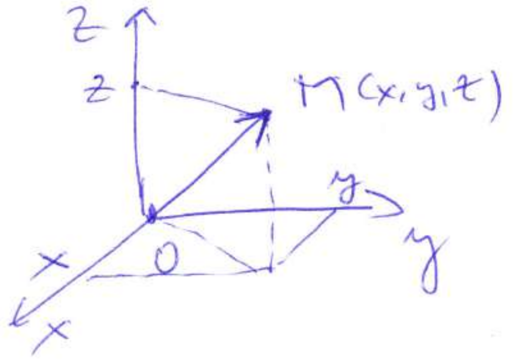
# ANALITIČKA GEOMETRIJA U PROSTORU

## * TAČKA




$\vec{r}_M = \vec{OM} = (x, y, z)$

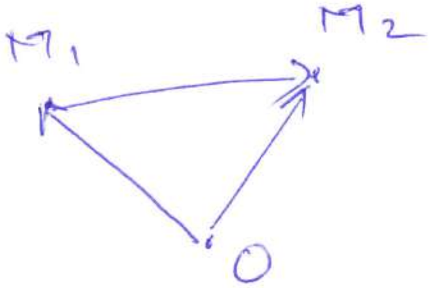
↑ VEKTOR POLOŽAJA TAČKE M

RASTOJANJE IZMEĐU $M_1(x_1, y_1, z_1)$ I $M_2(x_2, y_2, z_2)$

$$d(M_1, M_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\vec{M_1M_2} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

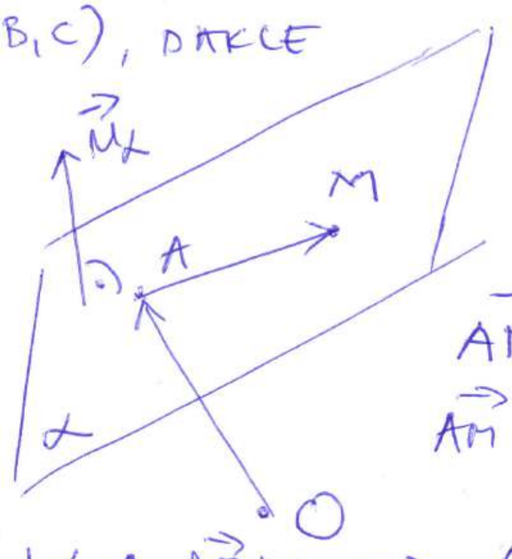
$$d(M_1, M_2) = |\vec{M_1M_2}|$$




## * RAVAN

POSMATRAJMO RAVAN $\alpha$ KOJA SADRŽI DATU TAČKU $A(x_1, y_1, z_1)$ I NORMALNA NA DATI VEKTOR $\vec{n}_\alpha = (A, B, C)$, DAKLE

$\underline{\alpha}$:
$A(x_1, y_1, z_1) \in \alpha$
$\vec{n}_\alpha(A, B, C) \perp \alpha$.




$\vec{n}_\alpha$ - VEKTOR NORMALE

$\vec{AM} \in \alpha \Leftrightarrow \vec{AM} \parallel \alpha$

$\vec{AM} \parallel \alpha$

NEKA JE $M(x, y, z)$ PROIZVOLJNA TAČKA RAVNI $\alpha$, $M \in \alpha$

$\vec{n}_\alpha \perp \alpha \land \vec{AM} \parallel \alpha \Rightarrow \vec{AM} \perp \vec{n}_\alpha$

$$\overrightarrow{AM} \perp \vec{u}_\alpha \iff \overrightarrow{AM} \cdot \vec{u}_\alpha = 0$$

$$\overrightarrow{AM} = (x-x_1, y-y_1, z-z_1)$$

$$\overrightarrow{AM} \cdot \vec{u}_\alpha = (x-x_1, y-y_1, z-z_1)\cdot(A, B, C) =$$
$$= A(x-x_1) + B(y-y_1) + C(z-z_1)$$

$$\overrightarrow{AM} \cdot \vec{u}_\alpha = 0 \iff \boxed{A(x-x_1)+B(y-y_1)+C(z-z_1)=0}$$

SKALARNI OBLIK JEDNAČINE RAVNI $\alpha$

$$\Downarrow$$

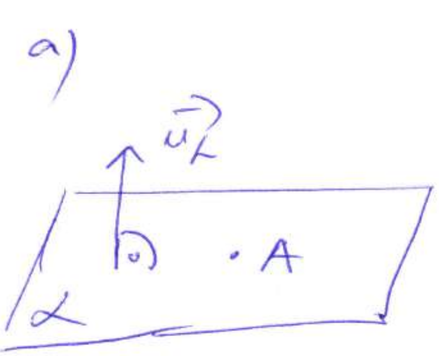
$$\boxed{Ax + By + Cz + D = 0}$$

OPŠTI OBLIK JEDNAČINE RAVNI $\alpha$

PRIMER a) ODREDITI JEDNAČINU RAVNI $\alpha$ KOJA SADRŽI TAČKU $A(1,0,1)$ I NORMALNA JE NA VEKTOR $\vec{u}_\alpha = (1,1,-1)$.

b) PROVERITI DA LI TAČKA $S(1,2,3)$ PRIPADA RAVNI $\alpha$.

Reš. a)




$\vec{u}_\alpha = (1,1,-1)$

$A(1,0,1)$

$$\alpha: 1\cdot(x-1) + 1\cdot(y-0) - 1\cdot(z-1) = 0$$

$$\alpha: x \cancel{-1} + y - z \cancel{+1} = 0$$

$$\boxed{\alpha: x + y - z = 0}$$

$S(1,2,3) \in \alpha$ ?

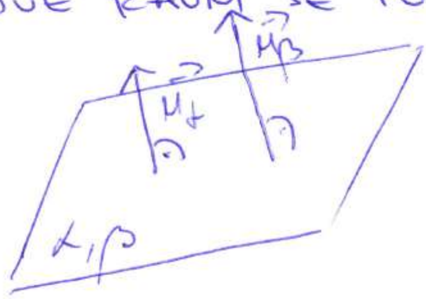
$$1 + 2 - 3 = 0 = 0 \checkmark \Rightarrow \boxed{S \in \alpha}$$

S ZADOVOLJAVA JEDNAČINU RAVNI.

(*) MEĐUSOBNI POLOŽAJ DVE RAVNI

$$\alpha: A_1x + B_1y + C_1z + D_1 = 0, \quad \beta: A_2x + B_2y + C_2z + D_2 = 0$$

$1^\circ$ DVE RAVNI SE POKLAPAJU




$\vec{u}_\alpha \parallel \vec{u}_\beta \ \wedge \ D_1 = k \cdot D_2$

ZA NEKO $k \in \mathbb{R}$

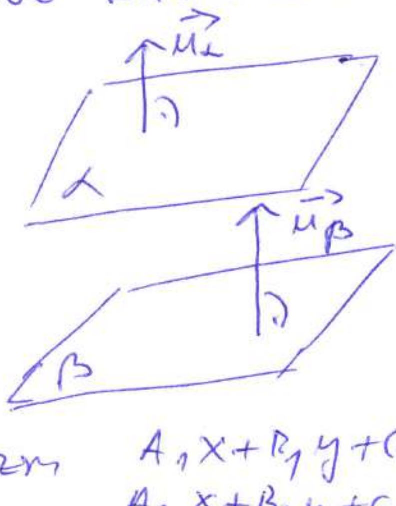
$k \neq 0$

$$\begin{aligned} A_1x + B_1y + C_1z + D_1 &= 0 \\ A_2x + B_2y + C_2z + D_2 &= 0 \end{aligned}$$

OVAJ LIN. SIST. JEDNAČINA JE DVOSTRUKO NEODREĐEN!

$$\boxed{\begin{gathered} \vec{u}_\alpha = k \cdot \vec{u}_\beta \ \wedge \ D_1 = k \cdot D_2 \\ \exists k \in \mathbb{R}, \ k \neq 0 \end{gathered}}$$

$2^\circ$ DVE RAVNI SU PARALELNE




$$\boxed{\begin{gathered} \text{POSTOJI } k \in \mathbb{R}, \ k \neq 0 \\ \vec{u}_\alpha = k \cdot \vec{u}_\beta \ \wedge \ D_1 \neq k \cdot D_2 \end{gathered}}$$

SISTEM $\begin{aligned} A_1x + B_1y + C_1z + D_1 &= 0 \\ A_2x + B_2y + C_2z + D_2 &= 0 \end{aligned}$ JE KONTRADIKTORAN!

$3^\circ$ DVE RAVNI SE SEKU

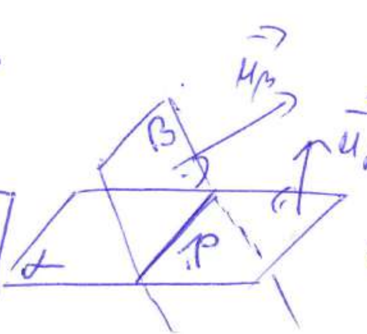
$$\begin{aligned} A_1x + B_1y + C_1z + D_1 &= 0 \\ A_2x + B_2y + C_2z + D_2 &= 0 \end{aligned}$$

JEDNOSTRUKO NEODREĐEN!

$\vec{u}_\alpha \not\parallel \vec{u}_\beta$

$\Updownarrow$

$\boxed{\vec{u}_\alpha \times \vec{u}_\beta \neq 0}$




PRAVA

$\downarrow$

$\alpha \cap \beta = p$

PRIMER (P4)

a) ISPITATI MEĐUSOBNI POLOŽAJ RAVNI

$\alpha: x+y+z=1$ i $\beta: 2x-y+z=2$.

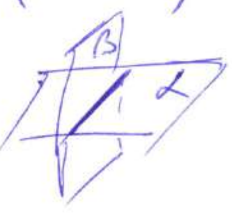
b) ODREDITI PARAMETAR $p$, TAKO DA TAČKA $T(1,p,2)$ PRIPADA RAVNI $\alpha$.

Reš. a) $\vec{n}_\alpha = (1,1,1)$, $\vec{n}_\beta = (2,-1,1)$

$$\vec{n}_\alpha \times \vec{n}_\beta = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix} = \vec{i}\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}$$

$$= \vec{i}(1+1) - \vec{j}(1-2) + \vec{k}(-1-2) = (2,1,-3) \neq \vec{0}$$

$\Downarrow$




RAVNI SE SEKU!

b) $T(1,p,2) \in \alpha$

$\Downarrow$

$1+p+2=1 \iff p=1-3 \iff \boxed{p=-2}$

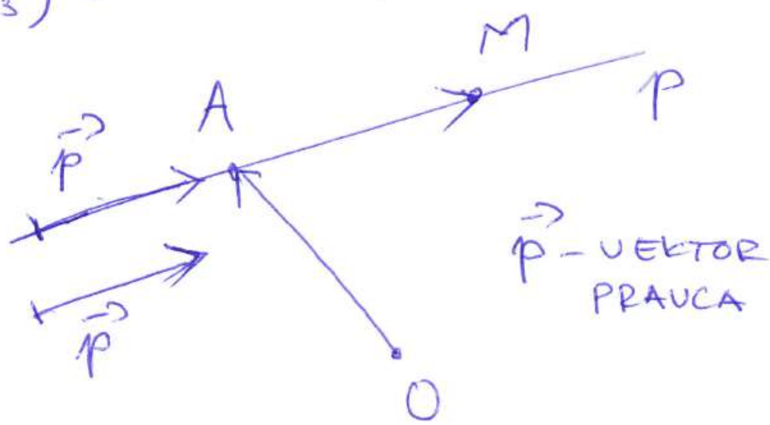
(*) PRAVA

POSMATRAJMO PRAVU $p$ KOJA PROLAZI KROZ TAČKU $A(x_1,y_1,z_1)$ I PARALELNA JE DATOM VEKTORU $\vec{p}(p_1,p_2,p_3)$.

$p$

$A(x_1,y_1,z_1) \in p$

$\vec{p}(p_1,p_2,p_3) \parallel p$




NEKA JE $M(x,y,z)$ PROIZVOLJNA TAČKA PRAVE $p$.

POŠTO JE $\vec{AM} \parallel \vec{p}$, POSTOJI $t \in \mathbb{R}$, TAKO DA JE $\vec{AM} = t \cdot \vec{p}$, $t \in \mathbb{R}$

$\vec{AM} = (x-x_1, y-y_1, z-z_1)$

$(x-x_1, y-y_1, z-z_1) = (tp_1, tp_2, tp_3)$

$\Downarrow$

(P5)

$$\begin{aligned} x-x_1 &= t\cdot p_1 \\ y-y_1 &= t\cdot p_2 \\ z-z_1 &= t\cdot p_3 \end{aligned} \Rightarrow \frac{x-x_1}{p_1}=t,\quad \frac{y-y_1}{p_2}=t,\quad \frac{z-z_1}{p_3}=t \Rightarrow \boxed{\frac{x-x_1}{p_1}=\frac{y-y_1}{p_2}=\frac{z-z_1}{p_3}}$$

KANONIČKI OBLIK JEDN. PRAVE $p$

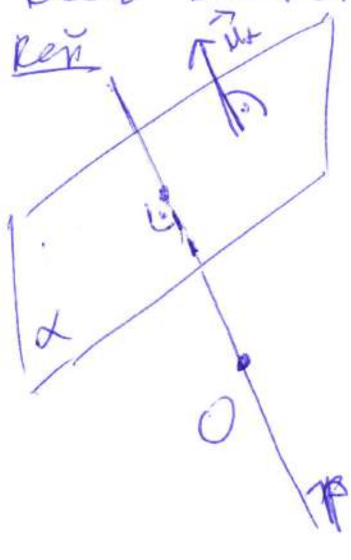
$$\Downarrow$$

$$\boxed{\begin{aligned} x &= x_1 + t p_1 \\ y &= y_1 + t p_2 \\ z &= z_1 + t p_3 \end{aligned}}$$

PARAMETARSKI OBLIK JEDN. PRAVE $p$

PRIMER ODREDITI JEDNAČINU PRAVE $p$ KOJA JE NORMALNA NA RAVAN $\alpha: x-y+2z=1$ I PROLAZI KROZ KOORDINATNI POČETAK.

Reš.




$\vec{n}_\alpha=(1,-1,2)$

~~$p\perp n_\alpha$~~ $p\perp\alpha \Rightarrow \vec{p}\parallel\vec{n}_\alpha$

MOŽEMO UZETI DA JE $\vec{p}=\vec{n}_\alpha=(1,-1,2)$

$O(0,0,0)\in p$
$\vec{p}=(1,-1,2)$ $\Rightarrow \dfrac{x-0}{1}=\dfrac{y-0}{-1}=\dfrac{z-0}{2}$

$$p: \boxed{\frac{x}{1}=\frac{y}{-1}=\frac{z}{2}}$$