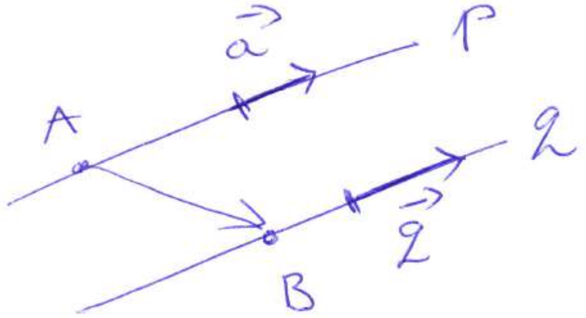
⊛ MEĐUSOBNI POLOŽAJ DVE PRAVE

$p: \frac{x-x_1}{a_1} = \frac{y-y_1}{a_2} = \frac{z-z_1}{a_3}$, $q: \frac{x-x_2}{b_1} = \frac{y-y_2}{b_2} = \frac{z-z_2}{b_3}$

$\vec{a} = (a_1, a_2, a_3) \leftarrow$ VEKT. PRAVCA $\rightarrow \vec{b} = (b_1, b_2, b_3)$

$A(x_1, y_1, z_1) \in p$ $B(x_2, y_2, z_2) \in p$

1° PRAVE SU PARALELNE



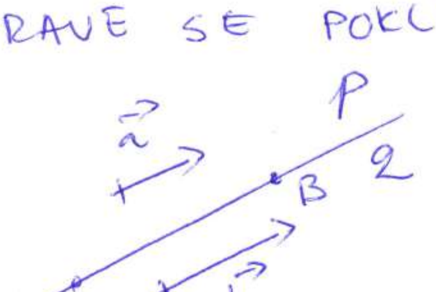
$p \cap q = \emptyset$

$\vec{AB} = (x_2-x_1, y_2-y_1, z_2-z_1)$

$\vec{AB} \nparallel \vec{a}$, $\vec{a} \parallel \vec{b}$ $\Leftrightarrow$ $\boxed{\vec{AB} \times \vec{a} \neq 0 \;\wedge\; \vec{a} \times \vec{b} = 0}$

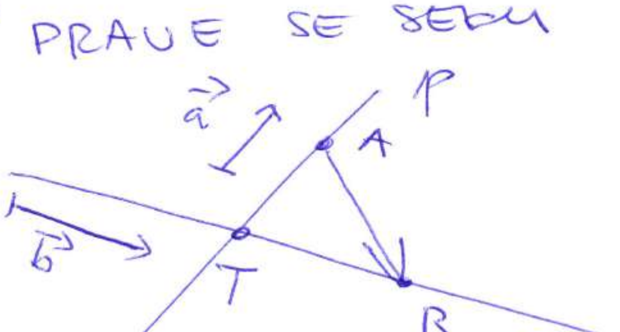
POTREBNI I DOVOLJNI USLOVI

2° PRAVE SE POKLAPAJU



$\vec{a} \parallel \vec{b}$, $\vec{AB} \parallel \vec{a}$ $\Leftrightarrow$ $\boxed{\vec{a} \times \vec{b} = 0 \;\wedge\; \vec{AB} \times \vec{a} = 0}$

3° PRAVE SE SEKU



$p \cap q = \{T\}$

↑ PRESEČNA TAČKA

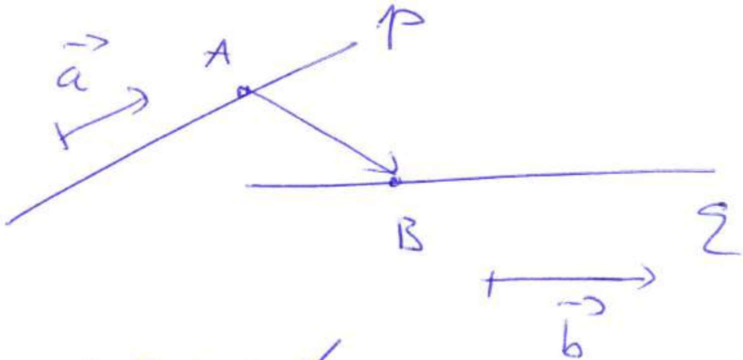
$\vec{AB}, \vec{a}$ i $\vec{b}$ MORAJU BITI KOMPLANARNI

$\wedge$

$\vec{a} \nparallel \vec{b} \Leftrightarrow (p \nparallel q)$

$$\boxed{\begin{array}{c}[\vec{AB}, \vec{a}, \vec{b}] = 0 \\ \wedge \\ \vec{a} \times \vec{b} \neq 0\end{array}} \Leftrightarrow p \text{ i } q \text{ SE SEKU}$$

4° PRAVE SU MIMOILAZNE



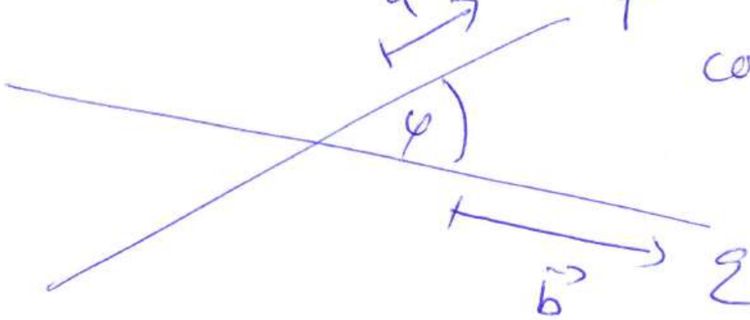
$p \cap q = \emptyset$

$\vec{AB}$, $\vec{a}$ i $\vec{b}$ NE LEŽE U ISTOJ RAVNI (NISU KOMPLANARNI)

$\Updownarrow$

$$\boxed{[\vec{AB}, \vec{a}, \vec{b}] \neq 0}$$

• UGAO IZMEĐU PRAVIH p i q SE DEFINIŠE KAO UGAO IZMEĐU NJIHOVIH VEKTORA PRAVACA, PRI ČEMU JE TAJ UGAO OŠTAR ILI PRAV $\left(0 \le \varphi \le \frac{\pi}{2}\right)$.



$$\cos\varphi = \frac{|\vec{a} \cdot \vec{b}|}{|\vec{a}| \cdot |\vec{b}|}$$

PRIMER: POKAZATI DA SE PRAVE

$p: \frac{x-2}{1} = \frac{y-2}{3} = \frac{z-3}{1}$ i $q: \frac{x-2}{1} = \frac{y-3}{4} = \frac{z-4}{2}$ SEKU.

$$p: \begin{array}{l}\vec{a} = (1,3,1)\\ A = (2,2,3)\end{array} \qquad q: \begin{array}{l}\vec{b} = (1,4,2)\\ B = (2,3,4)\end{array}$$

P3

$$\overrightarrow{AB} = (0,1,1)$$

$$[\overrightarrow{AB}, \vec{a}, \vec{b}] = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 4 & 2 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 0$$

$\cdot(-1)$

$$[\overrightarrow{AB}, \vec{a}, \vec{b}] = 0$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & 1 \\ 1 & 4 & 2 \end{vmatrix} = \vec{i}(6-4) - \vec{j}(2-1) + \vec{k}(4-3)$$

$$= (2,-1,1) \neq (0,0,0)$$

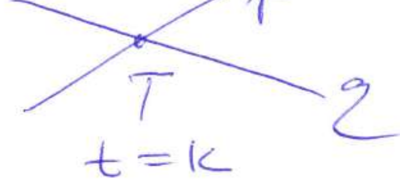
DAKLE, $[\overrightarrow{AB}, \vec{a}, \vec{b}] = 0 \wedge \vec{a} \times \vec{b} \neq 0$, ŠTO ZNAČI

DA SE PRAVE p I q SEKU.

REŠENJE NA DRUGI NAČIN:

$$p: \begin{cases} x = 2 + t \\ y = 2 + 3t \\ z = 3 + t \end{cases}, \quad q: \begin{cases} x = 2 + k \\ y = 3 + 4k \\ z = 4 + 2k \end{cases}$$

$$p \cap q = \{T\}$$



POSMATRAJMO SISTEM

$$\begin{array}{l} \cancel{2}+t = \cancel{2}+k \\ 2+3t = 3+4k \\ 3+t = 4+2k \end{array} \iff \begin{array}{l} t = k \\ 2+3t = 3+4t \\ 3+t = 4+2t \end{array} \iff \begin{array}{l} t = k \\ t = -1 \\ 3-1 = 4+2\cdot(-1) \end{array}$$

$$\iff \begin{array}{l} t = -1 \\ k = -1 \end{array} \implies T = (2-1, 2+3(-1), 3-1) = (1,-1,2)$$

(*) MEĐUSOBNI ODNOS PRAVE I RAVNI

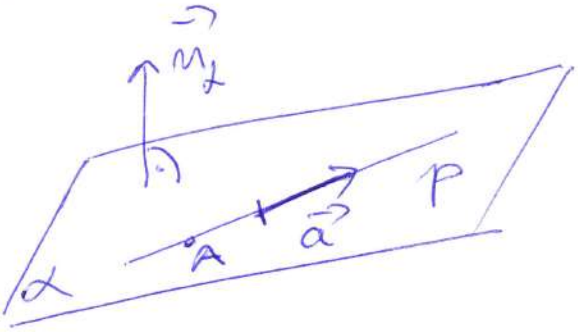
NEKA SU DATE:

$$p: \frac{x-x_1}{a_1} = \frac{y-y_1}{a_2} = \frac{z-z_1}{a_3} \quad \wedge \quad \alpha: Ax+By+Cz+D=0$$

$\vec{a} = (a_1, a_2, a_3)$ $\qquad\qquad$ $\vec{n}_\alpha = (A, B, C)$

$A(x_1, y_1, z_1) \in p$

1° PRAVA LEŽI U RAVNI



$A \in \alpha$ $\qquad$ $\cancel{Ax_1+By_1+Cz}$

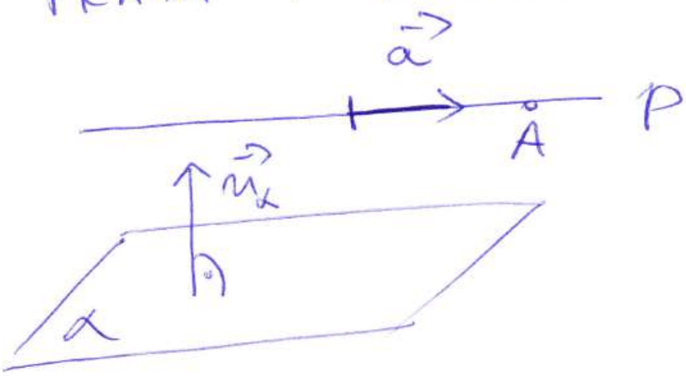
$\wedge$

$\vec{a} \perp \vec{n}_\alpha$

$\Updownarrow$

$$\boxed{\begin{gathered} Ax_1+By_1+Cz_1+D=0 \\ \wedge \\ \vec{a}\cdot\vec{n}_\alpha = 0 \end{gathered}}$$

2° PRAVA I RAVAN SU PARALELNE



$A \notin \alpha$

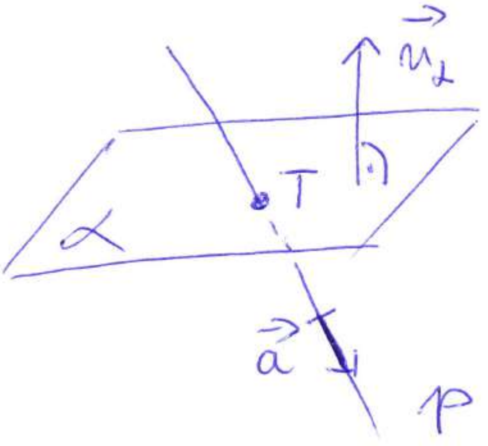
$\wedge$

$\vec{n}_\alpha \perp \vec{a}$

$\Updownarrow$

$$\boxed{\begin{gathered} Ax_1+By_1+Cz_1+D \neq 0 \\ \wedge \\ \vec{a}\cdot\vec{n}_\alpha = 0 \end{gathered}}$$

3° PRAVA PRODIRE RAVAN



$$\vec{a} \not\perp \vec{n}_\alpha$$

$$\Updownarrow$$

$$\boxed{\vec{a} \cdot \vec{n}_\alpha \neq 0}$$

- PROVERA MEĐUSOBNOG ODNOSA PRAVE I RAVNI MOŽE DA SE ANALIZIRA I POMOĆU SISTEMA LINEARNIH JEDNAČINA:

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{a_2}$$

$$\frac{y - y_1}{a_2} = \frac{z - z_1}{a_3} \qquad (3 \times 3)$$

$$Ax + By + Cz + D = 0$$

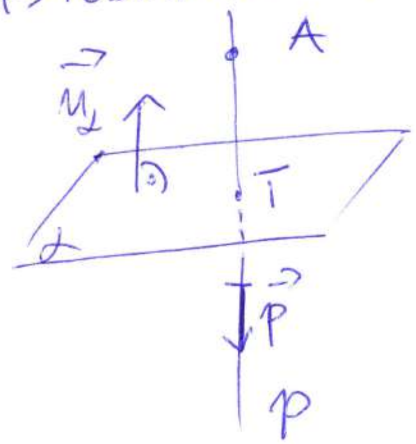
1° NEODREĐEN SISTEM (∞ MNOGO REŠENJA) $\Rightarrow$ PRAVA LEŽI U RAVNI

2° KONTRADIKTORAN SISTEM (NEMA REŠ) $\Rightarrow$ PRAVA I RAVNI SU PARALELNE

3° ODREĐEN SISTEM (JEDNO REŠ.) $\Rightarrow$ PRAVA PRODIRE RAVAN

RASTOJANJA

- RASTOJANJE TAČKE OD RAVNI

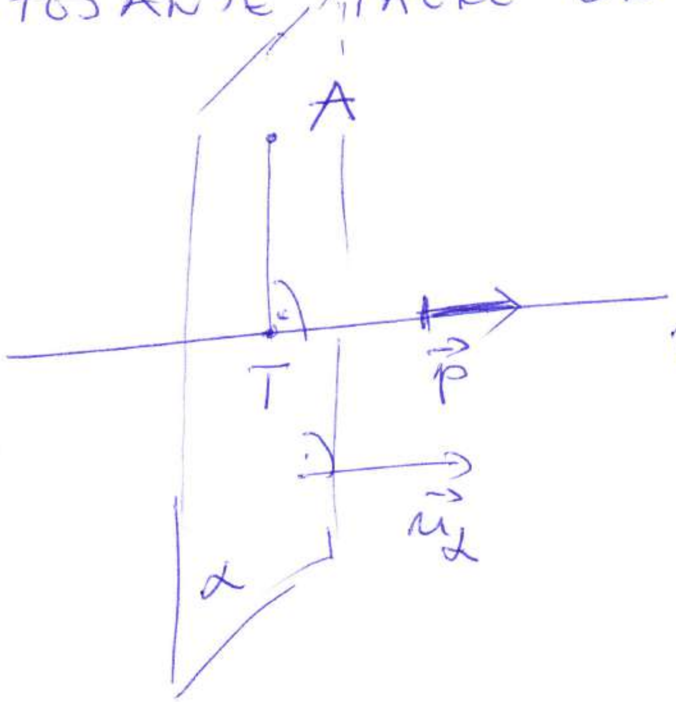


$p \perp \alpha \quad (\vec{p} = \vec{n}_\alpha)$

$p \cap \alpha = \{T\}$

$d(A,\alpha) = d(A,T)$

- RASTOJANJE TAČKE OD PRAVE



1) POSTAVI SE POMOĆNA RAVAN α: $A \in \alpha \wedge \alpha \perp p$

2) $\alpha \cap p = \{T\}$ ODREDIMO T

3) $d(A,p) = d(A,T)$