A Polynomial Time Algorithm for the Lambek Calculus with Brackets of Bounded Order

Max Kanovich, Stepan Kuznetsov, Glyn Morrill, Andre Scedrov

The Lambek Calculus $\mathbf{L}^*$

[Lambek 1958, 1961]

$$A \to A$$

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$$\frac{\Gamma \to A \quad \Delta \to B}{\Gamma, \Delta \to A \cdot B} \qquad \frac{\Gamma, A, B, \Delta \to C}{\Gamma, A \cdot B, \Delta \to C}$$

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- A fragment of non-commutative intuitionistic linear logic.

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- A fragment of non-commutative intuitionistic linear logic.
- We consider the variant of the Lambek calculus that allows empty antecedents.

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The Lambek Calculus with Brackets ($\mathbf{Lb}^*$)

[Morrill 1992, Moortgat 1995]

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- Brackets introduce *controlled non-associativity.*
- Cut elimination proved by Moortgat [1996].

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  In linguistic applications, R is small.
- Pentus' algorithm extended to $\mathbf{Lb}^*$, running time $\mathrm{poly}(N, 2^R, N^B)$ **[this talk].**
  Here B is the bracket nesting depth.

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  $\mathrm{ord}(p_i) = 0$; $\mathrm{ord}(A \cdot B) = \max\{\mathrm{ord}(A), \mathrm{ord}(B)\}$;
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  If we translate Lambek formulae into linear logic, R is the maximal alternation depth of $⅋$'s and $\otimes$'s.

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- $N = \|\Pi \to A\|$, counted as the total number of connectives (incl. bracket modalities) and brackets in the sequent.
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- Theoretical interest: a combination of proof net and finite automata techniques

Proof Nets for **Lb***

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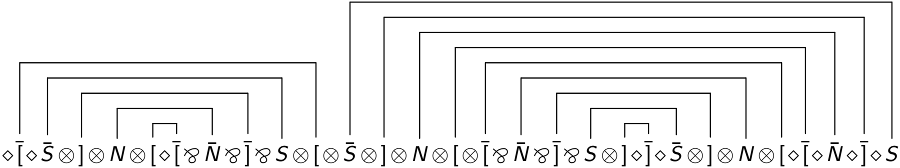
Axiom links:



$$\diamond \bar{[} \diamond \bar{S} \otimes] \otimes N \otimes [\diamond \bar{[} \parr \bar{N} \parr \bar{]} \parr S \otimes [\otimes \bar{S} \otimes] \otimes N \otimes [\otimes \bar{[} \parr \bar{N} \parr \bar{]} \parr S \otimes] \diamond \bar{]} \diamond \bar{S} \otimes] \otimes N \otimes [\diamond \bar{[} \diamond \bar{N} \diamond \bar{]} \diamond S$$

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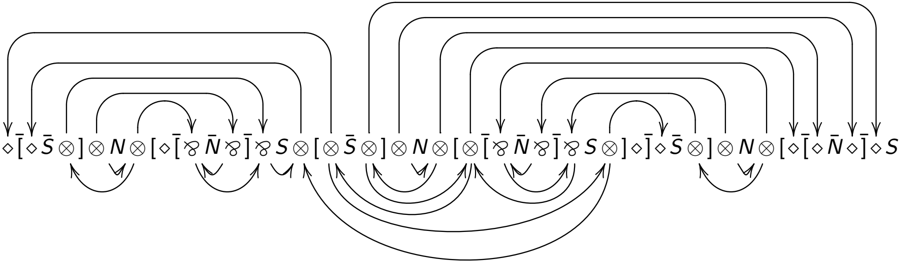
Acyclicity [Pentus 1998]:



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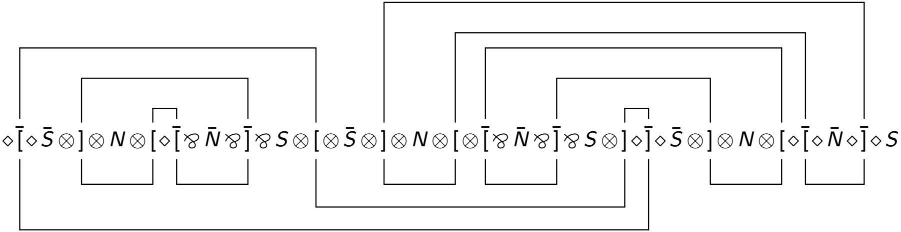
Sisterhood [Fadda and Morrill 2005]:



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- Sisterhood condition: $\mathcal{E}$ should respect pairing of brackets.

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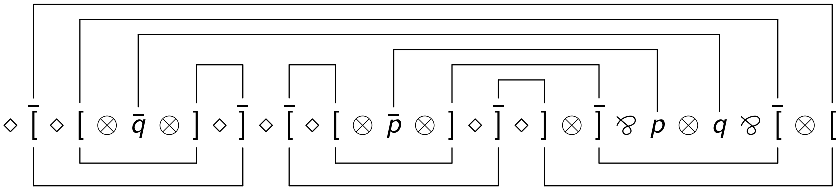
- The only possible proof net violates sisterhood:



Encoding Proof Nets

- $\mathcal{E} \rightsquigarrow c(\mathcal{E}) \in \{c_1, \ldots, c_n\}^n$:
  if the i-th and the j-th atoms are connected by an axiom link, then the i-th letter of $c(\mathcal{E})$ is e_j and the j-th letter is e_i.

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- There exists, and can be effectively constructed, a polynomial-size context free grammar for $P_1 \cap P_2$ (see [Ginsburg 1966]).
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Encoding Proof Nets

- $\mathcal{E} \rightsquigarrow c(\mathcal{E}) \in \{c_1, \ldots, c_n\}^n$:
  if the i-th and the j-th atoms are connected by an axiom link, then the i-th letter of $c(\mathcal{E})$ is e_j and the j-th letter is e_i.
- $P_1 = \{c(\mathcal{E}) \mid \mathcal{E}$ is a proof net, but maybe violating sisterhood$\}$
  There exists, and can be effectively constructed, a $\mathrm{poly}(n, 2^R)$-size context free grammar for P_1 [Pentus 2010].
- For a language P_2, such that $c(\mathcal{E}) \in P_2$ iff $\mathcal{E}$ respects sisterhood, there exists, and can be effectively constructed, a $\mathrm{poly}(n, n^B)$-size finite automaton for P_2.
- There exists, and can be effectively constructed, a polynomial-size context free grammar for $P_1 \cap P_2$ (see [Ginsburg 1966]).
- The non-emptiness of $P_1 \cap P_2$ (our goal) is checked in polynomial time.
- Notice that P_1 and P_2 are *finite,* so $P_1 \cap P_2$ is trivially context free. The real achievement is *polynomiality* of the grammar.

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- Feasible fragments of other enrichments of the Lambek calculus (even generally undecidable).

Thank you!