow{\text{Tr}_{XY}^A f} Y}$$



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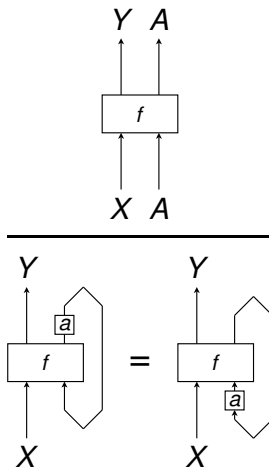
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Trace

$$\frac{X \otimes A \xrightarrow{f} Y \otimes A}{X \xrightarrow{\text{Tr}_{XY}^A f} Y}$$



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Fixpoint

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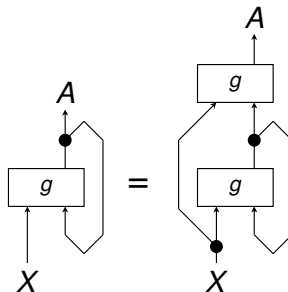
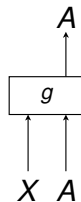
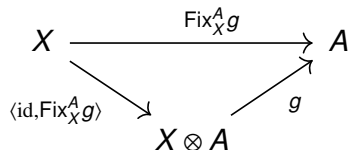
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$$X \otimes A \xrightarrow{g} A$$



Strategies as morphisms

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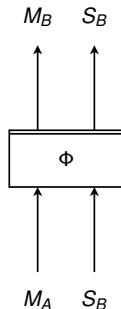
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$$A \xrightarrow{\Phi} B$$

Strategies as morphisms

$$M_A \otimes S_B \xrightarrow{\Phi} \Delta(M_B \otimes S_B)$$



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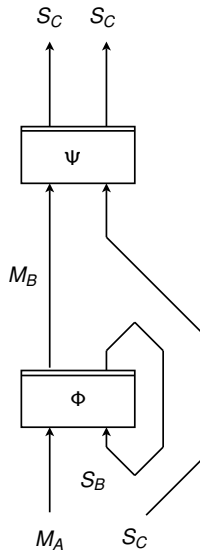
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Composition of strategies

$$(\Phi; \Psi)_{a\gamma c\gamma'} = \sum_{\beta b\beta} \Phi_{a\beta b\beta} \cdot \Psi_{b\gamma c\gamma'}$$



Dubious

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Statement 1

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Dubious
Statement 5

Nash equilibrium

Dubious

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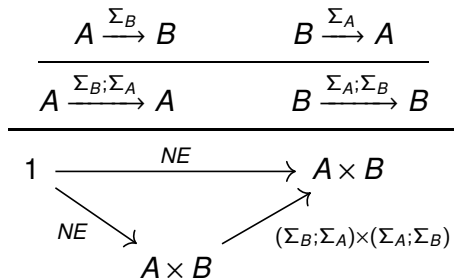
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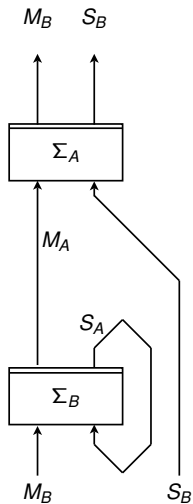
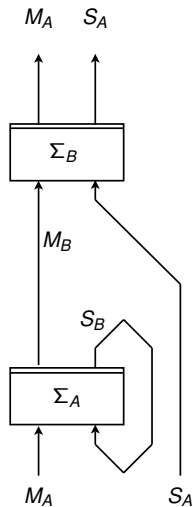
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Statement 5



Nash equilibrium



Dubious

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Dubious
Statement 1

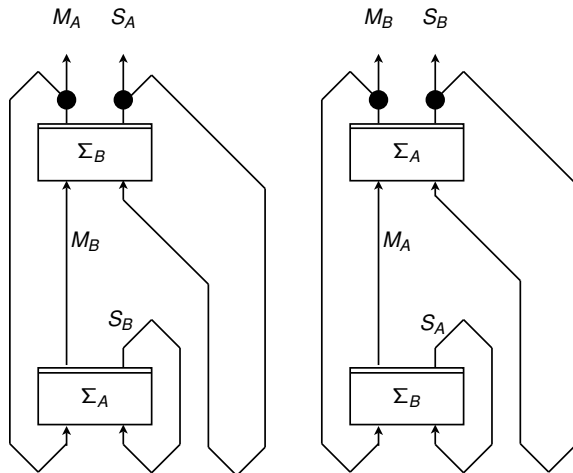
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Nash equilibrium



Dubious

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Dubious
Statement 1

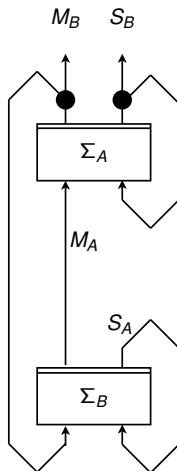
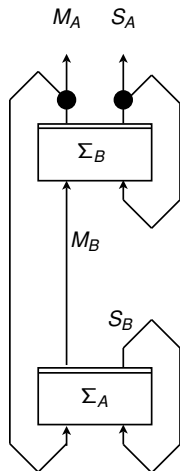
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Nash equilibrium



Dubious

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Explicit beliefs

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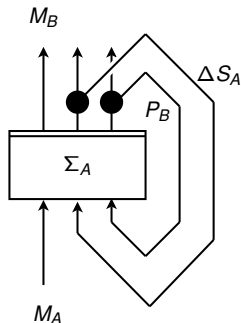
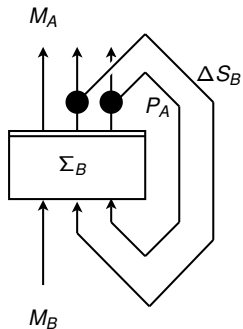
Dubious
Statement 4

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$$S_A = P_A \times \Delta S_B$$

$$S_B = P_B \times \Delta S_A$$

Explicit beliefs equilibrium



Dubious

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Outsmarting

Dubious

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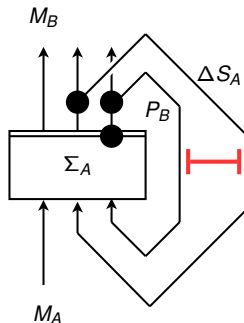
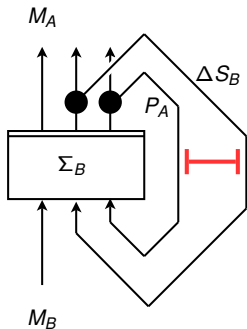
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Outsmarting

Dubious

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- ▶ static types (Harsanyi)
 - ▶ play their preferences
 - ▶ observe the same
 - ▶ cannot agree to disagree (Aumann)
- ▶ dynamic types
 - ▶ must disagree
 - ▶ deceive, posture, mimic. . .
 - ▶ create false posteriors

Dubious Statement 3

Dubious

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Equilibrium (even coordinated)
consists of individual fixed points
and **can perpetrate false beliefs.**

Corollary

Outsmarting pumps complexity.

Dubious

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Outline

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Outsmarting pumps complexity

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- ▶ Simple games require complex computation.

Matching pennies

	0	1
0	1, -1	-1, 1
1	-1, 1	1, -1

Dubious

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Penalty Kick

Dubious

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Same game

Rules

- ▶ repeated infinitely
- ▶ loser has bounded expected payoff

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Not losing is easy

Dubious

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Alice's reasoning

Suppose that I play

- ▶ 1 with a frequency $p \in [0, 1]$,
- ▶ 0 with a frequency $1 - p$.

Then

- ▶ $p < \frac{1}{2}$, Bob gets $(1 - p) - p = 1 - 2p > 0$ playing 1,
- ▶ $p > \frac{1}{2}$, Bob gets $p - (1 - p) = 2p - 1 > 0$ by playing 0,
- ▶ $p = \frac{1}{2}$, Bob's gets $1 - 2p = 2p - 1 = 0$.

Not losing is easy

Bob's reasoning

(same)

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Not losing is easy

Both players must randomize

If either player's moves are predictable,
then the other one can win.

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But what does it mean to randomize?

- ▶ Suppose that Bob plays

01

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But what does it mean to randomize?

- ▶ Suppose that Bob plays

01

- ▶ Alice predicts that Bob will play 0, and she plays 0.

Dubious

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But what does it mean to randomize?

- ▶ Suppose that Bob plays

01

- ▶ Alice predicts that Bob will play 0, and she plays 0.
 - ▶ But what if Bob plays 1?

Dubious

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But what does it mean to randomize?

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- ▶ Suppose that Bob plays

01

- ▶ Alice predicts that Bob will play 0, and she plays 0.
 - ▶ But what if Bob plays 1?

- ▶ If Alice thinks probabilistically, she notices that a fair coin is as likely to generate the above string as e.g.

1101000100110101001011100100000100000010

or any other sequence of 40 bits.

Probability does not distinguish events

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Probabilities

- ▶ talk about *ensembles*
- ▶ cannot tell apart *individual strings*

For probabilistic players mixing is obsolete

- ▶ If Bob believes that Alice thinks probabilistically, then he has no reason to randomize
 - ▶ (since Alice cannot tell)

Dubious

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For probabilistic players mixing is obsolete

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- ▶ If Bob believes that Alice thinks probabilistically, then he has no reason to randomize
 - ▶ (since Alice cannot tell)
- ▶ If Alice believes that Bob thinks probabilistically, then she has no reason to randomize
 - ▶ (since Bob cannot tell)

Sleeping players

- ▶ If the players *agree* to use only
 - ▶ probability theory, and
 - ▶ knowledge logic

then they will both commit fixed strategies,
and go to sleep.



Alice wakes up



"Let me look for regularities in Bob's strategy!"

Dubious

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Trying to win

- ▶ Check if the frequency of 0 and 1 is $\frac{1}{2}$.

Dubious

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Trying to win

- ▶ Check if the frequency of 0 and 1 is $\frac{1}{2}$.
 - ▶ detect that 00000000 ... is not random
 - ▶ not that 0101010101 ... is not random

Dubious

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Trying to win

Dubious

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- ▶ Check if the frequency of 0 and 1 is $\frac{1}{2}$.
 - ▶ detect that 00000000 ... is not random
 - ▶ not that 0101010101 ... is not random
- ▶ Check if the frequencies of 00, 01, 10 and 11 are $\frac{1}{4}$
 - ▶ detect that 010101010101 ... is not random
 - ▶ not that 0001101100011011 ... is not random

Trying to win

- Check if the frequency of each $b_1 b_2 \cdots b_n$ is $\frac{1}{2^n}$.

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Trying to win

- ▶ Check if the frequency of each $b_1 b_2 \cdots b_n$ is $\frac{1}{2^n}$.
 - ▶ The regularity within each 2^n will be detected (after infinite amount of time)

Dubious

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Trying to win

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- ▶ Check if the frequency of each $b_1 b_2 \cdots b_n$ is $\frac{1}{2^n}$.

- ▶ The regularity within each 2^n will be detected (after infinite amount of time)
- ▶ The regularity of

011011100101110111100010011010101111001101...

will remain undetected for every n

Trying to win

Dubious

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- ▶ Check if the frequency of each $b_1 b_2 \cdots b_n$ is $\frac{1}{2^n}$.

- ▶ The regularity within each 2^n will be detected (after infinite amount of time)
- ▶ The regularity of

011011100101110111100010011010101111001101...

will remain undetected for every n

- ▶ 0,1,2,3... in the binary notation, concatenated

Trying to win

General problem: Regular substrings

Bob may

- ▶ randomize even bits, and use a rule for the odd bits
- ▶ randomize a substring and use a rule for the rest

Dubious

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Statement 1

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Trying to win

General problem: Regular substrings

Bob may

- ▶ randomize even bits, and use a rule for the odd bits
- ▶ randomize a substring and use a rule for the rest

General task

Alice must

- ▶ check ***every substring*** for regularities

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Question

Dubious

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But what is regularity?

Regularity is programmability

Dubious

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- ▶ Let \mathcal{L} be a programming language, e.g.
 - ▶ finite state machines
 - ▶ Turing machines
 - ▶ Python, Scala, Java ...
- ▶ Let $h : \mathcal{L}^* \rightarrow \mathcal{L}^*$ be an \mathcal{L} -program interpreter.

Regularity is programmability

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Statement 1

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- ▶ Let \mathcal{L} be a programming language, e.g.
 - ▶ finite state machines
 - ▶ Turing machines
 - ▶ Python, Scala, **Java**...
- ▶ Let $h : \mathcal{L}^* \rightarrow \mathcal{L}^*$ be an \mathcal{L} -program interpreter.

Regularity is compressibility

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Statement 1

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01 =
 $h\left(\text{for}(i = 0; i < 20; i++)\{\text{print } 01\}\right)$

Regularity is compressibility

Dubious

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$$011011100101110111100010011010101111001101 = h\left(\textit{for}(i = 0;; i++)\{\textit{print } i\}\right)$$

Randomness is incompressibility

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Statement 5

$$1101000100110101001011100100000100000010 = h(\textit{print } 1101000100110101001011100100000100000010)$$

(Monoidal computer)

Dubious

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Definition

Monoidal computer is a strict symmetric monoidal category \mathbb{C} with

- (i) data services
- (ii) a distinguished type \mathbb{P} ("of programs")

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(Monoidal computer)

Definition

Monoidal computer is a strict symmetric monoidal category \mathbb{C} with

(iii) *universal evaluators* u^{AB} such that

$$\forall f \exists F \quad \begin{array}{c} B \\ \uparrow \\ \boxed{f} \\ \uparrow \\ A \end{array} = \begin{array}{c} B \\ \uparrow \\ \begin{array}{c} \diagup \quad \diagdown \\ \boxed{u^{AB}} \\ \diagdown \quad \diagup \end{array} \\ \uparrow \\ \begin{array}{c} \triangleleft \quad \triangleright \\ \boxed{F} \\ \triangleright \quad \triangleleft \end{array} \\ \uparrow \\ A \end{array}$$

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(Monoidal computer)

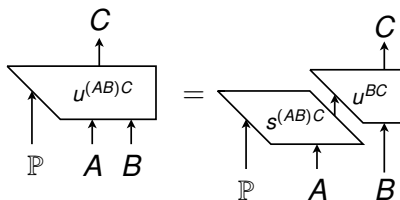
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Definition

Monoidal computer is a strict symmetric monoidal category \mathbb{C} with

(iv) *partial evaluators* $s^{(AB)C}$ such that



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Strategy for matching pennies

Dubious

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Definition

An \mathcal{L} -detector is an \mathcal{L} -function $h : \mathcal{Z}^* \rightarrow \mathcal{Z}^*$ such that

$$h(\vec{x}) = \vec{y} \implies \ell(\vec{x}) < \ell(\vec{y})$$

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Statement 1

Dubious
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Strategy for matching pennies

Dubious

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Definition

An \mathcal{L} -detector is an \mathcal{L} -function $h : \mathcal{Z}^* \rightarrow \mathcal{Z}^*$ such that

$$h(\vec{x}) = \vec{y} \implies \ell(\vec{x}) < \ell(\vec{y})$$

A bitstring $\vec{y} \in \mathcal{Z}^*$ is h -regular at the level n if

$$\exists \vec{x}. h(\vec{x}) \sqsupseteq \vec{y} \wedge \ell(\vec{y}) - \ell(\vec{x}) > n$$

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Strategy for matching pennies

Dubious

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Definition

An \mathcal{L} -detector is an \mathcal{L} -function $h : \mathcal{Z}^* \rightarrow \mathcal{Z}^*$ such that

$$h(\vec{x}) = \vec{y} \implies \ell(\vec{x}) < \ell(\vec{y})$$

A bitstring $\vec{y} \in \mathcal{Z}^*$ is h -regular at the level n if

$$\exists \vec{x}. h(\vec{x}) \sqsupseteq \vec{y} \wedge \ell(\vec{y}) - \ell(\vec{x}) > n$$

The h -regularity degree of \vec{y} is

$$\sigma_h(\vec{y}) = \max \{ n \mid \exists \vec{x}. h(\vec{x}) \sqsupseteq \vec{y} \wedge \ell(\vec{y}) - \ell(\vec{x}) \geq n \}$$

Dubious
Statement 1

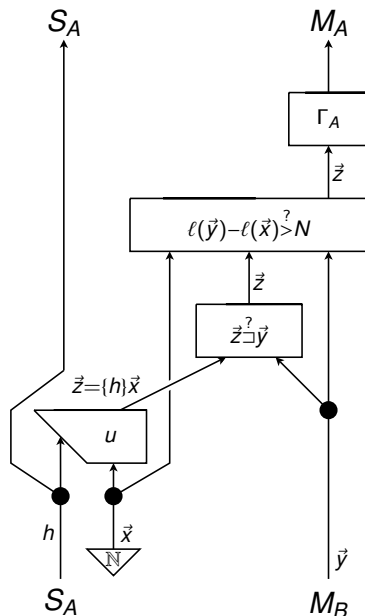
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Strategy for matching pennies



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Universal detector

Dubious

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Definition

An \mathcal{L} -detector $u : \mathcal{Z}^* \rightarrow \mathcal{Z}^*$ is *universal* if u -testing can detect all strings that are detected as h -regular with respect to any \mathcal{L} -detector h .

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Definition

An \mathcal{L} -detector $u : \mathcal{Z}^* \rightarrow \mathcal{Z}^*$ is *universal* if u -testing can detect all strings that are detected as h -regular with respect to any \mathcal{L} -detector h .

More precisely, for every \mathcal{L} -detector $h : \mathcal{Z}^* \rightarrow \mathcal{Z}^*$ there is a constant c_h such that for every bitstring \vec{y} holds

$$\sigma_h(\vec{y}) \leq c_h + \sigma_u(\vec{y})$$

Universal detector

Dubious

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Proposition

If \mathcal{L} is a Turing complete language, then there is a universal \mathcal{L} -detector.

Dubious
Statement 1

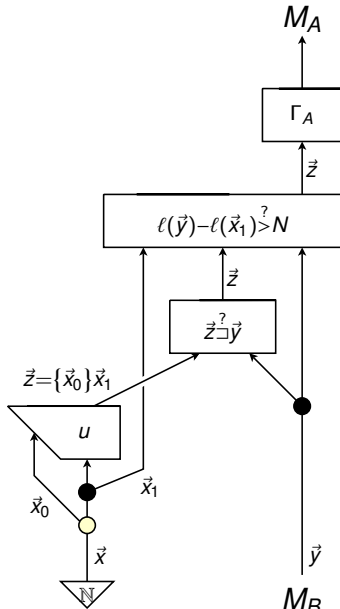
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Universal strategy for matching pennies



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Dubious Statement 4

Dubious

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**Winning pennies can be arbitrarily hard.
(It requires a universal computer.)**

Outline

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Learning strategy

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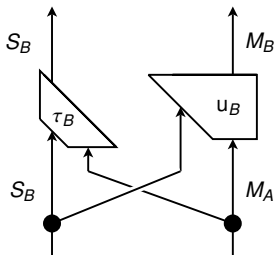
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Adaptation strategy?

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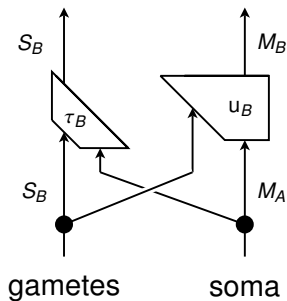
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Hereditary adaptation vs Random mutation

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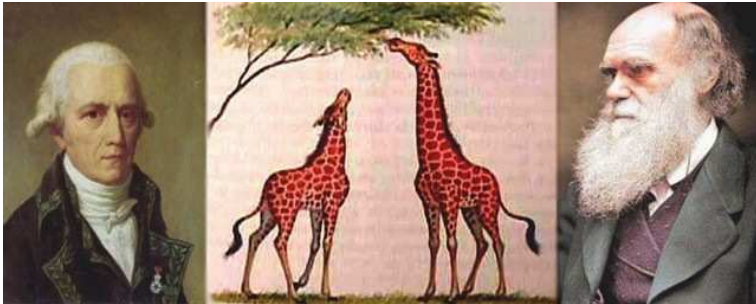
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Central Dogma of Molecular Biology

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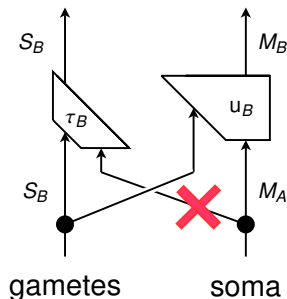
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Central Dogma of Molecular Biology

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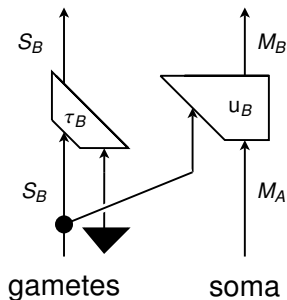
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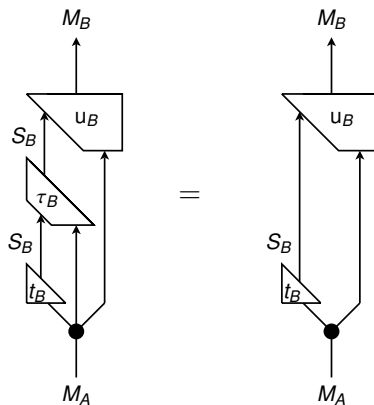
Graduate strategy

Dubious

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Proposition

Every learning strategy τ_B has a fixed point, i.e. a state (type) t_B at which nothing more can be learned:



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Learning equilibrium

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Corollary

For every learning profile there is a graduate profile, where the players cannot improve their winnings by learning.

Dubious Statement 5

Dubious

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Dubious
Statement 1

Dubious
Statement 2

Dubious
Statement 3

Dubious
Statement 4

Dubious
Statement 5

**All algorithmic learning strategies
reach equilibrium in one step.**

Corollary

All ontogenetic adaptations that are inheritable can be expressed within one generation.

Dubious

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Statement 2

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Statement 3

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Corollary

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Statement 5

All ontogenetic adaptations that are inheritable can be expressed within one generation.

- ▶ E.g., the epigenetic flows do not present filogenetic adoption of ontogenetic adaptations, but just triggers to express previously developed and stored filogenetic adaptations.

The Problem of Overfitting

- ▶ statistical overfitting: narrow statistic, eager training
- ▶ evolutionary overfitting: locked into an environment
- ▶ market model overfitting: windfall
- ▶ control overfitting: blind feedback

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The Problem of Overfitting

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- ▶ statistical overfitting: narrow statistic, eager training
- ▶ evolutionary overfitting: locked into an environment
- ▶ market model overfitting: windfall
- ▶ control overfitting: blind feedback
 - ▶ healthy heart is a chaotic system: interfering feedback loops
 - ▶ convergence is heart attack:
stability preempts adaptation

Dubious Statements 6–11

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Too much reflection leads to overfitting.

Dubious Statements 6–11

Dubious

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Too much reflection leads to overfitting.

In the long run, ignorance is bliss.