POST EQUATIONS AND INEQUATIONS

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A Post algebra of order r or an $r-$Post algebra is an algebra $(P, \wedge, \vee, e_0, e_1, ..., e_{r-1})$, where $(P, \wedge, \vee, 0, 1)$ is a bounded distributive lattice, the elements e_i satisfy $e_0 = 0 < e_1 < \cdots < e_{r-1} = 1$ and every element $x \in P$ has a unique representation of the form $x = \bigvee_{i=0}^{r-1}(x^i \wedge e_i)$, where $(x^0, x^1, \ldots, x^{r-1})$ is an orthonormal system, i.e. $\bigvee_{i=0}^{r-1} x^i = 1, \quad x^i \wedge x^j = 0 \quad (i, j = 0, \ldots, r-1, \ i \neq j)$. The sequence $e_0, \ldots, e_{r-1}$ is called the chain of constants of P and $x^o, \ldots, x^{r-1}$ are disjunctive components of x.

The pseudo-complement x^* of an element $x \in P$ is defined as

$$x^* = max\{y \mid xy = 0\}.$$

If $z \in P$ and there exists an element $\overline{z} \in P$ satisfying the conditions $z \vee \overline{z} = 1$ and $z \cdot \overline{z} = 0$, then $\overline{z}$ is called the complement of z.

Theorem 1. *(Epstein) If f is a Post polynomial in the variables $x_1, \ldots, x_n$, then*

$$f(x_1, \ldots, x_n) = \bigvee_{(a_1,\ldots,a_n) \in C_r^n} f(a_1, \ldots, a_n) x_1^{a_1} \cdots x_n^{a_n}. \tag{4}$$

Theorem 2. *A Post equation in one unknown*

$$\bigvee_{i=0}^{r-1} c_i x^i = 0$$

is consistent if and only if

$$\prod_{i=0}^{r-1} c_i = 0.$$

The function f is called evanescible if equation $f(X) = 0$ is consistent, i.e. has solutions $X \in P^n$.

S. Marinković, D. Banković, Subsumptive general solutions and parametric general solutions of Post equations, Information Sciences 187 (2012), 121-128.

Boolean equation

$$ax \vee bx' = 0 \Leftrightarrow b \leq x \leq a'$$

$$\begin{aligned} u_j(x_{j+1}, \ldots, x_n) \leq & x_j \leq v_j(x_{j+1}, \ldots, x_n) \quad (j = 1, \ldots, n-1), \\ u_n \leq & x_n \leq v_n \end{aligned}$$

Theorem 3. *The set of solutions of consistent Post equation* $\bigvee_{i=0}^{r-1} c_i x^i = 0$ *is the interval* $[\eta, \zeta]$ *if and only if*

$$\left(\bigvee_{h=0}^{i-1} c_h^* \right) \left(\bigvee_{j=i+1}^{r-1} c_j^* \right) \leq c_i^* \quad (i = 1, \ldots, r-2).$$

Is there, for any interval $[u, v]$ in Post algebra P, an evanescible Post function f such that the set of solutions of equation $f(x) = 0$ is this interval?

Theorem 4. *Let* $u, v \in P$ *such that* $u \leq v$. *There exists an evanescible Post function* $f : P \to P$ *such that*

$$f(x) = 0 \Longleftrightarrow u \leq x \leq v.$$

Boolean equation

$$ax \vee bx' = 0 \Leftrightarrow (\exists t) x = a't \vee bt'.$$

Let $\varphi_1, \ldots, \varphi_n : P^n \to P$ be Post functions and set $\Phi = (\varphi_1, \ldots, \varphi_n)$. The formulas

$$x_i = \varphi_i(t_1, \ldots, t_n) \ (i = 1, \ldots, n),$$

or in vector form $X = \Phi(T)$, determine a *parametric general solution* of equation $f(X) = 0$ if and only if

$$f(X) = 0 \Longleftrightarrow (\exists T \in P^n) X = \Phi(T).$$

Theorem 5. *For any* $\Phi = (\varphi_1, \ldots, \varphi_n)$, *where* $\varphi_i : P^n \to P$ *are Post functions, formula* $X = \Phi(T)$ *determines a parametric general solution of equation*

$$\prod_{A \in C_r^n} \bigvee_{i=1}^{n} (x_i + \varphi_i(A)) = 0.$$

where operation $+$ is defined in the following way $x + y = \bigvee_{i=0}^{r-1} x^i (y^i)'$, $x, y \in P$. One can prove $x = y \Longleftrightarrow x + y = 0$.

D. Banković, Post inequations, Journal of Multiple-Valued Logic and Soft Computing , in press.

Theorem 6. *Let f be a Post polynomial in variables $x_1, \dots, x_n$ and $f(X) \neq 0$ be a consistent inequation. If $p = \Psi(u)$ expresses the general solution of p-equation*

$$\bigvee_{j=0}^{r-1} p^j \prod_A \left((f(A))^j\right)^* = 0 \tag{6}$$

and $X = \Phi(T, p)$ expresses the general solution of the X-equation

$$f(X) = p,$$

then, for every $X \in P^n$,

$$f(X) \neq 0 \Leftrightarrow (\exists p)(\exists u)(\exists T)\big(p \neq 0 \wedge p = \Psi(u) \wedge X = \Phi(T, p)\big). \tag{7}$$

$$\begin{aligned} f(X) \neq 0 \quad &\Leftrightarrow (\exists p \in P)(p \neq 0 \wedge f(X) = p) \\ &\Leftrightarrow (\exists p \in P)(p \neq 0 \wedge (\exists X) f(X) = p \wedge (\exists T \in P^n) X = \Phi(p, T)) \\ &\Leftrightarrow (\exists p \in P)(p \neq 0 \wedge \textstyle\bigvee_{j=0}^{r-1} p^j \prod_A \left((f(A))^j\right)^* = 0 \wedge (\exists T \in P^n) X = \Phi(p, T)) \\ &\Leftrightarrow (\exists p \in P)(p \neq 0 \wedge (\exists u \in P) p = \Psi(u) \wedge (\exists T \in P^n) X = \Phi(p, T)) \\ &\Leftrightarrow (\exists p \in P)(\exists u \in P)(\exists T \in P^n)(p \neq 0 \wedge p = \Psi(u) \wedge X = \Phi(p, T)). \end{aligned}$$

Corrolary 1. *Let $f(x)$ be a Post polynomial in the variable x. Let $z_i = f(e_i)$, $y_i = \prod_{k=0}^{r-1} (z_k^i)^* \quad (i = 1, \dots, r-1)$,*

(8)

$$\begin{aligned} \psi(t, p) \quad = &\textstyle\bigvee_{i=0}^{r-1} \big((z_i + p)^* e_i \vee (z_i + p)^{**} (z_{i_1} + p)^* e_{i_1} \\ &\vee (z_i + p)^{**} (z_{i_1} + p)^{**} (z_{i_2} + p)^* e_{i_2} \vee \cdots \\ &\vee (z_i + p)^{**} (z_{i_2} + p)^{**} (z_{i_2} + p)^* \cdots (z_{i_{r-2}} + p)^{**} (z_{i_{r-1}} + p)^* e_{i_{r-1}}\big) t^i, \end{aligned}$$

where

$$(\forall i \in \{0, 1, \dots, r-1\}) \ \{i, i_1, \dots, i_{r-1}\} = \{0, 1, \dots, r-1\}.$$

and

$$\begin{aligned} \phi(u) \quad = &\textstyle\bigvee_{j=0}^{r-1} \big(y_j^* e_j \vee y_j^{**} y_{j_1}^* e_{j_1} \vee y_j^{**} y_{j_2}^{**} z_{j_2}^* e_{j_2} \vee \cdots \\ &\vee y_j^{**} y_{j_2}^{**} y_{j_2}^* \cdots y_{j_{r-2}}^{**} y_{j_{r-1}}^* e_{j_{r-1}}\big) u^j, \end{aligned} \tag{9}$$

where

$$(\forall j \in \{0, 1, \dots, r-1\}) \ \{j, j_1, \dots, j_{r-1}\} = \{0, 1, \dots, r-1\}.$$

If the inequation $f(x) \neq 0$ is consistent then, for every $x \in P$,

$$f(x) \neq 0 \Leftrightarrow (\exists p)(\exists u)(\exists t)(p \neq 0 \wedge p = \phi(u) \wedge x = \psi(t, p)). \tag{10}$$

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Chefarzt PD Dr. med. M. Weber

Nov 30 (3 days ago)

to C.Hamm, H.Moellmann, H.Nef, C.Liebetrau, V.Mitrovic, drapostolovic, dragic, dr.axel.schlitt, axelschlitt, andreas.luchner

Dear all,

I am extremely happy, that I can forward to you the information that the manuscript of the NoRisk study Incremental value of high sensitive troponin T (hsTnT) in addition to the revised cardiac index for perioperative risk stratification in non-cardiac surgery has been accepted for publication in the EHJ. It has been a lot of work from all of you to finally complete the study and to get the manuscript on the way. **For the last revision the help of Prof. Bankovic was particularly helpful**.

Thanks to you all!

Best wishes for the first Advents week end,

Best regards

Michael

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