FUNDAMENTAL PROBLEMS IN IMAGE FORMATION AND RESTORATION

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Content

- Before all...
- Image formation
- General imaging problems
- Fourier transform and spatial frequency domain
- Image restoration
- Inverse Ill-posed problems
- Physics of radiological imaging
- Case study-mammograms-The Project
“In the beginning God created the heavens and the earth. The earth was formless and void, and darkness was over the surface of the deep, and the Spirit of God was moving over the surface of the waters....
...Then God said, “Let there be light”; and there was light. God saw that the light was good; and God separated the light from the darkness...
What is Light?

- Light has a **dual nature**.
  - **Wave** properties.
  - **Particle** properties

\[ \nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \]

\[ \nabla^2 \vec{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \]

It would seem very strange if there were not a sharp distinction between objects and waves in our everyday world. Yet this appears to be the nature of light.
“Every Physicist thinks that he knows what photon is. I spent my life to find out what a photon is and I still don’t know it.”

…but do not be confused
Image Formation
The existence of light—or other forms of electromagnetic radiation is an essential requirement for an image to be created, captured, and perceived.

Types of Images:

• Reflection Images;
• Emission Images;
• Absorption Images;

The Greek word Optike—"Theory of vision"
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CEEPUS Lecture
Image Acquisition, Formation, and Digitization

An image as a visual two dimensional (2D) representation of an object produced by an imaging system.
Remember: We are mainly interested in the characteristics of the object by deriving information from the image!

Objective versus subjective information
A legitimate question to ask:

“When faced with a practical imaging processing problem, which techniques should I use and in which sequence?”

Naturally, there is no universal answer to this question. Most image processing solutions are problem specific and usually involve application of several algorithms – in a meaningful sequence – to achieve the desired goal.

My recommendation is at the first step - “Clean” the image from the influence of the imaging system!!!
How we usually think? Spherical Cow?

“Make everything as simple as possible, but not simpler.”

Albert Einstein

A MODEL

AP(P)ROXIMATION!
A MODEL
How is an image formed?

Image Plane (resulting Image)

Imaging System

Object Plane (scene Object Function)

Point source of light

Input Distribution (o) → Imaging System PSF(o) + n → Output Distribution (s)
How is mathematically described an image formation?

Image = PSF \ast object + noise

\[ g = h \ast f + n \]

- **PSF** - a characteristic of the imaging device and is a deterministic function;

- **Object function** - describes object surface or its internal structure;

- **Noise** - a stochastic function which is a consequence of all the unwanted external disturbances

- **Convolution operator** - which ‘smears’ (convolves) one function with another
Our task - Image restoration

Image restoration is based on the attempt to improve the quality of an image through knowledge of the physical processes which led to its formation ...

...i.e. to find object function $o$, or the original information $f(x,y)$ from the image function $g(x,y)$

Do DECONVOLUTION or DEBLURING or INVERSE or just make UNDO!
Linear imaging systems

\[ g(x, y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h(x, y; x', y') f(x', y') \, dx' \, dy' \]

Linear superposition integral
The Point-Spread Function

The Point Spread Function (PSF) describes the response of an imaging system to a point source or point object.

\[
f(x', y') = \delta(x' - x_0, y' - y_0)
\]

\[
g(x, y) = \int \int \delta(x' - x_0, y' - y_0)h(x, y; x', y')dx'dy'
\]

\[
g(x, y) = h(x, y; x_0, y_0)
\]
A very large number of image formation processes are well described by the process of convolution. If a system is Linear Shift-Invariant then the image formation is *necessarily* described by convolution.
An example of the LSI system and the convolution integral

The scan was acquired with uniform speed over the patient.

The derived signal is proportional to the gamma activity emanating from that region of the body beneath the aperture.
Inverse problems

- Complex links among measured image quantities and object parameters:

  the cause-effect connection of investigated phenomenon is inverse;

  **Data → Model parameters**

- A characteristic of the object plays a role of "cause", and the observed data of the image, such as brightness - "effect"

- Non-local properties: the average value of a quantity \( q(x, y, z) \) is measured across volume \( \Delta V \)

\[
G = \int_{\Delta V} g(x, y, z) dV
\]

where \( \Delta V \) is usually a volume, limited by a small space angle, a narrow layer

To obtain a local value, the measurements along different angles are needed.
The harmonic content of signals: The fundamental idea of Fourier analysis is that any signal, be it a function of time, space or any other variables, may be expressed as a weighted linear combination of harmonic (i.e. sine and cosine) functions having different periods or frequencies.
FOURIER TRANSFORM Quite generally, we can transform the information with any scan line signal into a series of sinusoidal functions of the appropriate amplitude and spatial frequency (the spatial frequency spectrum) and vice-versa, we can synthesize any spatial signal by summing its harmonic components.
Fourier Transform examples con’d

Superposition of two waves—a beat pattern

Fourier transform of the beat pattern

Different signals and its Fourier transform pairs
Filtering

Original image and its Fourier transform

High-pass filtered transform
High pass filtered image

Low-pass filtered image
Low-pass filtered Fourier transform

High-pass filtered image

Low-pass filtered image

Original image and its Fourier transform

High-pass filtered transform
High pass filtered image

Low-pass filtered image
Low-pass filtered Fourier transform

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CEEPUS Lecture
An imaging system operates on the constituent input harmonics and its quality can be assessed by its ability to transmit the input harmonics to the output.

**The convolution theorem**

\[ \mathcal{F}\{f(x, y) * h(x, y)\} = F(k_x, k_y)H(k_x, k_y) \]

The Fourier transform of the convolution of the two functions is equal to the product of the individual transforms.
The optical transfer function (OTF)

\[ g(x, y) = f(x, y) \ast h(x, y) \]

\[ \mathcal{F}\{g(x, y)\} = \mathcal{F}\{f(x, y) \ast h(x, y)\} \]

\[ G(k_x, k_y) = F(k_x, k_y)H(k_x, k_y) \]

\( H(k_x, k_y) \) is called the OTF

The OTF is the frequency –domain equivalent of the PSF i.e. OTF derives its name from the fact that it determines how the individual spatial frequency pairs \((k_x, k_y)\) are transferred from input to output.
The Naive Solution

\[ F(k_x, k_y) = \frac{G(k_x, k_y)}{H(k_x, k_y)} = Y(k_x, k_y)G(k_x, k_y) \]

\[ f(x, y) = \mathcal{F}^{-1}\{Y(k_x, k_y)G(k_x, k_y)\} \]

\[ Y(k_x, k_y) = \frac{1}{H(k_x, k_y)} \text{ Inverse filter} \]

and the end of my presentation ...but
Well-Posedness

Definition due to Hadamard, 1915: Given mapping $A: X \rightarrow Y$, equation $Ax = y$ is well-posed provided

• (Existence)

• (Uniqueness)

; and

• (Stability)

Equation is ill-posed if it is not well-posed.
An ill-posed problem means that the large data sets may contain a surprisingly small amount of information about the object.
Ill-posed problems-Differentiation-(Edge detection)
Regularization

Remedy for ill-posedness (or ill-conditioning, in discrete case).

**Informal Definition:** “Imposes stability on an ill-posed problem in a manner that yields accurate approximate solutions, often by incorporating prior information”.

- Regularization theory provides a sound mathematical basis for solving the problem

**Key idea is to introduce a’priori information** (size of noise e.g) and assumptions about size and smoothness of desired solution!!!
Back to our case:

\[ g(x, y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h(x - x', y - y') f(x', y') \, dx' \, dy' + n(x, y) \]

\[ g(x, y) = f(x, y) \ast h(x, y) + n(x, y) \]
\[ \mathcal{F}\{g(x, y)\} = \mathcal{F}\{f(x, y) \ast h(x, y) + n(x, y)\} \]
\[ G(k_x, k_y) = F(k_x, k_y) H(k_x, k_y) + N(k_x, k_y) \]
\[ \hat{F}(k_x, k_y) = Y(k_x, k_y) G(k_x, k_y) = \frac{G(k_x, k_y)}{H(k_x, k_y)} + \frac{N(k_x, k_y)}{H(k_x, k_y)} = F(k_x, k_y) + \frac{N(k_x, k_y)}{H(k_x, k_y)} \]
DEMONSTRATION OF NOISE INFLUENCE

Blurred original noiseless

Blurred original with white noise

PSF

MTF

Restored original

Restored original
Possible solutions:

- **Inverse Filter**;
- **The Wiener Filter**;
- **Constrained deconvolution**;
- **Blind deconvolution**;
- **Iterative deconvolution and Lucy-Richardson algorithm**

- Matlab functions:
  - `deconvwnr`;
  - `deconvreg`;
  - `deconvblind`;
  - `deconvlucy`;

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Acceptable solutions;

\[ n(x, y) = g(x, y) - f(x, y) \odot h(x, y) \approx n(x, y) \]
Physics of radiological imaging

Transmission X-ray radiography, which has been used for over 100 years, is based on the partially removing of X-rays in material, which depends on thickness $x$ and the material-dependent removing length $\lambda$ through D’Alembert’s Law.

**Exponential Attenuation:**

$$I(x) = I(0) \exp\left(-\frac{x}{\lambda}\right)$$
Attenuation: Monochromatic X-rays

Water Phantom

100 keV
1000 photons

410 photons

Attenuation = 0.20/cm

1 cm
20%

1 cm
20%

1 cm
20%

1 cm
20%

1 cm
20%

1000
800
640
512
410
INTERACTIONS BETWEEN X-RAYS AND MATTER

TYPES OF INTERACTIONS which contribute to the removing of the primary X-ray photons and their consequences at the image quality of radiographs

MODEL OF INTERACTIONS - billiard balls collisions

- Coherent (Classical) Scattering: Infrequent
- Photoelectric Absorption:
- Compton Scattering
Coherent (Classical) Scattering

- Atom/electrons react to electromagnetic waves: absorbs energy, which “excites” atom
- Photon later reemitted with same energy
- “Wavelike” behavior
- Infrequent and mainly with low energy x-rays
- No dose deposited
- Insignificant effect on image
PHOTOELECTRIC EFFECT

- Interaction with inner electron (K-shell)

- End products:
  1) Energetic photoelectron
     \[ KE = E_{\text{x-ray}} - BE \]
  2) Characteristic radiation
  3) Ionized atom

- Fate of Energy: electron and char photons deposit all their energy near site of photoelectric event: it is an Absorption Interaction
PHOTOELECTRIC EFFECT-SUMMARY

- Dominant interaction in tissue only for < 30 keV
- In tissue, probability decreases with \((\text{keV})^3\) and increases with \((Z)^3\):
- Good for “quality”: creates contrast via strong dependence on \(Z\), and no scatter produced
- “Absorption” event: all energy deposited as dose near site of interaction—“bad” for dose
- Insufficient penetration at lower kVp for acceptable patient dose
COMPTON SCATTERING

Interaction: with "outer" electron:

ie: BE << smaller than x-ray energy

3 End products:
1) Scattered x-ray (reduced energy)
2) Recoil electron with some kinetic energy
3) Ionized atom
COMPTON SCATTER (Con’t)

• Dominant interaction for most diag energies

• Collision (“billiard ball”) interaction: prob of scatter mostly related to concentration of electrons (electron density, e/cm$^3$)

• ‘Bad” for quality: little soft tissue contrast; much scatter produced

• ‘Good” for dose: most energy carried away
SUMMARY OF INTERACTIONS

The diagram illustrates the linear attenuation coefficient for different materials and energy levels. The materials include Water and Compact Bone, and the energy levels are represented in keV (kiloelectronvolts). The attenuation coefficients are shown for classical, photoelectric, and Compton interactions.
IMAGE FORMATION - spatial variation of some physical quantity

Task:

Find Signal: (the difference between structures of interest and background!)

Radiograph of a disk-shaped object

Micro-calcification in the breast glandular tissue

Contrast
Sharpness
noise

• X-ray fluence
• Optical density of film
• Grey-scale value on the monitor

"Structures of interest"

"Background"
Characteristics of the imaging chain, which affect image quality and dose
A Compromise

e.g. diagnostic information vs. radiation dose

Image quality
Radiation exposure

Early experiments in radiation

ALARA

My father was a radiologist and assures me that radiation is NOT hazardous

My father was a radiologist and assures me that radiation is NOT hazardous

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Some examples which affect image quality

Magnification and resolution; distortion; size distortion vs position in beam, shape distortion; position distortion
Some examples cont’d

Focal Spot Blur; Screen Blur; Motion Blur
Resolution (Blur): Detail Visibility

- X-rays
- Object
- Image Receptor
- Radiographic Image

Density vs. Distance

Blur Width vs. Contrast
What is a good (or valid) measure of image quality?

Contrast

SNR

Image Quality

Noise

Resolution

MTF

NPS

(Sharpness)
Why Mammography image?

A great challenge because of small signals and different shapes.

The goal of mammography is the early detection of breast cancer.
THE PROJECT
Phantom Mammo AT
or even simpler test bar pattern
IMAGES FOR NOISE , PSF , LSF AND MTF ESTIMATION
The test image

Some results

Profile of the test image

Profile of the Blurred and Noisy Image

Blurred and Noisy Image

+ noise
The Deblurred image NSR

The Deblurred image using autocorr.
THANK YOU FOR YOUR ATTENTION!