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Contents

Alternative axiomatization of NFU

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Keywords:
New Foundations with atoms, Axiom of choice, Axiomatization

The main notion of theory NF(U) is that of stratification. In order to show that a formula is stratified, one must assign types to its variables and check whether they satisfy certain conditions. Assigning types is a rather straightforward procedure, and continues to be so when a language is extended by introducing abstraction terms and it is specified how to assign types to them. However, types of some terms have certain undesirable properties. The most prominent example are Kuratowski's ordered pairs, where the type of the ordered pair is two types higher than the types of its projections (ordered pairs, as defined by Kuratowski, are not type-leveled).

Our goal is to explicitly define type-leveled ordered pairs. In order to do that, we suggest adding, along with the axiom of infinity, a specific version of the axiom of choice to the theory NFU. Namely, Tarski's theorem about choice, which we call *Tarski's axiom*. Tarski's axiom cannot be stated right away, so we first need to introduce few notions using Kuratowski's ordered pairs. After that, we are able to state Tarski's axiom and can directly use it to define type-leveled ordered pairs. Then the theory NFU + Inf + Tarski's axiom can be developed further using type-leveled ordered pairs, which is a big simplification in comparison to the theory NFU + Inf + AC with Kuratowski's ordered pairs.

Acknowledgment

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Fixed-Template Promise Model Checking Problems

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Keywords:
Model Checking Problem, First-Order Logic, Promise Constraint Satisfaction Problem, Computational Complexity

The fixed-template constraint satisfaction problem (CSP) can be seen as the problem of deciding whether a given primitive positive first-order sentence is true in a fixed structure (also called model). In recent work [1], we study a class of problems that generalizes the CSP simultaneously in two directions: we fix a set $\mathcal{L}$ of quantifiers and Boolean connectives, and we specify two versions of each constraint, one strong and one weak. Given a sentence which only uses symbols from $\mathcal{L}$, the task is to distinguish whether the sentence is true in the strong sense, or it is false even in the weak sense.

We classify the computational complexity of these problems for the existential positive equality-free fragment of first-order logic, i.e., $\mathcal{L} = \{\exists, \wedge, \vee\}$, and we prove some upper and lower bounds for the positive equality-free fragment, $\mathcal{L} = \{\exists, \forall, \wedge, \vee\}$.

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Pure Implicational Intuitionistic Logic

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Keywords:
Intuitionistic Implication, Universal Logic, Metalogic.

We study here inutitionitic logic with only one connective, implication. We previously studied logics with only negation, comparing in particular from this point of view intuititionistic negation with classical negation [1]. If we consider intuitionistic implication alone, it can be defined in a very simple way:

$$T, a \vdash b \text{ iff } T \vdash a \to b$$

This is not necessarily well-known (but appears in some way in [7]) and moreover it seems that up to now nobody has studied a system of logic with intuitionistic implication alone. An important thing is to specify which framework is used. We are working here in the perspective of universal logic [2], that means we are working in an abstract and general perspective where a system of logic is first of all considered as a structure [4].

We consider here the structure $\mathcal{PIIL} = \langle \mathcal{F}; \vdash \rangle$ where:

(1) $\mathcal{F}$ is $\langle \mathbb{F}; \to \rangle$, where $\mathcal{F}$ is the absolutely free algebra generated by $\to$ from a set of atomic formulas $\mathbb{A}$

(2) $\vdash$ is a binary relation between theories and formulas, i.e. $\vdash \subseteq \mathcal{P}(\mathbb{F}) \times \mathbb{F}$.

(3) for any endomorphism ϵ, $T \vdash a$ iff $\epsilon(T) \vdash \epsilon(a)$

(4) $a \vdash a$

(5) If $T \vdash a$ and $T \subseteq U$, then $U \vdash a$

(6) If $T \vdash a$ and $U, a \vdash b$, then $T, U \vdash b$

(7) $T, a \vdash b$ iff $T \vdash a \to b$

Items (1) and (2) are the basic framework, (3) is the structural axiom [8], (4), (5), (6) are the Tarskian axioms [10], (7) is the proper axiom for intutionistic implication.

Let us note that if we change the framework, properties change. For example if instead of considering a consequence relation with only one formula on the right, we consider theories also on the right (a so-called "multiple conclsuion logic" [9]), the properties of the connective change. This is also the case, as pointed out in [1], if we change the structure of the domain, considering for example an algebra which is not an absolutely free algebra. When we are talking about "axiom" here, we are talking about axiom in a model-theoretical sense, not in a proof-theoretical sense [5].

What is interesting with the axiom (7) is that it establishes a direct connection between a connective and the consequence relation, a correspondence identifying a connective with the consequence relation, a correspondence between logic and metalogic. So the study of $\mathcal{PIIL}$ is interesting both for a better understanding of intuitionistic implication and of universal logic ("universal logic" can be considered as a name for metalogic, cf. [6]). To examine this, let us divide the axiom of intuitionistic implication in two parts:

$$\text{If } T, a \vdash b \text{ then } T \vdash a \to b \quad (\to\ right)$$

$$\text{If } T \vdash a \to b \text{ then } T, a \vdash b \quad (\to\ left)$$

The $(\to\ right)$ axiom is going from metalogic to logic, a "logification" of the consequence operator. The $(\to\ left)$ axiom is going from logic to metalogic, a "metalogification" of the connective of implication.

We can consider the three following axioms for implication:

(8) $\vdash a \to a$

(9) if $T \vdash a$, then $T \vdash b \to a$

(10) if $T \vdash a \to b$ and $T \vdash b \to c$, then $T \vdash a \to c$

and show that under some circunstances it is possible to define $\mathcal{PIIL}$ in a equivalent way, using (7)-(10) instead of (4)-(7).

There are a lot of interplays between the consequence relation and intuitionistic implicition. It is for example possible to prove that the three following axioms are equivalent:

(A) if $T \vdash a$ and $U \vdash b$, then $T, U \vdash c$

(B) $T, a, b \vdash c$

(C) $T \vdash a \to (b \to c)$

In this work, we furthermore study some abstract features of $\mathcal{PIIL}$: the relation between maximal and relative maximal theories and the relation between two forms of compacity. These notions are abstract notions that can be defined in an abstract way without using connectives. When we have some connectives they may coincide or not (cf. [3]).

We then study some proof systems and some semantics for $\mathcal{PIIL}$, using a general abstract form of the complenetess theorem [3].

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Soundness and completeness of a probabilized natural deduction system

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Keywords:

inference rules, probability logic, natural deductions, soundness, completeness.

We present the semantics and syntax of a probabilized system of natural deductions, denoted by **NKprob**(k), where $k > 0$ is a given natural number, and prove the soundness and completeness theorem. More precisely, the formulas of **NKprob**(k) are of the form A^r, where A is any propositional formula and $r \in \{0, \frac{1}{k}, \ldots, \frac{k-1}{k}, 1\}$, meaning that the probability of formula A is greater than or equal to r. Mentioned system could be considered as a natural deduction system with high probabilities, conceptualized by two ideas — Gentzen's natural deductions systems (see [10]) and Suppes' approach to high probability propositions (see [17], [18]), where, in that case, formulae would be of the form $A^{1-n\varepsilon}$, for some natural number n and small real $\varepsilon > 0$ (see also [1], [3], [5], [7], [8]).

A **NKprob**(k)–model, defining the characteristics of sentence probability as considered by Carnap and Popper (see [9], [11], [12], [13], [14], [15], [16]), can be introduced as any mapping $p : For \to [0, 1]$ satisfying the following conditions:

(i) $p(A) = 1$, for each classical tautology A;

(ii) if $A \leftrightarrow B$ is a classical tautology, then $p(A) = p(B)$, and

(iii) if $p(A \wedge B) = 0$, then $p(A \vee B) = p(A) + p(B)$, for any formulae A and B.

Regarding the syntax of **NKprob**(k), here we will present some of the inference rules:

$$(1)\quad \frac{}{A^0}, \text{ for any formula } A$$

$$(2)\quad \frac{A^r\ B^s\ (\neg A \vee \neg B)^1}{(A \vee B)^{r+s}}$$

$$(3)\quad \frac{\begin{array}{cc} [A^r] & [(\neg A)^{1-r}] \\ \vdots & \vdots \\ \bot^{\frac{1}{k}} & \bot^{\frac{1}{k}} \end{array}}{\bot^{\frac{1}{k}}}$$

Note that the rule (3) makes the notion of consistency more intuitive.

Finally, we construct a maximal consistent extension of a given theory which plays the most important role in the proof of completeness theorem. The basic idea used in the construction of maximal consistent extension is adding A^r or $\neg A^{1-r}$ so that the given theory stays consistent (see [2], [4], [5], [6], [7]).

The obtained system, sound and complete with respect to mentioned models, is simple, with the general form of inference rules as follows:

$$\frac{A^r \quad B^s}{C^t}$$

where t depends on r and s, treating relationships between propositional connectives and probabilities. This system enables to define exactly a logical consequence relation between probabilized propositional formulae: $A^r, B^s \vdash C^t$ with intended meaning that C with probability greater than or equal to t follows from A with probability greater than or equal to r and B with probability greater than or equal to s.

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Various notions of computability of subsets of topological and metric spaces

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Keywords:
computable topological space, dense computable sequence, computable metric space, computable set, semicomputable set, computability from above

In the first part, we consider computable topological spaces, and show that in such a setting, we can find a dense computable sequence in any nonempty computable set.

In the second part, we report on our findings concerning the interplay of various notions of computability, such as computability from above and semicomputability, with respect to different properties of ambient computable metric spaces. Our results include the following:

Theorem 1: Let $\mathcal{X} = (X, \mathcal{T}, I)$ be a computable topological space and let S be a nonempty computable subset of X. Then there exists a computable sequence $(x_i)_i$ in $\mathcal{X}$ such that the closure of its image is equal to S.

Theorem 2: Let $\mathcal{X} = (X, d, \alpha)$ be a computable metric space such that (X, d) is complete and $M \subseteq X$. M is computable from above in $\mathcal{X}$ if and only if there exists a computable set N in $\mathcal{X}$ such that $M \subseteq N$.

Theorem 3: Let $\mathcal{X}$ be a computable metric space. If a compact subset K is semicomputable in $\mathcal{X}$, then K is computable from above in $\mathcal{X}$. The converse does not hold in general.

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Computable type of certain quotient spaces

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Keywords:

Computable topological space, Computable set, Semicomputable set, Quotient space.

Topology plays an important role in determining the relationship between different levels of computability of sets in computable topological spaces. In particular, semicomputable sets with certain topological properties are necessarily fully computable. This is expressed in the notion of *computable type:* a space A is said to have computable type if every semicomputable set homeomorphic to A must be computable. Some known examples of spaces with computable type are topological manifolds, chainable and circulary chainable continua and finite graphs ([3, 2, 4]).

We explore computable type of quotients of Euclidean spaces, motivated by the known fact that both the pair (B^n, S^{n-1}) of the unit ball and its boundary and the quotient space $B^n/S^{n-1} \cong S^n$ have computable type ([1]). Our aim is to, given a (locally Euclidean) space A with computable type, describe a subset B (or, more generally, an equivalence relation on A) such that the corresponding quotient space has computable type. We will present some positive results related to this, as well as some interesting counterexamples.

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Probabilistic-Temporal Logic with Actions

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Keywords:

Probabilistic logic, Temporal logic, Completeness.

Temporal logics are formal systems that allow reasoning about sentences referring to time and they find many applications in computer science [7]. The most standard division of temporal logics is into linear and branching time. In linear temporal logics (LTL) each moment of time has a unique possible future, while in temporal logics with brunching time for each moment there can be two or more possible futures. Propositional branching time temporal logic with the standard operators $\bigcirc$ (next), U (until) and A (universal path operator) is introduced in [8], usually called Computation tree logic (CTL). Full computation tree logic (CTL^*) is introduced in [15].

Uncertain reasoning has emerged as one of the main fields in artificial intelligence, with many different tools developed for representing and reasoning with uncertain knowledge. A particular line of research concerns the formalization in terms of logic, and the questions of providing an axiomatization and decision procedure for *probabilistic logic* attracted the attention of researchers and triggered investigation about formal systems for probabilistic reasoning [1, 9, 10, 11, 14, 16]. The probabilistic logic for reasoning about degrees of confirmation (LPP_2^{conf}) is introduced in [3]. The language of the logic allows statements as "Probability of A is at most one half" and "B confirms A with degree of at least one half" which means that the posterior probability of A on the evidence B is greater than the prior probability of A by at least one half. Degree of confirmation is measured as $c(A, B) = \mu(A|B) - \mu(A)$, where $\mu(A|B) = \frac{\mu(A\cap B)}{\mu(B)}$ if $\mu(B) \neq 0$ or undefined if $\mu(B) = 0$.

The probabilistic logic for reasoning about actions in time ($pCTL_A^*$) is developed in [2]. The language of the logic extend the language of PAL [19, 20] by employing the full power of CTL^* and probabilistic operators from logic LPP_2

[14], where we can formalize statements as "Probability that the precondition of the action A will hold in the next moment is at least one half".

In this talk we extend the logic $pCTL^*_A$ with new probabilistic operators from [3] to allow measuring how some actions confirms the other action in time.

Our main results are sound and strongly complete (every consistent set of formulas is satisfiable) axiomatization. We prove strong completeness using an adaptation of Henkin's construction, modifying some of our earlier methods [4, 6, 5, 14, 16]. Our axiom system contains infinitary rules of inference. In the infinitary rule for temporal part of the logic the premises and conclusions are in the form of so called k-nested implications. This form of infinitary rules is a technical solution already used in probabilistic, epistemic and temporal logics for obtaining various *strong necessitation* results [12, 13, 17, 18]. In the axiomatization we use this form of rules only on the part of temporal logic, because we do not allow iteration of probability operators.

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A good method of transforming Veltman into Verbrugge models

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Keywords:
interpretability logic, Verbrugge semantics, w-bisimulations, weak bisimulation games, modal equivalence.

Generalised Veltman semantics for interpretability logic, or nowadays called Verbrugge semantics (in honor of Rineke Verbrugge), was developed to obtain certain non-derivability results since Veltman semantics for interpretability logic is not fine-grained enough for certain applications. It has turned out that this semantics has various good properties (see e.g. [4] and [5]). A. Dawar and M. Otto developed a *models-for-games method* in [2], which provides conditions from which a Van Benthem characterisation theorem over a particular class of models immediately follows. M. Vuković and T. Perkov proved in [6] that this result can be extended to Veltman models for the interpretability logic IL. They used bisimulation games on Veltman models for interpretability logic to prove that. To prove similar result for Verbrugge semantics, one needs to define bisimulations and bisimulation games for Verbrugge semantics (and also their finite counterparts, n-bisimulations and n-bisimulation games).

It turns out that the notion of bisimulation for Verbrugge semantics as defined in [7] and [8] is not good enough. It can easily be shown that two n-bisimilar worlds are n-modally equivalent, but a standard result that the converse is true (if we take a finite set of propositional variables), does not hold. So, we have defined in [3] a new notion of weak bisimulations (or short, w-bisimulations), their corresponding games called weak bisimulation games and their finite approximations: n-w-bisimulations and weak n-bisimulation games.

We will present these new notions and then we will show that for Verbrugge semantics, w-bisimulation is strictly stronger than the modal equivalence. That is, there are two modally equivalent worlds in two Verbrugge models that are not w-bisimilar. In order to do that, first we will present two Veltman models which were used by Čačič and Vrgoč in [1] and which are counterexamples for Veltman semantics. Then we will present a method for ''lifting" two Veltman models to Verbrugge models, and apply it to the presented counterexamples for Veltman semantics. Finally, we will show the main result: our method preserves (in a way) bisimulations. More precisely, two worlds are bisimilar as worlds in Veltman models if and only if they are w-bisimilar as worlds in Verbrugge models that were obtained by applying our method to the Veltman ones.

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Probabilistic Systems in $LPP_{\mathcal{A}}$ language

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Keywords:
Probability logic; Markov process.

Probabilistic (modal) propositional logics (LPP) provide an internal, local perspective on probability systems, such as Markov processes, Harsanyi spaces (game theory), Aumann spaces (economics), Coalgebras over measurable spaces, Transition systems, Bayesian nets (artificial intelligence) etc. Rather than standing outside a probability system, LPP formulas are evaluated inside the system, and consequently, LPP languages offer a natural framework for describing 'dynamic' aspects of the corresponding random processes. The main difficulties associated with development of a strongly complete axiomatization come from non-compactness of semantical consequences, as well as from an infinitary nature of σ-additivity. We shall outline some key steps in developing a very general infinitary probabilistic propositional logic $LPP_{\mathcal{A}}$, where $\mathcal{A}$ is a countable transitive set, mostly an admissible set. The primary emphasis will be on combining logical methods to construct weak models, and (nonstandard) measure-theoretic techniques to obtain strong models from weak ones.

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The logic ILP for intuitionistic reasoning about probability

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Keywords:
Intuitionistic logic , Probability .

We introduce the logic ILP suitable for intuitionistic reasoning about probabilities. We use operators of the form $P_{\geqslant s}\alpha$ with the intended meaning "It is proven that the probability of α is at least s". We describe the corresponding class of models which are intuitionistic Kripke models equipped with the appropriate probability requirements. We give sound a axiomatic system and prove decidability of ILP. .

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Properties of Time-Sensitive Distributed Systems: Verification and Complexity

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Keywords: Time-Sensitive Distributed Systems, Multiset Rewriting, Complexity

We develop a Multiset Rewriting language with explicit time for the specification and analysis of various properties of Time-Sensitive Distributed Systems (TSDS). In particular, we focus on a class of systems called *Progressing Timed Systems* (PTS) [1], where intuitively only a finite number of actions can be executed in a bounded time period. Such systems are formalized using explicit time constraints to specify system actions, system goals and properties such as compliance.

We consider desirable properties of TSDSes that are specified over sets of traces of system rules with possible interference from the environment. A "good trace" is an infinite trace of system rules in which the goals are satisfied perpetually. We formalize various desirable properties of TSDSes: *realizability* [1] (there exists a good trace), *survivability* [1] (where, in addition, all admissible traces are good), *recoverability* [2] (all compliant traces do not reach points-of-no-return), and *reliability* [2] (the system can always continue functioning using a good trace).

We consider the relations among these properties and their computational complexity. We prove that for this class of systems the properties of recoverability and reliability coincide and are PSPACE-complete. Furthermore, if we impose a bound on time (as in bounded model-checking), we show that for PTS the reliability property is in the Π_2^p class of the polynomial hierarchy, a subclass of PSPACE. We also show that the bounded survivability is both NP-hard and coNP-hard.

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Probabilistic Reasoning about Typed Combinatory Logic

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Keywords:
Probabilistic reasoning, combinatory logic, simple types, classical propositional logic.

In [6, 7] we have introduced a Logic of Combinatory Logic (LCL), a formal system for reasoning about simply typed combinatory terms. The LCL is obtained by extending the simply typed combinatory logic with classical propositional connectives, and corresponding axioms and rules. In [7] we have presented the syntax, axiomatization and semantics of LCL. The language of the logic LCL is generated by the following syntax

$$\alpha, \beta := M : \sigma \mid \neg\alpha \mid \alpha \wedge \beta$$

where $M : \sigma$ is type assignment statement typable from some basis Γ in simply typed combinatory logic, M is a combinatory term and σ is a simple type. The axiomatic system of LCL is obtained by combining the axiomatic system for classical propositional logic and type assignment system for simply typed combinatory logic. The semantics for LCL, inspired by Kripke-style semantics for lambda calculus with types introduced in [8, 5], is based on an extensional applicative structure containing special elements that correspond to primitive combinators.

We have proved that the given axiomatization is sound and complete with respect to the proposed semantics. Further, we proved that the logic LCL is a conservative extension of the simply typed combinatory logic.

Our goal is to use the logic LCL to develop a formal system for probabilistic reasoning about typed combinatory terms. The idea of formal system for reasoning about simply typed lambda terms and lambda terms with intersection types is presented in [2, 3]. These models are based on the well-known models of lambda calculus, i.e. terms models ([4]) and filter models ([1]). However, these models have shown not to be suitable for propositional reasoning about typed terms. For this reason, we have developed the logic LCL.

We define PCL, a probabilistic system for simply typed combinatory terms, as a probabilistic logic over LCL. Formulas of PCL are layered into two sets: basic formulas and probabilistic formulas. Basic formulas are LCL-formulas. The set of probabilistic formulas is generated by the following syntax

$$\phi, \psi := P_{\geq a}\alpha \mid \neg\phi \mid \phi \wedge \psi$$

where α is an LCL-formula and $s \in \mathbb{Q} \cap [0,1]$. The semantics of PCL are defined as Kripke-style semantics where each world represents one LCL model. The axiomatic system for PCL is obtained from the axiomatic system for probability logic and axiomatic system for LCL.

For future work, we plan to prove that the given axiomatization of PCL is sound and complete with respect to the proposed semantics of PCL. Further, we plan to investigate probabilistic extensions of other typed calculi.

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Intensional negation

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Keywords:
Gilmore-Feferman theory, paradoxes, intensional negation, theory of concepts.

This talk proposes the idea of using the Gilmore-Feferman solution to the set-theoretic paradoxes as the foundation for a theory that deals with the formal properties of concepts and the relation of concept application (as envisioned by Gödel, e.g. in [3]). The solution to the paradoxes offered by Gilmore [2] and Feferman [1] is based on an interpretation of the relation of membership as a partial relation. It will be argued that such reinterpretation makes this relation similar to the relation of concept application.

According to the original characterization of membership, if S is defined by the formula $F(x)$, it holds that:

$$x \in S \iff F(x).$$

The relation of nonmembership $\notin$, or membership in a complement of a set, is defined:

$$x \notin S =_{def} \neg x \in S.$$

Since in classical logic, for every object x either $F(x)$ or $\neg F(x)$ holds, it follows that every object belongs either to a set defined by $F(x)$ or to its complement.

Gilmore-Feferman theory, on the other hand, allows for the possibility that an object is contained neither in a set nor in its complement. In this theory, the complement of a set is not defined by negating the membership in a corresponding set. Instead, the membership in a complement of a set is introduced as a primitive relation.

This follows from a more general restriction according to which not every formula can be taken to define a set. Only the so-called *positive formulas* are assumed to have this role. These are the formulas in which negation stands only in front of the atomic subformulas without $\in$ or $\notin$.

The characterization of the membership relation is changed accordingly. The original equivalence characterizing membership is replaced by the following two:

$$x \in S \iff F^{+}(x)$$

$$x \notin S \iff F^{-}(x)$$

where $F^{+}(x)$ stands for a *positive approximation* of the formula $F(x)$ and $F^{-}(x)$ for a positive approximation of the formula $\neg F(x)$.

A positive approximation of any formula can be found using the classical logical laws and replacing its subformulas of the form $\neg x \in S$ with $x \notin S$, and those of the form $\neg x \notin S$ with $x \in S$. A formula is not necessarily taken to be equivalent to its positive approximation: it is only so to the extent to which the denial of the membership of an object in a set can be understood as the statement of its membership in its complement. This excludes the possibility of defining a set only by denying the membership of its elements in other sets. Instead, some positive conditions for participating in a set have to be specified. The paradoxes, such as Russell's, that are based on the described negative definitions are thus resolved. Using the fixed-point argument, Gilmore has shown that this theory has a model, and is thus consistent (cf. [2]).

In the talk, I propose using this theory for setting up the foundation for a type-free theory of concepts. The reason why it might make an appropriate foundation for the concept theory is the fact that the majority of concepts are not meaningfully applicable to every object, that is, to some objects neither they nor their complements apply. The sentences describing a nonmeaningful application of concepts would be those that cannot be regarded as equivalent to their positive approximations.

The formulas which have such sentences as instances can be taken to misrepresent the structure of the corresponding concept. This can be justified by the requirement that a definition of a concept specifies some positive properties it contains. An apparent definition that cannot be reduced to any such positive definition would not be acceptable. This accords well with Gödel's idea that one of the main tasks for the theory of concepts is to describe the way the concepts are formed from simpler ones using logical connectives which provide them with structure. The paradoxes, which are in this interpretation the consequence of meaningless applications of particular concepts, can be taken to have shown that the role of negation in this process cannot be identified with its role in set-formation and is in need of further scrutinization.

The Gilmore-Feferman theory, understood as characterizing the relation of concept application, might thus offer an adequate solution to the intensional paradoxes (such as *the paradox of the concept of concepts nonapplicable to themselves*), which was regarded by Gödel as the crucial step in the formulation of the theory of concepts.

At the same time, it could suggest particular identity criteria for concepts. An intensional criterion suggested by the theory would be that two concepts are identical if the formulas expressing them have the same positive approximation.

This would insure that the concepts identified by the theory share the same structure and have the same concepts participating in their formation.

The features of the envisioned theory that make it an intensional counterpart to the set theory and its possible developments will be discussed.

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Capturing Term Algebra Computations in Matching Logic

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Keywords:
matching logic, term algebra, proof objects.

Matching Logic (ML) [10, 6, 5] is a unifying foundational logic for formal programming language semantics. It has a minimal design, using only 8 primitive constructs:

$$\varphi ::= \sigma \mid x \mid X \mid \varphi_1\,\varphi_2 \mid \bot \mid \varphi_1 \to \varphi_2 \mid \exists x.\, \varphi \mid \mu X.\, \varphi$$

where in $\mu X.\, \varphi$ we require that φ is positive in X, i.e., X is not nested in an odd number of times on the left-hand side of an implication $\varphi_1 \to \varphi_2$. This syntactic requirement is to make sure that φ is monotone with respect to the set X, and thus the least fixpoint $\mu X.\, \varphi$ exists.

Each pattern is interpreted as the subset of elements that match it. *Element variables* x are matched by a singleton set, while *set variables* X are matched by a subset of M, where M is the carrier set.

The pattern $\bot$ is matched by the empty set (and hence $\top$ is matched by M). The implication pattern $\varphi_1 \to \varphi_2$ is matched by the elements that do not match φ_1 or match φ_2. The pattern $\varphi_1\ \varphi_2$ is an *application* and its interpretation is given by means of a function $M \times M \to \mathcal{P}(M)$, which is pointwise extended to a function $\mathcal{P}(M) \times \mathcal{P}(M) \to \mathcal{P}(M)$. A pattern $\exists x.\, \varphi$ is matched by the instances of φ when x ranges over M. In particular, $\exists x.x$ is matched by M. The pattern $\mu X.\, \varphi$ is matched by the least fixpoint of the functional defined by φ when X ranges $\mathcal{P}(M)$.

```
theory DEF
Symbols:   def
Notations: ⌈φ⌉ ≡ def φ
Axioms:    (DEFINEDNESS)  ∀x. ⌈x⌉
Notations:
  ⌊φ⌋ ≡ ¬ ⌈¬φ⌉              // totality
  φ1 = φ2 ≡ ⌊φ1 ↔ φ2⌋        // equality
  φ1 ⊆ φ2 ≡ ⌊φ1 → φ2⌋        // set incl.
  x ∈ φ ≡ x ⊆ φ              // memb.
endtheory
```

Figure 1: DEF

An example of ML specification (theory) is given in Fig. 1. The theory DEF defines one symbol *def*, one notation $\lceil\varphi\rceil \equiv \mathit{def}\ \varphi$, and one axiom $\forall x.\ \lceil x\rceil$ named

(Definedness) and that states that every element x is defined. Indeed, x is always matched by one element and thus is not $\bot$. Totality $\lfloor\varphi\rfloor$, on the other hand, holds iff φ is total (equals to $\top$). Equality and set inclusion are defined from totality. Membership $x \in \varphi$ has the same meaning as $x \subseteq \varphi$, but we still write $x \in \varphi$ as it fits well the intuition that x is an element in φ.

Matching logic has a sound *fixed* Hilbert-style proof system (see, e.g., [5]) that supports formal reasoning for all specifications Γ.

ML is expressive enough to specify all properties within various logical systems such as FOL, separation logic, λ-calculus, (dependent) type systems, and modal μ-calculus. In particular, it supports operational semantics (term rewriting) and axiomatic semantics (Hoare triples).

theory LISTofNAT
Imports: DEF
Symbols: $Nat, List, zero, succ, nil, cons$
Notations:
$\exists x{:}s.\varphi \equiv \exists x.x \in [\![s]\!] \wedge \varphi$
$\forall x{:}s.\varphi \equiv \forall x.x \in [\![s]\!] \to \varphi$
Axioms:

(Inductive Domain) :	$[\![Nat]\!] = \mu N.zero \vee succ\ X$
	$[\![List]\!] = \mu L.nil \vee cons\ [\![Nat]\!]\ L$
(Function) :	$\exists y.y \in [\![Nat]\!] \wedge zero = y,$
	$\forall x.x \in [\![Nat]\!] \to \exists y.y \in [\![Nat]\!] \wedge succ\ x = y;$
	$\exists y.y \in [\![List]\!] \wedge nil = y,$
	$\forall x.x \in [\![Nat]\!] \wedge l \in [\![List]\!] \to \exists y.y \in [\![List]\!] \wedge cons\ x\ l = y$
(NoConfusion I) :	$zero \neq nil$
	$\forall x{:}Nat.\forall l{:}List.zero \neq cons\ x\ l$
	$\forall x{:}Nat.zero \neq succ\ x$
	$\forall l{:}List.nil \neq succ\ x$
	$\forall x{:}Nat.\forall l{:}List.nil \neq cons\ x\ l$
	$\forall n{:}Nat.\forall x{:}Nat.\forall l{:}List.succ\ n \neq cons\ x\ l$
(NoConfusion II) :	$\forall x{:}Nat.\forall x'{:}Nat.succ\ x = succ\ x' \to x = x'$
	$\forall x, x'{:}Nat.\forall l, l'{:}List.cons\, x\, l = cons\, x'\, l' \to x = x' \wedge l = l'$

endtheory

Figure 2: LISTofNAT

Given an algebraic many-sorted signature (S, F), the initial term (S, F)-algebra can be captured in matching logic up an isomorphism [4]. An example is the specification of lists of natural numbers given in Fig. 2. Such a specifications includes the definition of sorts $s \in S$, o their inhabitants sets $[\![s]\!]$, axioms specifying the function symbols F, inductive definition for the inhabitant sets using the least fixpoint binder (called also no-junk axioms), and no-confusion axioms for the term constructors.

In this talk, we show how to internalize many concepts regarding term algebra, including induction, unification and anti-unification [1, 2], within ML. By "internalization", we mean to map concepts, theorems, and the main reasoning methods in the term algebra, including the induction, onto formulas, logical theories/axioms, and formal proofs in matching logic.

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Emotion-based Norm Identification

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Norms are one of the important elements that agents need to consider for decision-making apart from their belief, desire and intention. Research on norm-based reasoning investigates how to resolve the conflicts between goal achievement and norm compliance, or how to design appropriate sanction-based norms in order to enforce desired system behavior. However, all this research assumes that agents are aware of norms so that agents can decide whether to comply with the norms or not. The area where people study how to realize this assumption is called norm awareness and identification. The problem of norm identification considers open multi-agent systems in which no central authority imposes or proposes norms, and considers how agents can identify norms that are already prevalent in a society or group, through their own experience and/or observations of others [2]. Currently, the norm-identification techniques rely on big data and machine learning. For example, Edmond Awad et al. extract ethical principles from dilemma vignettes using inductive logic programming (ILP) [1], and Stephen Cranefield and Ashish Dhiman identify norm candidates from a normative language using Markov Chain Monte Carlo (MCMC) search [2]. Alternatively, norms can also be identified based on the detection of emotion-based sanctioning signals. The work of Savarimuthu et al. claims that sanctioning signals may convey some negative or positive emotional force [3]. Together with the contexts, agents are able to infer the norms that triger their emotions. Nevertheless, how detected emotions convey information to observers is still mystery in their work. In psychology, it is argued that observers can glean information from others' emotional expressions via inference [4]. Typically, the information may indicate the inner states of the expresser such as what is valued and what is disvalued. This valuable information can be collected for the inference of social norms. For instance, if we see several persons get angry with another person for queue jumping, we can infer that they disapprove the act of queue jumping. In this abstract, we will present an idea to develop a logic-based framework that allows agents to reason about detected emotions for norm identification.

Compared with existing learning-based approaches that allow agents to identify norms during training before execution, our logic-based approach allows agents to identify norms directly during execution, making it more possible to deploy autonomous agents in open and unknown environments.

The model we use is a Kripke structure that represents a multi-agent system, which consists of agents, states, actions, agents' epistemic accessibility relation, a transition relation and an valuation function. Different people may have different emotional reactions to the same event, depending on the standards they use for evaluation. For example, someone gets joyful for breaking a cup because he can have a new one; someone gets distressed for for breaking a cup because he cannot use it anymore. In this abstract, we use value systems as agents' evaluation standards. A value can be seen as an abstract standard according to which agents have their preferences over states. In this sense, it is interpreted as a state property v. Moreover, a value can be regarded as a criterion for judging an action being praiseworthy or blameworthy in itself, irrespective of the result the action brings about. For example, helping people who are in need is always praiseworthy and sneering at people who are in need is always blameworthy. Comparing to other decision criteria such as goals and utilities, value systems are relatively stable over the life span of agents. Based on a psychological model of emotions, we can define an agent i's emotion $\mathrm{emt}_i(a, v)$ with respect to an action a and a value v as a state property. For example,

$$\mathrm{joy}_i(a, v) \overset{def}{=} B_i(v \wedge \langle -a \rangle \neg v) \wedge \mathrm{Val}_i(v),$$

which means that agent i is joyful with respect to action a and value v if and only if agent i believes that it is the case that v holds and v did not hold before action a was performed and agent i has value v. Because of our definitions of emotions, an agent's belief of another agent's emotion always implies his belief of another agent's value. Namely, given a multi-agent system $\mathcal{M}$ and a state s,

$$\mathcal{M}, s \models B_i\, \mathrm{emt}_j(a, v) \rightarrow B_i\, \mathrm{Val}_j(v).$$

In order for agents to identify norms from what agents' value about, the norms we refer to here are social norms and moral norms that have been accepted and used for judgment by emotion expressers. Following standard deontic logic, we represent a norm as $O\varphi$, read as "it is obligatory to be φ". How can we relate the belief of emotions and the belief of norms? One can imagine that different agents may have different ways to identify a norm. The first approach is a quantitative approach, denoted as $\mathfrak{Q}(t)$. An agent who applies this approach has a natural number t, representing the threshold above which an agent believes the existence of a norm. More precisely, the agent keeps a counter such that it is incremented when he gets to believe one more agent values about the same action or states of affairs as who it encountered before, and he believes a norm when the counter is above his threshold. Given a multi-agent system $\mathcal{M}$ and a state s, agent i uses a quantitative approach with a threshold t to identify a norm $O\varphi$ iff there exists a set of agents $\{1, \dots, t, t+1\}$ such that

$$\mathcal{M}, s \models B_i(\mathrm{Val}_1(\varphi) \wedge \ldots \wedge \mathrm{Val}_t(\varphi) \wedge \mathrm{Val}_{t+1}(\varphi)) \leftrightarrow B_i O\varphi,$$

meaning that agent i only believes a norm $O\varphi$ iff he believes there exists a set of agents with the number of $t+1$ having value φ. Alternatively, an agent's norm identification only relies on his belief of model agents' values, denoted as $\mathfrak{M}(R)$. We assume that there exists a group of model agents R whose value systems represent what the majority of the system has. An agent identifies a norm when he believes a model agent has a corresponding value. Given a multi-agent system $\mathcal{M}$ and a state s, agent i uses a model-based approach with a set of model agents R to identify a norm $O\varphi$ iff there exists $j \in R$ such that

$$\mathcal{M}, s \models B_i \operatorname{Val}_j(\varphi) \leftrightarrow B_i O\varphi,$$

meaning that agent i only believes a norm $O\varphi$ iff he believes there exists a model agent j having value φ. Because the belief of value systems can be implied from the belief of emotions, the condition of identifying a norm becomes the witness of enough agents or model agents expressing the corresponding emotions for the same situation, making $\mathfrak{Q}(t)$ and $\mathfrak{M}(R)$ emotion-based approaches for norm identification, which can be expressed as

$$\mathcal{M}, s \models B_i(\operatorname{emt}_1(a,v) \wedge \ldots \wedge \operatorname{emt}_t(a,v) \wedge \operatorname{emt}_{t+1}(a,v)) \leftrightarrow B_i O\varphi,$$

$$\mathcal{M}, s \models B_i \operatorname{emt}_j(a,v) \leftrightarrow B_i O\varphi.$$

Notice that our quantitative approach can be applied for an agent to identify norms through encountering the same situation and observing agents' emotion reactions in *different* time steps, if we assume that agents' belief of values systems cannot be changed by the performance of any physical actions. For example, instead of observing several agents show their ugly faces for a queue jumper at one time-step, agent i has observed that an agent shows his ugly face for a queue jumper for several times, and from that agent i concludes that it is forbidden to jump the queue. In order to use the model-based approach, an agent has to be aware the set of model agents before starting to identify norms.

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The disjunction property in a labelled multi-succedent intuitionistic calculus

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Proof theory, G3I, intuitionistic logic, disjunction property, intermediate logics.

The calculus G3I is a (G3-style) labelled [1, 2] intuitionistic multi-succedent sequent calculus with internalized Kripke semantics for intuitionistic logic [3, 4]. It has gained prominence as a sequent calculus useful to obtain results that require as uniform as possible a treatment for classical and intuitionistic propositional bases. Importantly, it enjoys full rule invertibility, so one can algorithmically, and in a purely local procedure, obtain a countermodel from any branch of the proof-search tree that does not end with an initial sequent. Thus, proofs of completeness are significantly streamlined.

Another upside of a labelled system is that one can extend the base intuitionistic logic formulated in G3I with rules of the appropriate (geometric) form [5] to obtain intermediate logics [3, 6]. As a consequence of the geometric format, all the structural properties of a system are retained.

Disjunction property (as well as that property under Harrop assumptions) is a fundamental result about intuitionistic calculi. Using a G3-style single-succedent calculus this result is almost immediate and therefore, given their deductive equivalence, we can indirectly see that this property also holds for G3I.

However, taking such an indirect route might not be satisfactory from a constructivist standpoint, and thus at odds with the system under consideration. Instead, we here develop a method of organizing labels into chains used to transform a derivation, in G3I, of a multi-succedent endsequent into one with only a single formula in the succedent of the endsequent. The disjunction property then follows immediately by invertibility. In addition to being more appropriate for the subject matter at hand, this procedure provides sharper results by offering an explicit algorithm for obtaining the required derivation.

Furthermore, some of the intermediate logics likewise posses the disjunction property ([7] provides a useful overview). Of these, Scott's logic [8, 9] corresponds to a property of Kripke frames [10, 11] which has not so far been captured via a set of geometric rules. We first fill this lacuna and offer a geometric representation of Scott's logic, demonstrate that it derives its characteristic axiom, and then show that our method can be extended to likewise successfully prove that the disjunction property holds of it.

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Connections between Logic and Geometry via Term Rewriting

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The short citation for the "2020 Rolf Schock Prize in logic and philosophy" says that it was awarded to Per Martin-Löf (shared with Dag Prawitz) "for the creation of constructive type theory." In a longer statement, the prize committee recalls that constructive type theory is "a formal language in which it is possible to express constructive mathematics" (...) "[which] also functions as a powerful programming language and has had an enormous impact in logic, computer science and, recently, mathematics." Indeed, by introducing a framework in which a formalisation of the logical notion of equality, via the so-called "identity type", it allows for a surprising connection between term rewriting and geometric concepts such as path and homotopy. As a matter of fact, Martin-Löf's type theory (MLTT) allows for making useful bridges between theory of computation, algebraic topology, logic, categories, and higher algebra, and a single concept seems to serve as a bridging bond: "path". The impact in mathematics has been felt more strongly since the start of Vladimir Voevodsky's program on the univalent foundations of mathematics around 2005, and one specific aspect which we would like to talk about here is the calculation of fundamental groups of surfaces. Taking from the Wikipedia entry on "homotopy group", calculation of homotopy groups is in general much more difficult than some of the other homotopy invariants learned in algebraic topology. Now, by using an alternative formulation of the "identity type" which provides an explicit formal account of "path" (and "path rewriting"), operationally understood as an invertible sequence of rewrites (such as Church's "conversion"), and interpreted as a homotopy, we wish to show examples of calculating fundamental groups of surfaces such as the circle, the torus, the 2-holed torus, the Klein bottle, and the real projective plane. We would like to suggest that these examples might bear witness to the impact of MLTT in mathematics by offering formal tools to calculate and prove fundamental groups. As for the impact in mathematics for the foundations of computer science, the connections between identity types and infinity-groupoids seems to help in the construction of models for higher-order lambda-calculus via homotopy theory.

The Multiplicative-Additive Lambek Calculus with Subexponential and Bracket Modalities: Undecidability and Decidable Fragments

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We give a proof-theoretic and algorithmic complexity analysis for systems introduced by Morrill to serve as the core of the CatLog categorial grammar parser. We consider two recent versions of Morrill's calculi, and focus on their fragments including multiplicative (Lambek) connectives, additive conjunction and disjunction, brackets and bracket modalities, and the ! subexponential modality. For both systems, we resolve issues connected with the cut rule and provide necessary modifications, after which we prove admissibility of cut (cut elimination theorem). We also prove algorithmic undecidability for both calculi, and show that categorial grammars based on them can generate arbitrary recursively enumerable languages. This is joint work with Max I. Kanovich and Stepan L. Kuznetsov [1]. We also consider fragments where the usage of subexponential is restricted by the so-called bracket non-negative/non-positive conditions. We prove that these fragments are decidable, and pinpoint their place in the complexity hierarchy. We also consider a more complicated, but more practically interesting problem of inducing (guessing) brackets. For this problem, we prove one decidability and one undecidability result, and leave some open questions for further research. This is joint work with Max I. Kanovich, Stepan G. Kuznetsov, and Stepan L. Kuznetsov [2].

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Crypto-Covid: Privacy challenges in BlockChain and Contact Tracing

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Data Privacy, Blockchain, Contact Tracing

One of the most urgent problems in the digital world is the problem of data privacy. In addition to the philosophical, legal and economic aspects of privacy, there is a need to study the technological aspect of privacy which is mainly based on mathematical models. We first analyze the development of mathematical models of privacy, as a continuation of research from [7]. We start with initial models for data privacy, like k-anonimity [8], l-diversity [6] and t-closeness [5]. We continue with more advanced models, like differential privacy [2], contextual integrity [1] and inverse privacy [4]. We then focus on the relationship between mathematical models of privacy and blockchain technology, considering two directions - privacy in blockchain and blockchain in privacy [9]. Finally, we explore digital contact tracing applications which became a very significant topic during the COVID-19 pandemic. We point out the problems in the functioning of these applications and present our solution for overcoming the problem of their interconnection through distributed overlay networks such as Chord and Synapse. The results of this research can be found in [3].

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Proof Theory for Intuitionistic Temporal Logic over Topological Dynamics

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Topological dynamics is a branch of dynamical systems theory which studies the asymptotic behaviour of continuous functions on topological spaces. A (topological) dynamic system is a topological space $\mathcal{X} = (X, \tau)$ equipped with a continuous function $S : X \longrightarrow X$. Based on Tarski's observations that modal logic can be evaluated in topological spaces [9], Artemov et al. introduced in 1997 a temporal logic that extends modal logic by the next operator $\bigcirc$ to reason about topological dynamic systems [1]. From a temporal point of view the continuous function S can be regarded as a time-function which maps points of the topological space from one time moment to the next. The next operator is therefore used to reason about the behaviour of S. The work of Artemov et al. was later continued by Kremer and Mints [6] by extending their system with the temporal operators $\Diamond$ called eventually and $\Box$ called henceforth. The resulting system is called Dynamic Topological Logic (DTL). The addition of $\Diamond$ and $\Box$ substantially increases the expressive power of DTL and allows one to formulate interesting properties of dynamical systems. The project to build a logic to reason about topological dynamics however suffered a setback when Konev et al. proved that DTL is not decidable [5]. As a consequence of this result the focus of the project has shifted from DTL to an intuitionistic variant of DTL called Intuitionistic Temporal Logic (ITL). This focus shift is motivated by the observation that intuitionistic logic has better computational properties than classical logic and so it is hoped that ITL is decidable. Indeed, first results about ITL are promising: In 2018, Fernández-Duque established decidability of a fragment of ITL called $\mathsf{ITL}_\Diamond$ which only contains the next and the eventually operator [4]. Importantly, henceforth and eventually are not interdefinable in ITL (in contrast to DTL) as the base logic of ITL is intuitionistic. The proof of decidability relies on model theoretic techniques, in particular on the construction of so-called quasi models. Later, Boudou et al. proved completeness of this fragment with respect to the class of topological dynamic systems [2] by using similar techniques.

While the semantical aspects of ITL have been studied quite extensively in recent years, there is little known about the proof theory of ITL. Our long term goal is to fill this gap and provide a satisfying proof theory for intuitionistic temporal logic. For a start, we aim to investigate the proof theory of $\mathsf{ITL}_\diamond$. Our project roughly consists of three main steps:

1. Define a sound and complete cyclic proof system for $\mathsf{ITL}_\diamond$.
2. Establish cut-elimination either syntactically or by an indirect argument.
3. Use the cut-free system to obtain a syntactic decidability proof and invesitigate the complexity of the validity problem.

At the point of writing this abstract we have completed step 1 and we are currently investigating the second step. In the following we describe in more detail each step.

For step 1 we define a cyclic proof system called $\mathsf{ITL}^c_\diamond$ which is based on a standard multi-conclusion sequent calculus for intuitionistic logic. This calculus is extended by rules for the next operator and the eventually operator. In particular, the rules for $\diamond$ are standard unfolding rules, which replace the formula $\diamond A$ by its equivalent unfolding $A \vee \bigcirc\diamond A$. The rules for $\diamond$ together with the cycle mechanism characterize the formula $\diamond A$ as the least fixed point of the function $X \mapsto A \vee \bigcirc X$. As henceforth is not definable in our language, there does not exist any form of fixed point alternation in $\mathsf{ITL}_\diamond$. This implies that characterizing successful repetitions in a cyclic proof is a much easier task than for other fixed point logics such as the modal mu-calculus. In particular, we do not require a focus mechanism for our system. Soundness of $\mathsf{ITL}^c_\diamond$ is established by a minimal counter model approach which is common in the literature (see for example [8]). For completeness we consider a Hilbert style proof system for $\mathsf{ITL}_\diamond$ which is proven to be complete with respect to the class of topological dynamic systems in [2] and show how to embed it into the cyclic calculus $\mathsf{ITL}^c_\diamond$. As a consequence of this technique we do not obtain cut-free completeness, as the cut-rule is needed to derive the modus ponens rule. An important goal of our work is therefore to also establish cut free completeness, which brings us to step 2.

For step 2 we plan to establish a cut-elimination result by providing a syntactic cut-elimination procedure similar to the continuous cut-elimination procedure of Savateev and Shamkanov in [7]. To that end we define a non-wellfounded proof system called $\mathsf{ITL}^n_\diamond$ for $\mathsf{ITL}_\diamond$. We first show how to unfold a cyclic proof into a non-wellfounded proof and vice versa, how to prune a non-wellfounded proof into a cyclic one. By doing so we establish soundness and completeness of the non-wellfounded system. Then a procedure is described to eliminate cuts in $\mathsf{ITL}^n_\diamond$.

Finally, for step 3, we plan to establish decidabilty of $\mathsf{ITL}_\diamond$ by translating the non-wellfounded calculus $\mathsf{ITL}^n_\diamond$ minus cut into a parity game called proof search game.

Our work is a continuation of the project to develop logics for reasoning about topological dynamics with good computational properties. We hope to provide a first insight into the proof theory of intuitionistic temporal logics and lay a foundation to investigate more complicated logics, in particular the logic ITL based on the full language with next, eventually and henceforth. The work on cut elimination is especially interesting, as surprisingly little can be found about this topic for cyclic proofs in general and we are interested in filling this gap. Furthermore, we hope to provide a new proof of decidability of $\mathsf{ITL}_\Diamond$ which, in contrast to [4], relies entirely on syntactic arguments.

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A note on countable additivity

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The "definition" of probability as long run frequency does not work. But even Kolmogorov used the "definition" heuristically because it makes the proof of the axioms of probability very easy - apart from the axiom of countable additivity. The common opinion is that limiting frequencies violate countable additivity due to very simple counterexamples. We prove that it is not the case. So limiting frequencies have no problems with any of the probability axioms. Their problem is that they may not exist, i.e. it is possible that an infinite sequence of experimental results has no limiting frequency (but c.f. the random sequence concept due to Martin-Löf).

Modeling Complex Systems in Rewriting Logic

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Rewriting Logic (RWL) is a logic of concurrency and change. It has a dual logical and computational interpretation; application of rules generates proofs or executions; one can search for proofs or reachable states. Using a case study concerning security of Industry 4.0 applications, we will present two techniques using RWL to manage model complexity: formal patterns and symbolic execution. Formal patterns are a means to use abstract models to design and reason about systems and to generate deployable representations by property preserving transformations. Symbolic execution allows representing families of system states and executions compactly, increasing converage of reasoning and decreasing size of search for reachable states of interest.

On the complexity of the Quantified Constraint Satisfaction Problem

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Keywords:
Quantified Constraints, CSP, Computational Complexity.

The Constraint Satisfaction Problem (CSP) is the problem of deciding whether there is an assignment to a set of variables subject to some specified constraints. In general this problem is NP-complete and one of the ways to make it solvable in polynomial time (tractable) is by restricting the set of allowed constraints. In 2017 it was proved [3, 2, 5, 4] that for any constraint language Constraint Satisfaction Problem over this language is either solvable in polynomial time, or NP-complete. For example, systems of linear equations and graph 2-colouring are solvable in polynomial time while graph 3-colouring is NP-hard [1].

The *Quantified Constraint Satisfaction Problem (QCSP)* is the generalization of the Constraint Satisfaction problem where both existential and universal quantifiers are allowed. Formally, the QCSP over a constraint language Γ is the problem to evaluate a sentence of the form

$$\forall x_1 \exists y_1 \forall x_2 \exists y_2 \dots \forall x_n \exists y_n \ (R_1(\dots) \wedge \dots \wedge R_s(\dots)),$$

where $R_1, \dots, R_s$ are relations from Γ. While CSP remains in NP for any Γ, QCSP(Γ) can be PSpace-hard, as witnessed by Quantified 3-Satisfiability [10] or Quantified Graph 3-Colouring [6]. Nevertheless, if Γ consists of linear equations modulo p then QCSP(Γ) is tractable [6]. For many years there was a hope that for any constraint language the QCSP is either in P, NP-complete, or PSpace-complete. Moreover, a very simple conjecture describing the complexity of the QCSP was suggested by Hubie Chen [8, 9]. However, in 2018 together with Mirek Olšák and Barnaby Martin we discovered constraint languages for which the QCSP is coNP-complete, DP-complete, and even Θ_2^P-complete, which refutes the Chen conjecture [7]. Additionally, we described the complexity for each constraint language on a 3-element domain with constants. It turned that the QCSP in this case is either solvable in polynomial time, or NP-complete, or coNP-complete, or PSpace-complete. Nevertheless, after we discovered so many complexity classes we did not hope to obtain a complete classification.

Recently, I obtained several results that make me believe that such a classification is closer than it seems. First, I obtained an elementary proof of the

PGP reduction, which allows to reduce the QCSP to the CSP. Second, I showed that QCSP over any language is other in Π_2^P or PSpace-complete, that is, there is a gap between Π_2^P and PSpace. Moreover, I found a criterion for the QCSP to be PSpace-hard. Finally, I discovered a constraint language on a 6-element domain such that QCSP over this language is Π_2^P-complete, and I believe this is the last complexity class such that the QCSP over some language is complete in this class.

In the talk I will discuss the above and some other results.

Acknowledgment

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5th workshop Formal Reasoning and Semantics (FORMALS 2022)

a satellite workshop of 11th conference Logic and Applications (LAP 2022)

Inter-University Center, Dubrovnik

26–29 September 2022

This workshop is organized within the research project Formal Reasoning and Semantics (FORMALS), supported by Croatian Science Foundation (HRZZ), under the project UIP-2017-05-9219.

The 1st, 3rd and 4th workshop (FORMALS 2018, 2020, 2021) were also co-located with Logic and Applications conference (LAP 2018, 2020, 2021) in Dubrovnik. The 2nd workshop (FORMALS 2019) was held at the Faculty of Teacher Education, University of Zagreb.

The present workshop consists of the project research group members' talks (T. Ban Kirigin, B. Perak, A. Hatzivelkos, L. Mikec), an invited talk (V. Nigam) and contributed talks (S. Bujačić Babić, Y. Petrukhin).

The workshop is organized in a hybrid form, part of the contributors being present in Dubrovnik, while others participate online.

We are grateful to the directors of LAP for agreeing this workshop to be a part of the conference.

On behalf of the FORMALS project research group,

Tin Perkov

Semi-Local Integration Centrality for Complex Networks

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Keywords:
centrality measure; node importance; complex networks; applications of graph data processing; lexical graph analysis.

Centrality is one of the fundamental concepts in graph theory and network analysis. Numerous centrality measures have been introduced to reflect various properties of complex networks such as connectivity, survivability, and robustness, and attempt to numerically evaluate the importance of nodes in a network.

In this work, we introduce Semi-Local Integration (SLI), which evaluates the integration of nodes within their neighbourhood. This centrality measure evaluates the importance of nodes according to how integrated they are in the local subnetwork. The measure considers both the *weighted degree* centrality of the node itself and the *weighted degree* of the adjacent nodes, as well as the number of cycles that are part of the neighbouring subnetwork of the node itself.

SLI centrality is particularly suitable for applications in dynamic and complex networks, where it could optimize the analysis of subnetwork structures, including friend-of-a-friend (FoF)-based networks such as social networks. We demonstrate the potential of applications of the SLI measure in the analysis of lexical networks [3, 4, 5], which form the basis of many natural language processing (NLP) tasks.

The Python function implementing the SLI measure is available in the GitHub repository [2].

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Syntactic Dependency Networks: Cognitive Aspects of Hierarhical Multi-Layer Structures

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Keywords:
Universal dependencies; Dependency graph analysis; Semantics; Syntax.

Summary

Lexical items, i.e. words are building blocks of linguistic structures such as phrases, sentences, paragraphs, texts, etc that encode the conceptual content perceived, or imagined, by human cognitive processing. The sequence of lexical items within a linguistic structure is organised by a set of syntactic relations construing a conceptual relation and representing an emergent semantic structure or a meaning. According to the Universal Dependencies (UD), a framework for morphosyntactic annotation of human language, which to date has been used to create treebanks for more than 100 languages (De Marneffe et al. 2022), the classification of syntactic relations offers a linguistic representation that is useful for morphosyntactic research, semantic interpretation, and for practical natural language processing across different human languages. A syntactic dependency, in general, is a binary phrasal asymmetric grammar relation with a lexical head and other lexical items as dependents of that head, represented in diagrams by an arrow from the head word to the dependent word. This work will present the types of UD dependency relations and their potential for multi-layered syntactic-semantic analysis of texts. The types of dependencies are classified according to the emergent cognitive aspects of the syntactic-semantic complexity represented in terms of the hierarchical emergent ontological schema of semantic roles and agent-based representation. The multi-layered formalization of the dependency

structures extracted from tagged corpora can be used for a graph representation of common knowledge of conceptual entities, attributes and processes as well as downstream NLP applications, such as lexical labelling, figurative speech identification and dictionary-based sentiment analysis systems.

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On p-Disapproval voting characterization

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Keywords:

Approval voting, Disapproval voting, characterization, dilemma

Voting procedure p-Disapproval voting is a generalization of standard Diss&Approval voting system (as presented by Alcantu and Laruell, see [1]), build upon the grading which utilizes a parameter value p. As previously shown (see [4]), such generalization enables the fulfillment of the strong version of the Compromise axiom, as presented in [3]. In this paper we are exploring the possibilities of axiomatic characterization of the p-Disapproval voting, following the method presented by Gonzalez, Laruelle and Solal [2].

With this in mind, we show that p-Disapproval vote satisfies standard Anonymity and Neutrality Axioms. Following the work of Gonzalez et al. we define the Independence of unconcerned voters and Independence of Pareto dominated candidates axioms, and show that p-Disapproval vote satisfies both of them. As a result, p-Disapproval vote belongs to the class of voting functions described in [2]. Furthermore, we introduce the notion of voting rule characterization *dilemma*, and explore the possibilities of construction a characterization *dilemma* for p-Disapproval vote.

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FORMALS contributions overview (2018–2022)

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Keywords:
interpretability logic, Veltman semantics, generalised Veltman semantics, modal logic.

In this talk no new results will be presented, instead an overview will be given of the results that the author contributed to during the FORMALS project. The following results will be mentioned:

- modal completeness of the interpretability logics $\mathbf{IL}\mathsf{R}$ ([4]) and $\mathbf{IL}\mathsf{P}_0$ ([4]);
- complexity of the interpretability logics **IL** ([7]), $\mathbf{IL}\mathsf{W}$ ([2]), and $\mathbf{IL}\mathsf{P}$ ([2]);
- advances in *labelling*, in particular the 'labelling systems' ([1]);
- conditional modal completeness of the interpretability logic $\mathbf{IL}\mathsf{W}_\omega$ ([5]);
- arithmetical soundness of the interpretability logic $\mathbf{IL}\mathsf{W}_\omega$ ([5]);
- determining relationships between some existential modal logics ([6]);
- implementing an algorithm for the inverse of standard translation ([3]).

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Automating Safety Proofs about Cyber-Physical Systems using Rewriting Modulo SMT

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Cyber-Physical Systems, such as Autonomous Vehicles (AVs), are operating with high-levels of autonomy allowing them to carry out safety-critical missions with limited human supervision. To ensure that these systems do not cause harm, their safety has to be rigorously verified. Existing works focus mostly on using simulation-based methods which execute simulations on concrete instances of logical scenarios in which systems are expected to function. The level of assurance obtained by these methods is, therefore, limited by the number of simulations that can be carried out. A complementary approach is to produce, instead, proofs that vehicles are safe for all instances of logical scenarios. We investigate how Rewriting modulo SMT applied to Soft Agents, a rewriting framework for the specification and verification of Cyber-Physical systems, can be used to generate such proofs in an automated fashion. In particular, rewrite rules specify the executable semantics of systems on logical scenarios instead of concrete scenarios. This is accomplished by generating at each execution step a set of (non-linear) constraints whose satisfiability are checked by using SMT-solvers. Intuitively, a model of such a set of constraints corresponds to a concrete execution on an instance of the corresponding logical scenario. We demonstrate how to specify and verify scenarios in this framework using an example involving a vehicle platoon. Finally, we investigate the trade-offs between how much of the verification is delegated to search engines (namely Maude) and how much is delegated to SMT-solvers (e.g., Z3).

Cut-free hypersequent calculus for a non-contingency version of S5

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Abstract

In this report, we modify Restall's cut-free hypersequent calculus for **S5** and obtain a cut-free hypersequent calculus for a non-contingency version of **S5**.

Sequent and hypersequent calculi for modal logics is a fruitful and well-developed area of research. (Almost) all standard modal logics have already had cut-free sequent or hypersequent calculi. The modal logic **S5** is especially remarkable in this sense. Although it does not have a cut-free sequent calculus, it has at least 8 different cut-free hypersequent calculi and several cut-free non-standard sequent calculi (see [1, 2] for more details). However, in the case of non-standard modalities (e.g., contingency or non-contingency) the situation is different.

Non-cut-free sequent calculi for some non-contingency logics were developed by Zolin [6, 7]. In particular, he considered the logic $\mathbf{S5}^{\triangleright}$, the non-contingency version of **S5**. Since there are a lot of cut-free calculi for **S5**, we think that this logic is a good starting point for the development of cut-free (hyper)sequent calculi for non-contingency logics.

First of all, we fix two modal languages, $\mathscr{L}_{\Box}$ and $\mathscr{L}_{\triangleright}$, with the alphabets $\langle \mathscr{P}, \neg, \rightarrow, \Box, (,) \rangle$ and $\langle \mathscr{P}, \neg, \rightarrow, \triangleright, (,) \rangle$, respectively, where $\mathscr{P} = \{p, q, r, p_1, \ldots\}$ is the set of propositional variables, $\Box$ is a necessity operator, and $\triangleright$ is a non-contingency operator. The other propositional connectives are introduced by definitions in a standard way. The notion of a formula in these languages is understood as usual. Let us define the translation function τ from $\mathscr{L}_{\triangleright}$ to $\mathscr{L}_{\Box}$ as follows: $\tau(\triangleright\phi) = \Box\tau(\phi) \vee \Box\neg\tau(\phi)$ (the propositional case remains unchanged). Then we can put $\mathbf{S5}^{\triangleright} = \{\phi \in \mathscr{L}_{\triangleright} \mid \tau(\phi) \in \mathbf{S5}\}$. We are able to define a translation function from $\mathscr{L}_{\Box}$ to $\mathscr{L}_{\triangleright}$: $\tau^*(\Box\phi) = \tau^*(\phi) \wedge \triangleright\tau^*(\phi)$ (the propositional case remains unchanged). As follows from [7], $\mathbf{S5} = \{\phi \in \mathscr{L}_{\Box} \mid \tau^*(\phi) \in \mathbf{S5}^{\triangleright}\}$.

Montgomery and Routley [3] (see also Zolin [7]) presented an axiomatic system for the logic $\mathbf{S5}^{\triangleright}$ which has all the classical axioms in the language $\mathscr{L}_{\triangleright}$ as well as the following ones:

($A^{\triangleright}_{\neg}$) $\triangleright p \leftrightarrow \triangleright\neg p$

($A^{\triangleright}_{\mathbf{T}}$) $p \to (\triangleright(p \to q) \to (\triangleright p \to \triangleright q))$

($A^{\triangleright}_{\mathbf{5}}$) $\triangleright\triangleright p$

Inference rules are as follows: (MP) $\dfrac{\phi \quad \phi \to \psi}{\psi}$, (Sub) $\dfrac{\phi}{\phi[\psi/p]}$, and (Dec) $\dfrac{\phi}{\triangleright\phi}$,
where $\phi[\psi/p]$ the result of a replacement of all the occurrences of a variable p in ϕ with ψ.

Our cut-free hypersequent calculus for $\mathbf{S5}^{\triangleright}$ is a modification of Restall's hypersequent calculus for **S5** [5] (we replace the rules for $\Box$ with the rules for $\triangleright$). The axiom and the rules of our calculus are presented below (the notion of a sequent is understood in a standard way, a hypersequent is a multiset of sequents, the letters $\Gamma, \Delta, \Pi, \Sigma$ stand for finite sets of $\mathscr{L}_{\triangleright}$-formulas, and the letters H, G stand for hypersequents).

$$(\text{Ax})\ \phi \Rightarrow \phi$$

$$(\text{Merge})\ \frac{\Gamma \Rightarrow \Delta \mid \Pi \Rightarrow \Sigma \mid H}{\Gamma, \Pi \Rightarrow \Delta, \Sigma \mid H} \qquad (\text{Cut})\ \frac{\Gamma \Rightarrow \Delta, \phi \mid H \qquad \phi, \Pi \Rightarrow \Sigma \mid G}{\Gamma, \Pi \Rightarrow \Delta, \Sigma \mid H \mid G}$$

$$(\text{EW}\Rightarrow)\ \frac{H}{\phi \Rightarrow \mid H} \qquad (\Rightarrow\text{EW})\ \frac{H}{\Rightarrow \phi \mid H} \qquad (\text{IW}\Rightarrow)\ \frac{\Gamma \Rightarrow \Delta \mid H}{\phi, \Gamma \Rightarrow \Delta \mid H} \qquad (\Rightarrow\text{IW})\ \frac{\Gamma \Rightarrow \Delta \mid H}{\Gamma \Rightarrow \Delta, \phi \mid H}$$

$$(\neg \Rightarrow)\ \frac{\Gamma \Rightarrow \Delta, \phi \mid H}{\neg\phi, \Gamma \Rightarrow \Delta \mid H} \qquad (\Rightarrow \neg)\ \frac{\phi, \Gamma \Rightarrow \Delta \mid H}{\Gamma \Rightarrow \Delta, \neg\phi \mid H}$$

$$(\to\Rightarrow)\ \frac{\Gamma \Rightarrow \Delta, \phi \mid H \qquad \psi, \Pi \Rightarrow \Sigma \mid G}{\phi \to \psi, \Gamma, \Pi \Rightarrow \Delta, \Sigma \mid H \mid G} \qquad (\Rightarrow\to)\ \frac{\phi, \Gamma \Rightarrow \Delta, \psi \mid H}{\Gamma \Rightarrow \Delta, \phi \to \psi \mid H}$$

$$(\triangleright \Rightarrow)\ \frac{\phi, \Gamma \Rightarrow \Delta \mid H \qquad \Pi \Rightarrow \Sigma, \phi \mid G}{\triangleright\phi \Rightarrow \mid \Gamma \Rightarrow \Delta \mid \Pi \Rightarrow \Sigma \mid H \mid G} \qquad (\Rightarrow \triangleright)\ \frac{\Rightarrow \phi \mid \phi \Rightarrow \mid H}{\Rightarrow\ \triangleright\phi \mid H}$$

We have proved the following theorems.

Theorem 1. *For any $\mathscr{L}_{\triangleright}$-formula ϕ, it holds that ϕ is provable in Zolin's axiomatic system for $\mathbf{S5}^{\triangleright}$ iff it is is provable in the hypersequent calculus for $\mathbf{S5}^{\triangleright}$.*

Theorem 2. *The rule* (Cut) *is admissible in the hypersequent calculus for $\mathbf{S5}^{\triangleright}$.*

As a kind of bonus, using the equalities $\triangleleft\phi = \neg\triangleright\phi$ and $\triangleright\phi = \neg\triangleleft\phi$, where $\triangleleft$ is a contingency operator, we are able to present the rules for this modality as well:

$$(\triangleleft \Rightarrow)\ \frac{\Rightarrow \phi \mid \phi \Rightarrow \mid H}{\triangleleft\phi \Rightarrow \mid H} \qquad (\Rightarrow \triangleleft)\ \frac{\phi, \Gamma \Rightarrow \Delta \mid H \qquad \Pi \Rightarrow \Sigma, \phi \mid G}{\Rightarrow \triangleleft\phi \mid \Gamma \Rightarrow \Delta \mid \Pi \Rightarrow \Sigma \mid H \mid G}$$

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