

# Beyond Geometric Validity: Two versions of relative validity

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# Overview

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- 3 Validity and Interpretation Strategies
- 4 Speech Acts
- 5 Dialogues, Reasons and Elementary Dialogues
- 6 Anthropological Validity

## Decision Making vs. Speech/Text Interpretation

- Decision making is concerned with “valid” sequences of inferences based on accepted information, argumentation schemas (topoi), priorities, argument formation rules etc.
- Speech/Text interpretation requires for a “definition” of the notion of argument, encompassing arguments that may be neither valid, nor even sequences of inferences.

## A “Wider” Definition, but...

A definition nonetheless. We still need to be able to distinguish between the parts of a dialogue which are argumentative and those that constitute just “noise”.

# Analytical Reconstruction of Argumentative Discourse

- We want to systematically reconstruct real-life dialogues, in order to bring their argumentative content to the surface.
- The argumentative reality contains too much noise; neither every utterance serves an argumentative purpose, nor, those who actually do, are always what they seem to be.
- We translate utterances to speech - acts, guided by the function of the utterance in the dialogue.
- In many cases, pragmatics offer us more than one translation, thus we need suitable criteria in order to decide which is the “correct” one.

## Versions of Validity

- *Geometrical Validity*: The standard notion of validity, referring to the conclusion being, in parts, included to the premises. This notion of validity is most commonly used in logic and mathematics.
- *Critical Validity*: The acceptance of some claim, based on the lack of expressed doubt towards it. This notion of validity is used in dialectics.
- *Anthropological Validity*: The identification of validity with persuasiveness of some argument for some claim, towards some specific audience. This notion of validity is inherently relativistic and is used in rhetoric.

# Interpretation Strategies

- Reconstructing the dialogue so that “partial” (logical) arguments become geometrically valid.
- Interpreting an utterance guided by the feedback offered by the opposing side, so that the former may be critically valid.
- Making interpretation choices guided by hypothesis regarding the rhetorical strategies of the arguer.

# Speech Acts Theory

- Introduced by John Searle in “Speech Acts: An Essay in the Philosophy of Language” (1970)
- “Speaking” is treated as “acting” and utterances are seen as having a propositional content, as well as a function within the argumentative discourse.
- Not all of them serve an argumentative purpose.
- Argumentatively “useful” speech acts are:
  - assertives
  - (positive) commissives
  - (negative) commissives
  - (challenge) directives
  - (clarification) directives



## Why Speech Acts?

- They offer a wide framework for the study of every version of human interaction
- By interpreting utterances on the grounds of their function, a formalism of speech acts enables us to formalize rhetorical strategies.

## Prerequisites - Language of Propositions and “Meta-language” of Argumentation Schemes

- First order predicate language  $\mathbb{L}$ .
- Contains “=” and “ $\in$ ”.
- Enumerably infinite constants denoting natural numbers.
- Argumentation schemes expressed informally as conditionals in the language of propositional calculus.
- The symbol “ $\equiv$ ” is used to denote the syntactic identity between two elements of  $\mathbb{L}$ .
- The set of functionality indicators for speech acts,  $\mathbf{S} = \{\mathbf{a}, \mathbf{cy}, \mathbf{cn}, \mathbf{dq}, \mathbf{dc}\}$ . We will use the informal symbol “ $=_s$ ” to denote that two indicators are syntactically identical.

## Prerequisites - Referring to Propositions and Argumentation Schemes in $\mathbb{L}$

- Pre-constructed injection from the elements of  $\mathbb{L}$  to the set of odd numbers,  $d(p), p \in \mathbb{L}$
- Pre-constructed injection from the set of the propositional types  $L$  of the form  $P_0 \wedge \dots \wedge P_m \rightarrow P$  to the set of even numbers,  $D(L)$ .
- By  $\Sigma$  we will denote a finite set of even numbers, usually the mutually accepted argumentation schemes.
- By  $\Pi$  we will denote a finite set of odd numbers, usually the mutually accepted premises.

# Speech Acts Formalism

## Definition

Let  $\sigma = \langle p, x \rangle$  be an ordered couple.  $\sigma$  will be called a **speech act**, if it satisfies at least one of the following conditions:

- ①  $x =_s \mathbf{a}$  and  $p \in \mathbb{L}$  (assertive speech act - claim)
- ②  $x =_s \mathbf{cy}$  and  $p \in \mathbb{L}$  (commisive speech act - acceptance)
- ③  $x =_s \mathbf{cn}$  and  $p \in \mathbb{L}$  (commisive speech act - doubt)

...

We will call  $x$  the function of  $\sigma$  and denote by  $\mathbf{F}(\sigma)$ , while  $p$  will be called the proposition of  $\sigma$  and will be denoted using  $\mathbf{P}(\sigma)$ .

# Speech Acts Formalism

## Definition

Let  $\sigma = \langle p, x \rangle$  be an ordered couple.  $\sigma$  will be called a **speech act**, if it satisfies at least one of the following conditions:

...

- ④  $x =_s \mathbf{dc}$  and  $p \in \mathbb{L}$  (directive speech act - challenge to defend a claim)

We will call  $x$  the function of  $\sigma$  and denote by  $\mathbf{F}(\sigma)$ , while  $p$  will be called the proposition of  $\sigma$  and will be denoted using  $\mathbf{P}(\sigma)$ .

# Speech Acts Formalism

## Remark

*For the rest of this presentation, we will focus on the first four cases of speech acts, i.e., claims, acceptances, doubts and challenges to defend some claim. However, our formalism may easily be expanded in order to include every kind of speech act.*

## Notation

*The set of all speech acts will be denoted by  $\mathbb{SA}$ .*

# Dialogues

## Definition

Let  $n, m \in \mathbb{N}$  and  $\Delta = \langle (a_i)_{i=0}^n, (b_i)_{i=0}^m \rangle$ , where, for every  $i = 0, \dots, n$  and  $j = 0, \dots, m$ ,  $a_i, b_j \in \mathbb{SA}$  and  $\mathbf{F}(a_i), \mathbf{F}(b_j) \in \{\mathbf{a}, \mathbf{cy}, \mathbf{cn}, \mathbf{dc}\}$ . Then,  $\Delta$  will be called a **dialogue**.

## Definition

Let  $n, m, \in \mathbb{N}$  and  $\Delta = \langle (a_i)_{i=0}^n, (b_i)_{i=0}^m \rangle$  be a dialogue, such that  $n = m$ . The **set of exchanges** of  $\Delta$  will be denoted by  $E(\Delta)$ , and it is defined as follows

$$\{(a_i, b_i) \mid i = 0, \dots, n\}$$

## Complete and Orderly Dialogues

### Definition

Let  $n, m \in \mathbb{N}$  and  $\Delta = \langle (a_i)_{i=0}^n, (b_i)_{i=0}^m \rangle$  be a dialogue, such that  $n = m$ . Then,  $\Delta$  will be called a **complete dialogue**. Moreover, if all of the following conditions hold for the exchanges of  $\Delta$ :

- ① For every  $i \in \mathbb{N}$  where  $i \leq n$ ,  $\mathbf{P}(a_i) = \mathbf{P}(b_i)$ .
- ② For every  $i \in \mathbb{N}$  where  $i \leq n$ , if  $\mathbf{F}(a_i) \in \{\mathbf{dc}, \mathbf{cy}, \mathbf{cn}\}$ , then  $\mathbf{F}(b_i) =_{\mathbf{s}} \mathbf{a}$ .
- ③ For every  $j \in \mathbb{N}$  where  $j \leq n$ , if  $\mathbf{F}(b_j) \in \{\mathbf{dc}, \mathbf{cy}, \mathbf{cn}\}$ , then  $\mathbf{F}(a_j) =_{\mathbf{s}} \mathbf{a}$ .

Then  $\Delta$  will be called an **orderly dialogue**.



## Complete and Orderly Dialogues

### Definition

*(Alternative Definition of Orderly Dialogues)* Let  $n \in \mathbb{N}$  and  $\Delta = \langle (a_i)_{i=0}^n, (b_i)_{i=0}^n \rangle$  be a complete dialogue.  $\Delta$  will be called an **orderly dialogue**, if for every  $i \in \mathbb{N}$  with  $i \leq n$ , it is true that  $P(a_1) = P(b_1)$  and, at least one of the  $F(a_i), F(b_i)$  is “equal” to  $a$ , where by “equal” we are referring to the relation “ $=_s$ ”.

### Proposition

*The two definitions of orderly dialogue are equivalent.*

## Complete and Orderly Dialogues

### Corollary

*Let  $n \in \mathbb{N}$  and  $\Delta = \langle (a_i)_{i=0}^n, (b_i)_{i=0}^n \rangle$  be a complete dialogue. If  $\Delta$  is an orderly dialogue, then, the following holds*

$$|\{i \in \mathbb{N} : i \leq n \text{ and } \mathbf{F}(a_i) =_{\mathbf{s}} \mathbf{a}\}| + |\{i \in \mathbb{N} : i \leq n \text{ and } \mathbf{F}(b_i) =_{\mathbf{s}} \mathbf{a}\}| \geq n$$

# Complete and Orderly Dialogues

## Definition

A complete dialogue  $\Delta = \langle (a_i)_{i=0}^n, (b_i)_{i=0}^n \rangle$  is **orderable**, if there exist re-orderings  $(a_{k_i})_{i=0}^n, (b_{k_i})_{i=0}^n$  of  $(a_i)_{i=0}^n, (b_i)_{i=0}^n$  respectively, such that  $\Delta' = \langle (a_{k_i})_{i=0}^n, (b_{k_i})_{i=0}^n \rangle$  is orderly.

## Proposition

For every  $n, m \in \mathbb{N}$ , where  $n \leq m \leq 2n$ , there exists a complete dialogue  $\Delta' = \langle (a'_i)_{i=0}^n, (b'_i)_{i=0}^n \rangle$  such that,  $\Delta'$  is not orderable and the following holds:

$$|\{i \in \mathbb{N} : i \leq n \text{ and } \mathbf{F}(a_i = \mathbf{a})\}| + |\{i \in \mathbb{N} : i \leq n \text{ and } \mathbf{F}(b_i = \mathbf{a})\}| = m.$$

## Complete and Orderly Dialogues

### Proposition

*Let  $n, m \in \mathbb{N}$  and  $\Delta = \langle (a_i)_{i=0}^n, (b_i)_{i=0}^m \rangle$  be a dialogue. Then, there exists  $n_0 \in \mathbb{N}$ , where  $n, m \leq n_0$  and  $\Delta' = \langle (a'_i)_{i=0}^{n_0}, (b'_i)_{i=0}^{n_0} \rangle$ , complete and orderly dialogue, such that the following hold:*

- ① *For every  $i = 0, \dots, n$ , there exists  $j \in \mathbb{N}$ ,  $j \leq n_0$ , such that  $a_i = a'_j$ .*
- ② *For every  $i = 0, \dots, m$ , there exists  $j \in \mathbb{N}$ ,  $j \leq n_0$ , such that  $b_i = b'_j$ .*

## Extensional and Intentional Reasons

### Definition

Let  $\Pi$  and  $\Sigma$  be a set of mutually accepted propositions and a set of mutually accepted argument schemes,  $p, p' \in \mathbb{L}$  and  $\sigma = \langle p', \mathbf{a} \rangle$ . Then,

- ① If  $p' \equiv (d(p) \in \Pi)$ , then,  $\sigma$  will be called an **extensional reason** for  $p$  with regard to  $\Pi$ .
- ② If there exist  $n \in \mathbb{N}$ ,  $n > 0$  and  $q_0, \dots, q_n \in \mathbb{L}$ , such that,  $q_0 \equiv (D((l_1 \wedge \dots \wedge l_n) \rightarrow l) \in \Sigma)$ , where  $q_1, \dots, q_n$  are instances of the propositional types  $l_1, \dots, l_n$  and  $p' \equiv ((q_0 \wedge \dots \wedge q_n) \rightarrow p)$ , then,  $\sigma$  will be called an **intentional reason** for  $p$  with regard to  $\Sigma$ .

## Extensional and Intentional Elementary Dialogues

### Definition

Let  $n \in \mathbb{N}$  and  $\Delta = \langle (a_i)_{i=0}^n, (b_i)_{i=0}^n \rangle$ , be an orderly dialogue.  $\Delta$  will be called **elementary dialogue** for  $P(a_0)$ , if one of the following conditions holds:

- ①  $n = 0$ ,  $F(a_0) = \mathbf{a}$  and  $F(b_0) = \mathbf{a}$
- ②  $n = 2$  the following hold:
  - ①  $F(a_0) = \mathbf{a}$  and  $F(b_0) = \mathbf{cn}$
  - ②  $P(b_1) = P(b_0)$ ,  $F(a_1) = \mathbf{a}$  and  $F(b_1) = \mathbf{dc}$
  - ③  $F(a_1) = \mathbf{a}$ ,  $F(b_1) \in \{\mathbf{cn}, \mathbf{cy}\}$  and  $a_2$  is an extensional reason for  $P(a_0)$
- ...

## Extensional and Intentional Elementary Dialogues

### Definition

...

③  $n = 2$  and the following hold:

①  $F(a_0) = \mathbf{a}$  and  $F(b_0) = \mathbf{cn}$

②  $P(b_1) = P(b_0)$ ,  $F(a_1) = \mathbf{a}$  and  $F(b_1) = \mathbf{dc}$

③  $F(a_1) = \mathbf{a}$ ,  $F(b_1) \in \{\mathbf{cn}, \mathbf{cy}\}$  and  $a_2$  is an intentional reason for  $P(a_0)$

Moreover, if  $\Delta$  satisfies condition 1., then we will say that it is a **immediate acceptance** for  $P(a_0)$ , while, if  $\Delta$  satisfies condition 2. (res. 3.), then it will be called **extensional** (res. **intentional**) **elementary dialogue** for  $P(a_0)$  with regard to  $\Pi$  (res.  $\Sigma$ ).

## Sub-dialogues

### Definition

Let  $n, m \in \mathbb{N}$  and  $\Delta = \langle (a_i)_{i=0}^n, (b_i)_{i=0}^m \rangle$  be a dialogue.  $\Delta'$  will be called a **sub-dialogue** of  $\Delta$ , if there exist  $k, l \in \mathbb{N}$ , where  $0 \leq k \leq l \leq n$ , such that  $\Delta' = \langle (a_i)_{i=k}^l, (b_i)_{i=k}^l \rangle$  and we denote as  $\Delta' \preceq \Delta$ . Moreover, if  $l - k < n$ , then, we will say that  $\Delta'$  is a **proper sub-dialogue** of  $\Delta$  and, we will denote as  $\Delta' \prec \Delta$ .



## Defense and Conclusive Defense

### Definition

Let  $n, m, k, l \in \mathbb{N}$ , where  $k - l \leq \min\{n, m\}$  and  $\Delta = \langle (a_i)_{i=0}^n, (b_i)_{i=0}^m \rangle$ ,  $\Delta' = \langle (a_i)_{i=0}^k, (b_i)_{i=0}^l \rangle$  be two dialogues, such that  $\Delta' \preceq \Delta$  and  $\Delta'$  is an elementary dialogue for some  $p \in \mathbb{L}$ . We will say that  $\Delta'$  **defends**  $p$  in  $\Delta$  (with regard to some set of mutually accepted premises  $\Pi$  or with regard to some set of mutually accepted argument schemes  $\Sigma$ ), if,  $\Delta'$  satisfies one of the following conditions:

- ①  $\Delta'$  is an immediate acceptance for  $p$ .

...

## Defense and Conclusive Defense

### Definition

...

- ②  $\Delta'$  is an extensional elementary dialogue for  $p$  with regard to  $\Pi$ , and it holds that  $d(p) \in \Pi$
- ③  $\Delta'$  is an intentional elementary dialogue for  $p$  with regard to  $\Sigma$  and, if  $\langle ((q_0 \wedge \dots \wedge q_{n'}) \rightarrow p), \mathbf{a} \rangle$  is the intentional reason of  $\Delta'$ , then, the following hold:
  - ①  $q_0$  is of the form  $z \in \Sigma$  and it holds that  $z \in \Sigma$ ,
  - ② for every  $0 \leq i \leq n$  (res.  $0 \leq i \leq m$ ), if  $a_i$  (res.  $b_i$ ) is of the form  $\langle q, \mathbf{dc} \rangle$ , where,  $q$  belongs to  $\{q_1, \dots, q_{n'}\}$ , there exists  $\Delta_q \preceq \Delta$ , such that  $\Delta_q$  defends  $q$ .

## Defense and Conclusive Defense

### Definition

Let  $n, m, k, l \in \mathbb{N}$ , where  $k - l \leq \min\{n, m\}$  and  $\Delta = \langle (a_i)_{i=0}^n, (b_i)_{i=0}^m \rangle$ ,  $\Delta' = \langle (a_i)_{i=0}^k, (b_i)_{i=0}^l \rangle$ , be two dialogues, such that  $\Delta' \preceq \Delta$  and  $\Delta'$  is an elementary dialogue for some  $p \in \mathbb{L}$ . We will say that  $\Delta'$  **defends**  $p$  **conclusively** in  $\Delta$ , with regard to some set  $\Pi$  of mutually accepted propositions (or, with some set  $\Sigma$  of mutually accepted argumentation schemes), if  $\Delta'$  defends  $p$  in  $\Delta$  and, for every  $i \in \{k + 1, \dots, n\}$ , if  $\mathbf{P}(a_i) \equiv p$ , then  $\mathbf{F}(a_i), \mathbf{F}(b_i) \notin \{\mathbf{dc}, \mathbf{cn}\}$ .

## Arguments and Complete Arguments

### Definition

*Let  $T = \langle V, E \rangle$  be a finite tree with a root, the vertices of which are elementary dialogues and  $q \in \mathbb{L}$ . Let, also,  $\text{Root}(T) = \Delta_0$  be an elementary dialogue for  $q$  and  $\Sigma$  be a set of mutually accepted argumentation schemes. If,*

- 1 *every vertex of  $T$  which is not a leaf of  $T$ , is an intentional elementary dialogue and*

*...*

# Arguments and Complete Arguments

## Definition

...

- ② *for every vertex of  $T$ , let it be  $\Delta_i$ , where  $i \in \mathbb{N}$  and  $i < |V|$ , if  $\Delta_i$  is not a leaf of  $T$ , and if  $\langle p_0 \wedge \dots \wedge p_n \rightarrow p, \mathbf{a} \rangle$  is the intentional reason of  $\Delta_i$ , where  $n \in \mathbb{N}$ , then, every child of  $\Delta_i$  is a reason for some*

*$q' \in \{p_0, \dots, p_n, (D((l_1 \wedge \dots \wedge l_n) \rightarrow l) \in \Sigma)\}$ , where  $l_1 \wedge \dots \wedge l_n \rightarrow l$ , the argument scheme of which  $p_0 \wedge \dots \wedge p_n \rightarrow p$  is an instance,*

*then,  $T$  will be called an **argument** for  $q$ . Moreover,*

## Arguments and Complete Arguments

### Definition

- ① *if for every vertice of  $T$  which is an intentional elementary dialogue, let it be  $\Delta_{i'}$  and, for every  $p \in \mathbb{L}$  which appears in the intentional reason of  $\Delta_{i'}$ , there exists a child of  $\Delta_{i'}$  which is a reason of  $p$  and*
  - ② *no vertice of  $T$  is an immediate acceptance,*
- then  $T$  will be called a **complete argument** for  $q$ .*

## Dialectical Proofs

### Definition

Let  $T = \langle V, E \rangle$  be an argument,  $p \in \mathbb{L}$ ,  $\Pi$  be a set of mutually accepted premises and  $\Sigma$  be a set of mutually accepted argumentation schemes. If,

- 1 for every leaf of  $T$  which is an extensional elementary argument, let it be  $\Delta_i$ , if  $q \in \Pi$  is the extensional reason of  $\Delta_i$ , then,  $q \in \Pi$  holds,

...

then,  $T$  will be called a **dialectical proof** from  $\Pi$  and  $\Sigma$  for  $p$  and, if  $T$  is a complete argument, then,  $T$  will be called a **complete dialectical proof** from  $\Pi$  and  $\Sigma$  for  $p$ .

## Dialectical Proofs

### Definition

Let  $T = \langle V, E \rangle$  be an argument,  $p \in \mathbb{L}$ ,  $\Pi$  be a set of mutually accepted premises and  $\Sigma$  be a set of mutually accepted argumentation schemes. If,

...

- ② every argumentation scheme, let it be  $L$ , with some instance of  $L$  being the proposition of the intentional reason of some vertice of  $T$  which is not a leaf,  $D(L) \in \Sigma$  holds,

then,  $T$  will be called a **dialectical proof** from  $\Pi$  and  $\Sigma$  for  $p$  and, if  $T$  is a complete argument, then,  $T$  will be called a **complete dialectical proof** from  $\Pi$  and  $\Sigma$  for  $p$ .



# Arguments in Dialogues

## Definition

*Let  $T$  be some argument and  $\Delta$  be some dialogue. We may say that  $T$  appears in  $\Delta$  if every node of  $T$  is a proper sub - dialogue of  $\Delta$ .*

# Anthropological Validity

- Our formalism requires of norms upon which it may be built.
- However, an ultra - relativistic and descriptive approach to rhetoric, doesn't offer us nothing more than a list of useful rhetorical "tricks".
- Thus, we work under the assumption that "persuassiveness" is dialectifiable.
- Audience demands are treated not as random vices but as the product of dialogues the members of the audience had in the past.

## Which Rhetoric?

- “Conquest” rhetoric views the audience as an absolutely passive entity, considered usually as a “construction” of the arguer’s mind.
- “Invitational” rhetoric focuses on cooperation between the opposing sides.
- We aim to formalize what we may call by “Rhetoric of Unwilling Cooperation”.

End

*Thank you for your attention!*