* DOKAZI NEKIH TVRĐENJA

P1

(T) $\vec{F}$ JE GRADIJENTNO V.P. $\Rightarrow$ $\operatorname{rot}\vec{F} = \vec{0}$.

DOKAZ NEKA JE $f(x,y,z)$ POTENCIJAL V.P.

$$\vec{F}(x,y,z) = (P(x,y,z), Q(x,y,z), R(x,y,z)).$$

TADA JE $\vec{F} = \nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$.

$$\operatorname{rot}\vec{F} = \operatorname{rot}\nabla f = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \end{vmatrix} =$$

$$= \vec{i}\left(\frac{\partial^2 f}{\partial y \partial z} - \frac{\partial^2 f}{\partial z \partial y}\right) - \vec{j}\left(\frac{\partial^2 f}{\partial x \partial z} - \frac{\partial^2 f}{\partial z \partial x}\right) + \vec{k}\left(\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial^2 f}{\partial y \partial x}\right)$$

$= (0, 0, 0)$, DAKLE $\operatorname{rot}\vec{F} = (0,0,0) = \vec{0}$.

(T) $\vec{F}$ JE BEZVRTLOŽNO $(\operatorname{rot}\vec{F} = \vec{0}) \Rightarrow \vec{F}$ JE KONZERVATIVNO.

DOKAZ NEKA JE $\vec{F}(x,y) = (P(x,y), Q(x,y))$ BEZVRTLOŽNO V.P., TADA JE

$$\operatorname{rot}\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P(x,y) & Q(x,y) & 0 \end{vmatrix} = \vec{i}(0-0) - \vec{j}(0-0) + \vec{k}\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right),$$

ODNOSNO $\operatorname{rot}\vec{F} = \vec{0} \Rightarrow \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0$.

NA OSNOVU GRINOVE TEOREME, ZA SVAKU ZATVORENU KRIVU C VAŽI:

$$\int_C \vec{F}\cdot d\vec{r} = \iint_D \underbrace{\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)}_{=0} dx\,dy = 0,$$ A TO ZNAČI

DA JE $\vec{F}$ KONZERVATIVNO V.P.

18.11.2022

P2

* GRAVITACIONO POLJE JE KONZERVATIVNO (GRADIJENTNO).

NA OSNOVU NJUTNOVOG ZAKONA O ~~GRAVITACIONO~~ GRAVITACIJI:

$$\vec{F}(x,y,z) = -\frac{GmM}{r^2}\cdot\vec{r}_0 \text{ , GDE SU}$$

↑ GRAVITACIONA SILA

$\vec{r}=(x,y,z)$

$\vec{r}_0 = \frac{\vec{r}}{|\vec{r}|}$, $r=|\vec{r}|$

G - GRAV. KONSTANTA

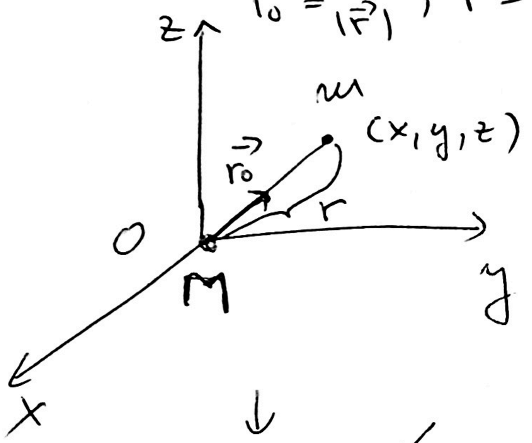
m - MASA MANJEG TELA

M - MASA VEĆEG TELA

r - RASTOJANJE IZMEĐU M I m

$\vec{r}_0$ - JEDINIČNI VEKTOR IZ TAČKE M PREMA TAČKI m

$|\vec{r}_0|=1$

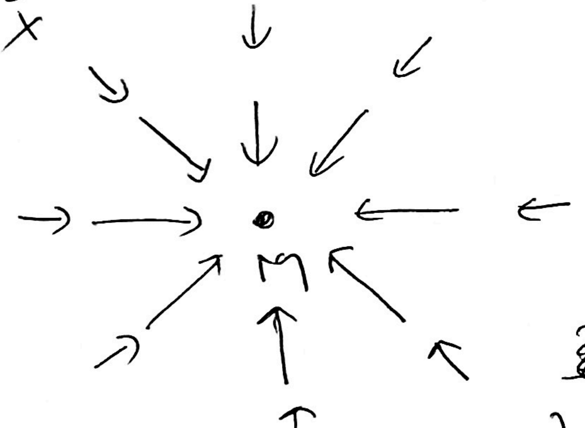


SKALARNA FUNKCIJA

$$f(x,y,z) = \frac{GmM}{r} \text{ JE}$$

POTENCIJAL V.P. $\vec{F}$, ZAISTA

$r=|\vec{r}|=\sqrt{x^2+y^2+z^2}$, TADA JE



GRAV. POLJE

$$\frac{\partial f}{\partial x} = \left(\frac{GmM}{\sqrt{x^2+y^2+z^2}}\right)'_x = GmM\frac{-x}{|\vec{r}|^3};$$

$$\frac{\partial f}{\partial y} = \left(\frac{GmM}{\sqrt{x^2+y^2+z^2}}\right)'_y = -GmM\frac{\cancel{2}y}{\cancel{2}(\sqrt{x^2+y^2+z^2})^3} = GmM\frac{-y}{|\vec{r}|^3};$$

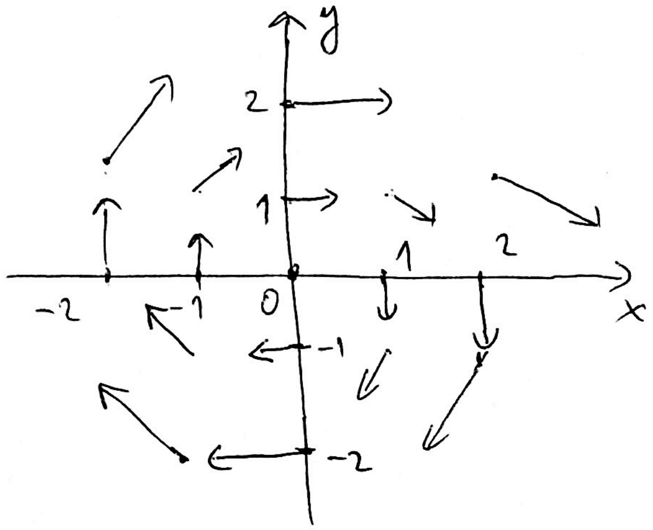
SLIČNO $\frac{\partial f}{\partial z} = GmM\frac{-z}{|\vec{r}|^3}$, DAKLE

$$\nabla f = \left(\frac{\partial f}{\partial x},\frac{\partial f}{\partial y},\frac{\partial f}{\partial z}\right) = GmM\left(\frac{-x}{|\vec{r}|^3},\frac{-y}{|\vec{r}|^3},\frac{-z}{|\vec{r}|^3}\right) =$$

$$= -GmM\cdot\frac{\vec{r}}{|\vec{r}|^3} = -GmM\cdot\frac{1}{r^2}\cdot\frac{\vec{r}}{|\vec{r}|} = -GmM\cdot\frac{1}{r^2}\cdot\vec{r}_0 = \vec{F}.$$

(*) VRTLOŽNO KRETANJE FLUIDA JE PRIMER V.P. KOJE NIJE KONZERVATIVNO.

POSMATRAJMO V.P. $\vec{F}(x,y)=(y,-x)$, KOJE MODELIRA STRUJANJE FLUIDA, $\vec{F}(x,y)$ JE BRZINA U TAČKI (x,y).



~~FLU~~ FLUID VRŠI KRUŽNO STRUJANJE OKO KOORDINATNOG POČETKA.

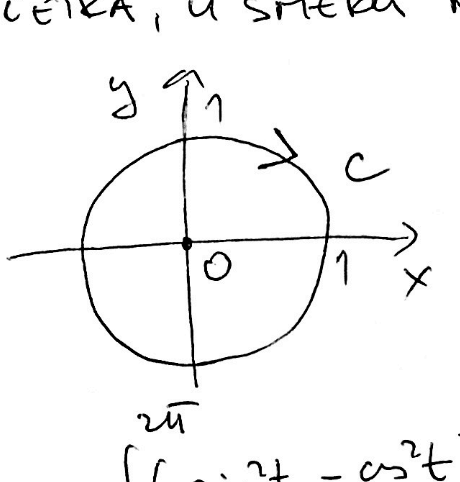
POŠTO JE

$$\operatorname{rot}\vec{F}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ y & -x & 0\end{vmatrix}=\vec{i}(0-0)-\vec{j}(0-0)+\vec{k}(-1-1)=$$

$$=(0,0,-2)\neq(0,0,0)$$

SLEDI DA $\vec{F}$ NIJE BEZVRTLOŽNO, ODNOSNO NA OSNOVU (T) NE MOŽE BITI NI KONZERVATIVNO.

POLJE VRŠI RAD, ROTIRAJUĆI ČESTICU OKO KOORDINATNOG POČETKA, U SMERU NEGATIVNOG UGLA.



$$-C:\begin{cases} x=\cos t & dx=-\sin t\,dt\\ y=\sin t & dy=\cos t\,dt\end{cases}$$
$$0\le t\le 2\pi$$

$$\underline{A}=\int_C \vec{F}\cdot d\vec{r}=-\int_0^{2\pi}(\sin t,-\cos t)\cdot(-\sin t,\cos t)\,dt$$

$$=-\int_0^{2\pi}(-\sin^2 t-\cos^2 t)\,dt=-\int_0^{2\pi}(-1)\,dt=\int_0^{2\pi}dt=t\Big|_0^{2\pi}=\underline{2\pi}>0$$