Formalizations of social choice theory in modal logic

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We focus on formalizations in modal logic.

Modal logics for social choice

We consider the following three logical systems for social choice:

- modal logic of judgment aggregation[1], which in particular formalizes preference aggregation, i.e. social welfare functions
- modal logic of social choice functions[2], which only choose winner from individual preferences
- logic of knowledge and voting[3], aimed to express some strategic aspects of voting

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[1] T. Ågotnes, W. van der Hoek, and M. Wooldridge. On the logic of preference and judgment aggregation. *Journal of Autonomous Agents and Multi-Agent Systems*, 22:4–30, 2011

[2] N. Troquard, W. van der Hoek, and M. Wooldridge. Reasoning about social choice functions. *Journal of Philosophical Logic*, 40:473–498, 2011

[3] Z. Bakhtiarinoodeh. *The Dynamics of Incomplete and Inconsistent Information: Applications of Logic, Algebra and Coalgebra*. PhD thesis, University of Lorraine, Nancy, 2017

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Judgment set R_i represents judgments of agent i, while $F(R)$ represents resulting collective judgment.

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If we are only interested in the winner (e.g. of an election), we consider social choice functions (SCF), which map each profile to an alternative (instead of an ordering of alternatives).

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For $C \subseteq N$, we denote $p_C := \bigwedge_{i \in C} p_i \wedge \bigwedge_{i \in N \setminus C} \neg p_i$ (“exactly voters from C judge that A holds”).

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Arrow's Theorem

Denote the formulas from previous examples as follows:

- $Pareto := \Box\blacksquare(p_1 \wedge \cdots \wedge p_n \rightarrow \sigma)$,
- $IIA := \Box\blacksquare \bigwedge_{C \subseteq N}(p_C \wedge \sigma \rightarrow \Box(p_C \rightarrow \sigma))$,
- $Dictatorial := F \Vdash \bigvee_{i \in N} \Box\blacksquare(p_i \rightarrow \sigma)$.

We can now express (instances of) Arrow's impossibility theorem (if there are more then two alternatives, there is no non-dictatorial SWF that satisfies the Pareto condition and IIA): if $|M| \geqslant 3$, then $\Vdash \neg(Pareto \wedge IIA \wedge \neg Dictatorial)$. Ågotnes et al. make some steps towards a formal Hilbert-style proof. Later, I developed a natural deduction system for JAL and provided a formal proof of Arrow's Theorem[4].

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[4] T. Perkov. Natural deduction for modal logic of judgment aggregation. *J. Log. Lang. Inf.*, 25:335–354, 2016

Modal logic of social choice functions

Troquard et al. developed a simpler system, aimed to formalize preference aggregation, instead of judgment aggregation in general, furthermore considering SCF's instead of SWF's, which enables further simplification.
Nevertheless, it is sufficiently expressive to formalize classical results of social choice theory, as demonstrated by Cinà and Endriss, who provided formal proofs of Arrow's Theorem and some other results using this system.[5]

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[5] G. Cinà and U. Endriss. Proving classical theorems of social choice theory in modal logic. *Journal of Autonomous Agents and Multi-Agent Systems*, 30:963–989, 2016

Syntax and semantics

We present a fragment used by Ciná and Endriss. As in the case of JAL, the language is parametrized by N and M. The language has:

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- $F, R \Vdash p_{i,x,y}$ iff $(x, y) \in R_i$ ("i prefers x to y"),

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Syntax and semantics

We present a fragment used by Ciná and Endriss. As in the case of JAL, the language is parametrized by N and M. The language has:

- a propositional variable $p_{i,x,y}$ for each $i \in N, x, y \in M$,
- a propositional variable x for each $x \in M$,
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- $F, R \Vdash \Diamond_C \varphi$ iff there is R' s.t. $R'_i = R_i$ for all $i \notin C$ and $F, R' \Vdash \varphi$. ("C can force φ by a change of vote, assuming others do not change the vote")

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- a social choice function F satisfies the Pareto condition iff $F \Vdash Pareto' := \bigwedge_{(x,y)\in X^2}(p_{1,x,y} \wedge \cdots \wedge p_{n,x,y} \to \neg y)$

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Similarly as in the case of JAL, Arrow's Theorem now formalizes as: if $|M| \geqslant 3$, then $\Vdash \neg(\mathit{Pareto}' \wedge \mathit{IIA}' \wedge \neg \mathit{Dictatorial}')$.

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- profiles and alternatives as propositional variables
- formulas are built using Boolean connectives, epistemic modalities K_i for each $i \in N$ and public announcement modalities $[\varphi]$ for each formula of the language

Semantics of LKV

- possible worlds w are epistemic alternatives, in fact they correspond to profiles, but there may be several copies of the same profile. Denote by $R(w)$ the profile corresponding to w.

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- $F, w \Vdash [\varphi]\psi$ iff $F, w \Vdash \varphi$ implies that ψ holds in the model restricted only to worlds in which φ holds ("after the public announcement of φ, ψ holds")