Probabilistic-Temporal Logic with Actions

Š. Dautović[1], Dragan Doder [2], Zoran Ognjanović [1]

[1] Mathematical Institute of SASA, Serbia
[2] Utrecht University, The Netherlands

LAP 2022

Overview

Goal:

Provide the probabilistic logic for reasoning about degrees of confirmation with actions in time

The main results:

- Sound and strongly complete axiomatization

Methods:

- Henkin-style method

Confirmation theory

- R.Carnap - Logical Foundations of Probability (1962)

Concept of confirmation

A hypotheses A is confirmed by evidence B means that the conditional probability of A in presence of B is greater than just the probability of A.

Confirmation theory

- R.Carnap - Logical Foundations of Probability (1962)

Concept of confirmation

A hypotheses A is confirmed by evidence B means that the conditional probability of A in presence of B is greater than just the probability of A.

- The quantitative concept of confirmation measures the degree of confirmation: $c(A, B) = r$ *"how much B confirms/disconfirms A"*

Confirmation theory

- R.Carnap - Logical Foundations of Probability (1962)

Concept of confirmation

A hypotheses A is confirmed by evidence B means that the conditional probability of A in presence of B is greater than just the probability of A.

- The quantitative concept of confirmation measures the degree of confirmation: $c(A, B) = r$ *"how much B confirms/disconfirms A"*

- Logic for reasoning about degrees of confirmation LPP_2^{conf}

  Difference measure: $c(A, B) = \mu(A|B) - \mu(A)$

Šejla Dautović, Dragan Doder and Zoran Ognjanović, *Reasoning About Degrees of Confirmation*, Logic and Argumentation - Third International Conference, CLAR 2020.

Probabilistic logics

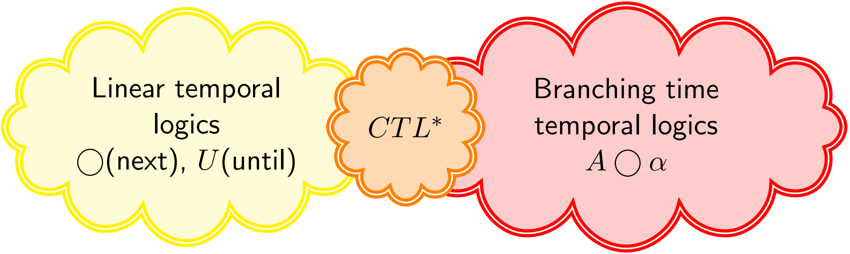
- Syntax: Extension of classical logic by applying probabilistic operators:
  - $3w(\alpha) + 2w(\beta) \leq 1$ (LWF)
    Not applicable for axiomatizing conditional probabilities
  - $P_{\geq r}\alpha$
- Semantics: Possible worlds + a probability measure
  - $M \models P_{\geq r}\alpha$ if $\mu(\{w \in W \mid v(w, \alpha) = true\}) \geq r$

Temporal logics

Linear temporal logics
$\bigcirc$(next), U(until)

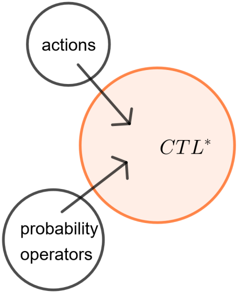
Branching time temporal logics
$A \bigcirc \alpha$

Temporal logics



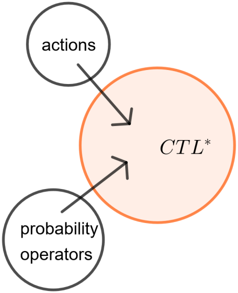
Starting points

- Logic for reasoning about degrees of confirmation LPP_2^{conf}
- Probabilistic logic for reasoning about actions in time $pCTL_A^*$



Starting points

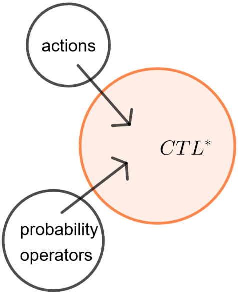
- Logic for reasoning about degrees of confirmation LPP_2^{conf}
- Probabilistic logic for reasoning about actions in time $pCTL_A^*$



van Zee, M., Doder, D., van der Torre, L., Dastani, M., Icard, T., Pacuit, E.: Intention as commitment toward time. Artif. Intell. 283, 103270 (2020)

Starting points

- Logic for reasoning about degrees of confirmation LPP_2^{conf}
- Probabilistic logic for reasoning about actions in time $pCTL_A^*$



Šejla Dautović, Dragan Doder, *Probabilistic Logic for Reasoning about Actions in Time*, Symbolic and Quantitative Approaches to Reasoning with Uncertainty, 15th European Conference, ECSQARU 2019.

Our logic $\mathrm{CTL}^*_{\mathrm{A,conf}}$

$$pCTL^*_A + LPP_2^{conf}$$

$P_{\geq\frac{1}{2}}(\bigcirc do(a))$

$c_{\leq\frac{1}{2}}(\beta, \gamma)$

Our logic $\mathrm{CTL}^*_{\mathrm{A,conf}}$

$$pCTL^*_A + LPP_2^{conf}$$

$$P_{\geq \frac{1}{2}}(\bigcirc do(a))$$

$$c_{\leq \frac{1}{2}}(\beta, \gamma)$$

$$c_{\geq 0}(\bigcirc pre(b), do(a))$$

"The execution of the action a confirms that the preconditions of the action b hold in the next moment with a degree of at least zero."

$\mathrm{CTL}^*_{\mathrm{A,conf}}$ - Syntax

- $\mathrm{CTL}^*_{\mathrm{A}}$-formulas: $\alpha, \beta \ldots$

$$\alpha ::= \chi \mid do(a) \mid \bigcirc\alpha \mid A\alpha \mid \alpha U \alpha \mid \alpha \wedge \alpha \mid \neg\alpha$$

where $\chi \in Prop = P \cup \{pre(a), post(a) \mid a \in Act\}$ and $a \in Act$.

$\mathrm{CTL}^*_{\mathrm{A,conf}}$ - Syntax

- $\mathrm{CTL}^*_{\mathrm{A}}$-formulas: $\alpha, \beta \ldots$

$$\alpha ::= \chi \mid do(a) \mid \bigcirc\alpha \mid A\alpha \mid \alpha U \alpha \mid \alpha \wedge \alpha \mid \neg\alpha$$

where $\chi \in Prop = P \cup \{pre(a), post(a) \mid a \in Act\}$ and $a \in Act$.

- Probabilistic formulas: $\phi, \psi, \ldots$

$$\phi ::= P_{\geq r}\alpha \mid c_{\geq s}(\alpha, \beta) \mid c_{\leq s}(\alpha, \beta) \mid \phi \wedge \phi \mid \neg\phi$$

where α and β are CTL^*_A formulas, $r \in [0,1]_{\mathcal{Q}}$ and $s \in [-1,1]_{\mathcal{Q}}$

$$For_{\mathrm{CTL}^*_{\mathrm{A,conf}}} = For_{\mathrm{CTL}^*_{\mathrm{A}}} \cup For_P$$

Definition ($\mathrm{CTL}^*_{\mathrm{A}}$-structure)

A $\mathrm{CTL}^*_{\mathrm{A}}$-structure $\mathbf{S}$ is tuple (S, R, v) where:

- S is a non- empty set of states;
- $v : S \to 2^{Prop}$ is a valuation function from states to sets of propositions;
- $R = \bigcup R_a$, $a \in Act$, R_a relation on $S \times S$, such that the following conditions hold:
  - (a) R is serial;
  - (b) For all action $a \in Act$: If sR_as' and sR_as'' then $s' = s''$;
  - (c) If $pre(a) \in v(s), a \in Act, s \in S$ then there exists $s' \in S, sR_as'$;
  - (d) If sR_as' then $post(a) \in v(s')$ and $pre(a) \in v(s)$.

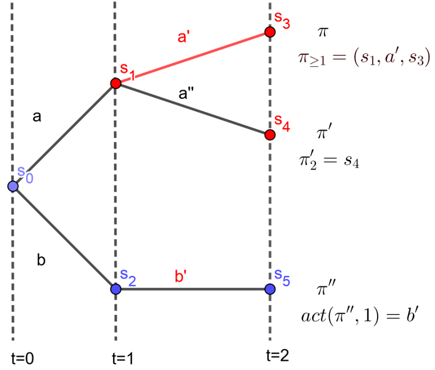
Definition (Path)

$\pi = (s_0, a_0, s_1, a_1, ...)$, such that for each i, $s_i R_{a_i} s_{i+1}$.

$$\pi_k = s_k$$

$$\pi_{\geq i} = (s_i, a_i, s_{i+1}, a_{i+1}, ...)$$

$$act(\pi, k) = a_k$$



- $\mathbf{S}, \pi \models_c do(a)$ iff $act(\pi, 0) = a$;
- $\mathbf{S}, \pi \models_c \bigcirc \alpha$ iff $\mathbf{S}, \pi_{\geq 1} \models_c \alpha$;

Definition ($\mathrm{CTL}^*_{\mathrm{A,conf}}$−structure)

A $\mathrm{CTL}^*_{\mathrm{A,conf}}$−structure is a tuple $M = \langle W, H, \mu, \sigma\rangle$ where:

- W is a nonempty set of worlds,
- $\langle W, H, \mu\rangle$ is a probability space, i.e.,
  - H is an algebra of subsets of W,
  - $\mu : H \longrightarrow [0, 1]$ is a finitely additive measure,
- σ provides for each world $w \in W$ a $\mathrm{CTL}^*_{\mathrm{A}}$-structure and a path , i.e., $\sigma(w) = (\mathbf{S}_w, \pi_w)$.

- $M \models c_{\geq r}(\alpha, \beta)$ if $\mu(\{w \in W \mid v(\beta, \mathbf{S}_w, \pi_w) = 1\}) > 0$ and $\mu(\{w \in W \mid v(\alpha, \mathbf{S}_w, \pi_w) = 1\} \mid \{w \in W \mid v(\beta, \mathbf{S}_w, \pi_w) = 1\}) - \mu(\{w \in W \mid v(\alpha, \mathbf{S}_w, \pi_w) = 1\}) \geq r$

Axiomatization

- Propositional reasoning
- Temporal logic
- Actions, pre and postconditions
- Reasoning about conditional probabilities

Axiomatization

- Propositional reasoning
- Temporal logic
- Actions, pre and postconditions
- Reasoning about conditional probabilities

- Infinitary inference rule:
  - From the set of premises

$$\{c_{\geq r-\frac{1}{k}}(\alpha,\beta) \mid k \in \mathcal{N}\}$$

$$\text{infer } c_{\geq r}(\alpha,\beta)$$

If the conditional probability is arbitrary close to r, it is at least r.

Results

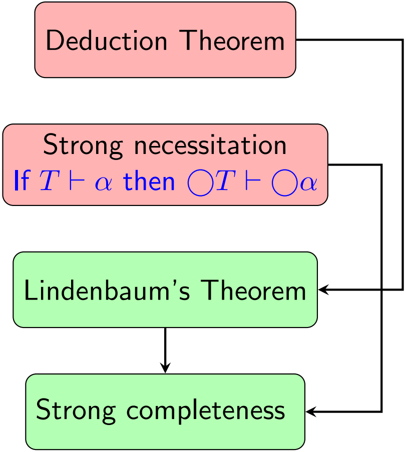
Theorem (Soundness and completeness)

A set of formulas T is consistent iff T is satisfiable.

- Adaptation of Henkin's construction

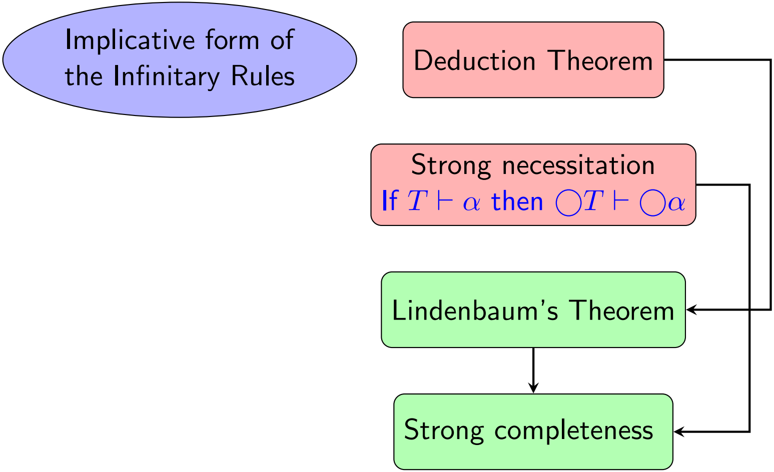
Technical details

- Henkin's construction



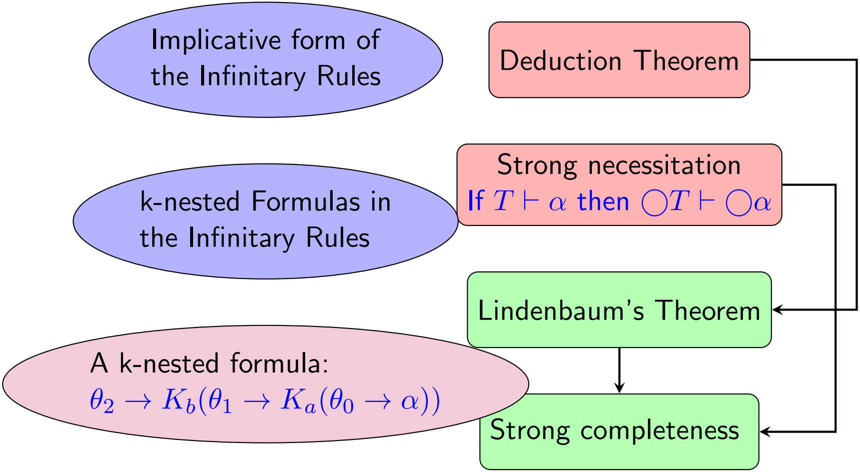
Technical details

- Henkin's construction



Technical details

- Henkin's construction



Thank you!