

1. Primenom rednog algoritma sa pragom  $\theta = 1$  odrediti jednu bazu skupa binarnih obeležja  $S = \{p_1, p_2, p_3, p_4, p_5\}$ ,  $p_1 = (110)$ ,  $p_2 = (011)$ ,  $p_3 = (101)$ ,  $p_4 = (100)$ ,  $p_5 = (001)$ .

2. Na vektorskom prostoru  $(\mathbb{R}^n, \mathbb{R}, +, \cdot)$  je funkcija  $d_{[n]} : \mathbb{R}^n \times \mathbb{R}^n \rightarrow [0, \infty)$  definisana sa

$$d_{[n]}((x_1, \dots, x_n), (y_1, \dots, y_n)) = \max_{i \in \{1, \dots, n\}} |x_i - y_i|.$$

- (a) Ispitati da li je funkcija  $d_{[n]}$  funkcija rastojanja na  $\mathbb{R}^n$ .  
 (b) Ispitati da li je funkcija  $d_{[n]}$  metrika na  $\mathbb{R}^n$ .  
 (c) Ispitati da li za funkciju  $d_{[n]}$  važi ultrametrička nejednakost

$$\forall \mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{R}^n, \quad d_{[n]}(\mathbf{x}, \mathbf{z}) \leq \max(d_{[n]}(\mathbf{x}, \mathbf{y}), d_{[n]}(\mathbf{y}, \mathbf{z})).$$

3. U  $\mathbb{R}^2$ , primenom rešavajućih funkcija

$$d_1(x, y) = -2x - 6y + 11, \quad d_2(x, y) = -3x + 7y - 1, \quad d_3(x, y) = 4x - 5y - 4, \quad d_4(x, y) = -x + 5y - 8,$$

po principu maksimuma (treći način), klasifikovati tačke

$$A_1(-3, 1), \quad A_2(5, -3), \quad A_3(-2, -4), \quad A_4(3, 2), \quad A_5(4, -2),$$

u odgovarajuće klase  $\omega_1, \omega_2, \omega_3$  i  $\omega_4$ .

1. U vektorskom prostoru  $\mathbb{R}^2$ , klase  $\omega_1, \omega_2$  i  $\omega_3$  su zadane svojim predstavnicima

$$z_1 = (-3, -2) \in \omega_1, \quad z_2 = (4, -1) \in \omega_2, \quad z_3 = (1, -6) \in \omega_3.$$

Klasifikovanjem na osnovu minimuma rastojanja (primenom jednostrukog etalona) klasifikovati tačke

$$t_1(-1, 1), \quad t_2(-3, 4), \quad t_3(1, 2), \quad t_4(4, -1).$$

Napisati jednačine razdelnih pravih klasa  $\omega_1, \omega_2$  i  $\omega_3$ .

2. Prirodni brojevi se klasifikuju u tri klase  $\omega_i, i \in \{1, 2, 3\}$  sa matricom gubitaka  $L = \begin{bmatrix} 0.1 & 3 & 2 \\ 5 & 0.2 & 6 \\ 4 & 8 & 0.2 \end{bmatrix}$ . Apriorne vero-

vatnoće izbora klase za svrstavanje objekta su 

$\omega_i$	$\omega_1$	$\omega_2$	$\omega_3$
$p(\omega_i)$	0.2	0.3	0.5

. Uslovne verovatnoće klasifikovanja objekta

$n \in \mathbb{N}$  u klase  $\omega_i, i \in \{1, 2, 3\}$  su definisane sa  $p(n | \omega_i) = \frac{p(\omega_i)}{n + i - 1}, i \in \{1, 2, 3\}$ . Primenom količnika uslovnih verovatnoća (kao rešavajućih funkcija) klasifikovati brojeve 5 i 10 u jednu od klasa  $\omega_i, i \in \{1, 2, 3\}$ .

3. U prostoru  $\mathbb{R}^2$  su date tačke  $x_1 = (5, -2), \quad x_2 = (-1, -1), \quad x_3 = (-5, -4), \quad x_4 = (3, 4),$   
 $x_5 = (-1, 4), \quad x_6 = (1, 3), \quad x_7 = (-1, 2), \quad x_8 = (3, 5).$

Funkcija rastojanja  $d : \mathbb{R}^2 \times \mathbb{R}^2 \rightarrow [0, \infty)$  je definisana sa  $d((a_1, b_1), (a_2, b_2)) = |a_1 - a_2| + |b_1 - b_2|$ . Poboľšanim algoritmom fiksnog praga klasifikovati tačke  $x_i, i \in \{1, \dots, 8\}$  sa fiksnim pragom  $FP = 8$ , korišćenjem funkcije rastojanja  $d$ .

# REŠENJA - KOLOKVIJUM 1

1. Sa  $N = |S| = 5$  i početnim vrednostima  $f_k(0) = (111)$ ,  $k \in \mathbb{N}$  dobijamo sledeće iterativne korake.

(k1)  $\theta^* = \theta = 1$ ;  $f_1(0) = (111)$ ;

$$(k1.1) \quad \|f_1(0) \cap p_1\| = \|(111) \cap (110)\| = \|(110)\| = 2 \geq 1 = \theta^* \\ \Rightarrow f_1(1) = f_1(0) \cap p_1 = (110);$$

$$(k1.2) \quad \|f_1(1) \cap p_2\| = \|(110) \cap (011)\| = \|(010)\| = 1 \geq 1 = \theta^* \\ \Rightarrow f_1(2) = f_1(1) \cap p_2 = (010);$$

$$(k1.3) \quad \|f_1(2) \cap p_3\| = \|(010) \cap (101)\| = \|(000)\| = 0 < 1 = \theta^* \\ \Rightarrow f_1(3) = f_1(2) = (010);$$

$$(k1.4) \quad \|f_1(3) \cap p_4\| = \|(010) \cap (100)\| = \|(000)\| = 0 < 1 = \theta^* \\ \Rightarrow f_1(4) = f_1(3) = (010);$$

$$(k1.5) \quad \|f_1(4) \cap p_5\| = \|(010) \cap (001)\| = \|(000)\| = 0 < 1 = \theta^* \\ \Rightarrow f_1(5) = f_1(4) = (010);$$

Skup  $B_1 = \{f_1(5)\} = \{(010)\}$  ne generiše skup  $S$ .

(k2)  $f_2(0) = (111)$ ;

$$(k2.1) \quad \theta^* = \theta + \|f_1(5) \cap f_2(0) \cap p_1\| = 1 + \|(010) \cap (111) \cap (110)\| \\ = 1 + \|(010)\| = 1 + 1 = 2;$$

$$\|f_2(0) \cap p_1\| = \|(111) \cap (110)\| = \|(110)\| = 2 \geq 2 = \theta^* \\ \Rightarrow f_2(1) = f_2(0) \cap p_1 = (110);$$

$$(k2.2) \quad \theta^* = \theta + \|f_1(5) \cap f_2(1) \cap p_2\| = 1 + \|(010) \cap (110) \cap (011)\| \\ = 1 + \|(010)\| = 1 + 1 = 2;$$

$$\|f_2(1) \cap p_2\| = \|(110) \cap (011)\| = \|(010)\| = 1 < 2 = \theta^* \\ \Rightarrow f_2(2) = f_2(1) = (110);$$

$$(k2.3) \quad \theta^* = \theta + \|f_1(5) \cap f_2(2) \cap p_3\| = 1 + \|(010) \cap (110) \cap (101)\| \\ = 1 + \|(000)\| = 1 + 0 = 1;$$

$$\|f_2(2) \cap p_3\| = \|(110) \cap (101)\| = \|(100)\| = 1 \geq 1 = \theta^* \\ \Rightarrow f_2(3) = f_2(2) \cap p_3 = (101);$$

$$(k2.4) \quad \theta^* = \theta + \|f_1(5) \cap f_2(3) \cap p_4\| = 1 + \|(010) \cap (101) \cap (100)\| \\ = 1 + \|(000)\| = 1 + 0 = 1;$$

$$\|f_2(3) \cap p_4\| = \|(101) \cap (100)\| = \|(100)\| = 1 \geq 1 = \theta^* \\ \Rightarrow f_2(4) = f_2(3) \cap p_4 = (100);$$

$$(k2.5) \quad \theta^* = \theta + \|f_1(5) \cap f_2(4) \cap p_5\| = 1 + \|(010) \cap (100) \cap (001)\| \\ = 1 + \|(000)\| = 1 + 0 = 1;$$

$$\|f_2(4) \cap p_5\| = \|(100) \cap (001)\| = \|(000)\| = 0 < 1 = \theta^* \\ \Rightarrow f_2(5) = f_2(4) = (100);$$

Skup  $B_2 = \{f_1(5), f_2(5)\} = \{(010), (100)\}$  ne generiše skup  $S$  jer je npr.  $p_2 = (011) \notin \{f_1(5), f_2(5), f_1(5) \cup f_2(5)\} = \{(010), (100), (110)\}$ .

(k3)  $f_3(0) = (111)$ ;

$$(k3.1) \quad \theta^* = \theta + \|f_1(5) \cap f_3(0) \cap p_1\| + \|f_2(5) \cap f_3(0) \cap p_1\| \\ = 1 + \|(010) \cap (111) \cap (110)\| + \|(100) \cap (111) \cap (110)\| \\ = 1 + \|(010)\| + \|(100)\| = 1 + 1 + 1 = 3;$$

$$\|f_3(0) \cap p_1\| = \|(111) \cap (110)\| = \|(110)\| = 2 < 3 = \theta^* \\ \Rightarrow f_3(1) = f_3(0) = (111);$$

$$(k3.2) \quad \theta^* = \theta + \|f_1(5) \cap f_3(1) \cap p_2\| + \|f_2(5) \cap f_3(1) \cap p_2\| \\ = 1 + \|(010) \cap (111) \cap (011)\| + \|(100) \cap (111) \cap (011)\| \\ = 1 + \|(010)\| + \|(000)\| = 1 + 1 + 0 = 2;$$

$$\|f_3(1) \cap p_2\| = \|(111) \cap (011)\| = \|(011)\| = 2 \geq 2 = \theta^* \\ \Rightarrow f_3(2) = f_3(1) \cap p_2 = (011);$$

$$(k3.3) \quad \theta^* = \theta + \|f_1(5) \cap f_3(2) \cap p_3\| + \|f_2(5) \cap f_3(2) \cap p_3\| \\ = 1 + \|(010) \cap (011) \cap (101)\| + \|(100) \cap (011) \cap (101)\| \\ = 1 + \|(000)\| + \|(000)\| = 1 + 0 + 0 = 1;$$

$$\|f_3(2) \cap p_3\| = \|(011) \cap (101)\| = \|(001)\| = 1 \geq 1 = \theta^* \\ \Rightarrow f_3(3) = f_3(2) \cap p_3 = (001);$$

$$(k3.4) \quad \theta^* = \theta + \|f_1(5) \cap f_3(3) \cap p_4\| + \|f_2(5) \cap f_3(3) \cap p_4\| \\ = 1 + \|(010) \cap (001) \cap (100)\| + \|(100) \cap (001) \cap (100)\| \\ = 1 + \|(000)\| + \|(000)\| = 1 + 0 + 0 = 1;$$

$$\|f_3(3) \cap p_4\| = \|(001) \cap (100)\| = \|(000)\| = 0 < 1 = \theta^* \\ \Rightarrow f_3(4) = f_3(3) = (001);$$

$$\begin{aligned}
(k3.5) \quad \theta^* &= \theta + \|f_1(5) \cap f_3(4) \cap p_5\| + \|f_2(5) \cap f_3(4) \cap p_5\| \\
&= 1 + \|(010) \cap (001) \cap (001)\| + \|(100) \cap (001) \cap (001)\| \\
&= 1 + \|(000)\| + \|(000)\| = 1 + 0 + 0 = 1; \\
\|f_3(4) \cap p_5\| &= \|(001) \cap (001)\| = \|(001)\| = 1 \geq 1 = \theta^* \\
\Rightarrow f_3(5) &= f_3(4) \cap p_5 = (001).
\end{aligned}$$

Skup  $B_3 = \{f_1(5), f_2(5), f_3(5)\} = \{(010), (100), (001)\}$  jeste baza skupa  $S$  jer je

$$\begin{aligned}
p_1 &= (110) = f_1(5), f_2(5) = (010) \cup (100), \\
p_2 &= (011) = f_1(5) \cup f_3(5) = (010) \cup (001), \\
p_3 &= (101) = f_2(5) \cup f_3(5) = (100) \cup (001), \\
p_4 &= (100) = f_2(5), \\
p_5 &= (001) = f_3(5).
\end{aligned}$$

2. (a) Dokazaćemo da za  $d_{[n]}$  važe aksiome funkcije rastojanja.

(d01) Za svako  $\mathbf{x} = (x_1, \dots, x_n) \in \mathbb{R}^n$  važi

$$d_{[n]}(\mathbf{x}, \mathbf{x}) = \max_{i \in \{1, \dots, n\}} |x_i - x_i| = \max_{i \in \{1, \dots, n\}} 0 = 0.$$

(d02) Za svako  $\mathbf{x} = (x_1, \dots, x_n) \in \mathbb{R}^n$  i  $\mathbf{y} = (y_1, \dots, y_n) \in \mathbb{R}^n$  je

$$d_{[n]}(\mathbf{x}, \mathbf{y}) = \max_{i \in \{1, \dots, n\}} |x_i - y_i| = \max_{i \in \{1, \dots, n\}} |y_i - x_i| = d_{[n]}(\mathbf{y}, \mathbf{x}).$$

(b) Dokazaćemo da  $d_{[n]}$  jeste metrika na  $\mathbb{R}^n$ . Aksiome (d01) i (d02) su dokazane pod (a), te preostaje da se dokažu aksiome (d03) i (d04).

(d03) Za svako  $\mathbf{x} = (x_1, \dots, x_n) \in \mathbb{R}^n$  i  $\mathbf{y} = (y_1, \dots, y_n) \in \mathbb{R}^n$  imamo

$$d_{[n]}(\mathbf{x}, \mathbf{y}) = \max_{i \in \{1, \dots, n\}} |x_i - y_i| = 0 \Rightarrow \forall i \in \{1, \dots, n\}, |x_i - y_i| = 0$$

$$\Rightarrow \forall i \in \{1, \dots, n\}, x_i = y_i, \Rightarrow \mathbf{x} = \mathbf{y}.$$

(d04) Posmatrajmo proizvoljne  $\mathbf{x} = (x_1, \dots, x_n) \in \mathbb{R}^n$ ,  $\mathbf{y} = (y_1, \dots, y_n) \in \mathbb{R}^n$  i  $\mathbf{z} = (z_1, \dots, z_n) \in \mathbb{R}^n$ . Iz poznate nejednakosti trougla  $|a - c| \leq |a - b| + |b - c|$  za proizvoljne  $a, b, c \in \mathbb{R}$  sledi da je

$$\forall i \in \{1, \dots, n\}, |x_i - z_i| \leq |x_i - y_i| + |y_i - z_i|, \Rightarrow \max_{i \in \{1, \dots, n\}} |x_i - z_i| \leq \max_{i \in \{1, \dots, n\}} \{|x_i - y_i| + |y_i - z_i|\}.$$

Dokažimo da je

$$\max_{i \in \{1, \dots, n\}} \{|x_i - y_i| + |y_i - z_i|\} \leq \max_{i \in \{1, \dots, n\}} |x_i - y_i| + \max_{i \in \{1, \dots, n\}} |y_i - z_i|,$$

odakle će iz tranzitivnosti relacije  $\leq$  slediti nejednakost trougla

$$d_{[n]}(\mathbf{x}, \mathbf{z}) = \max_{i \in \{1, \dots, n\}} |x_i - z_i| \leq \max_{i \in \{1, \dots, n\}} |x_i - y_i| + \max_{i \in \{1, \dots, n\}} |y_i - z_i| = d_{[n]}(\mathbf{x}, \mathbf{y}) + d_{[n]}(\mathbf{y}, \mathbf{z}).$$

Označimo  $a_i = |x_i - z_i| \geq 0$ ,  $b_i = |x_i - y_i| \geq 0$  i  $c_i = |y_i - z_i| \geq 0$  za sve  $i \in \{1, \dots, n\}$ . Iz nejednakosti trougla sledi da je

$$\forall i \in \{1, \dots, n\}, a_i \leq b_i + c_i.$$

Neka je  $\max_{i \in \{1, \dots, n\}} a_i = a_{a_m}$  za neko  $a_m \in \{1, \dots, n\}$ . Tada je

$$a_{a_m} \leq b_{a_m} + c_{a_m} \leq \max_{i \in \{1, \dots, n\}} b_i + \max_{i \in \{1, \dots, n\}} c_i,$$

$$\Rightarrow \max_{i \in \{1, \dots, n\}} a_i = a_{a_m} \leq b_{a_m} + c_{a_m} \leq \max_{i \in \{1, \dots, n\}} b_i + \max_{i \in \{1, \dots, n\}} c_i$$

$$\Rightarrow d_{[n]}(\mathbf{x}, \mathbf{z}) = \max_{i \in \{1, \dots, n\}} |x_i - z_i| \leq \max_{i \in \{1, \dots, n\}} |x_i - y_i| + \max_{i \in \{1, \dots, n\}} |y_i - z_i| = d_{[n]}(\mathbf{x}, \mathbf{y}) + d_{[n]}(\mathbf{y}, \mathbf{z}).$$

(c) Za funkciju  $d_{[n]}$  u opštem slučaju ne važi ultrimetrička nejednakost što ćemo dokazati kontraprimenom. Neka je  $n = 2$ ,  $\mathbf{x} = (x_1, x_2) = (1, 0)$ ,  $\mathbf{y} = (y_1, y_2) = (2, 0)$ ,  $\mathbf{z} = (z_1, z_2) = (4, 0)$ .

S jedne strane je

$$d_{[2]}(\mathbf{x}, \mathbf{z}) = \max\{|x_1 - z_1|, |x_2 - z_2|\} = \max\{3, 0\} = 3,$$

a s druge

$$\max\{d_{[2]}(\mathbf{x}, \mathbf{y}), d_{[2]}(\mathbf{y}, \mathbf{z})\} = \max\{\max\{|x_1 - y_1|, |x_2 - y_2|\}, \max\{|y_1 - z_1|, |y_2 - z_2|\}\}$$

$$= \max\{\max\{1, 0\}, \max\{2, 0\}\} = \max\{1, 2\} = 2,$$

$$\Rightarrow d_{[2]}(\mathbf{x}, \mathbf{z}) = 3 > 2 = \max\{d_{[2]}(\mathbf{x}, \mathbf{y}), d_{[2]}(\mathbf{y}, \mathbf{z})\}.$$

3. (A<sub>1</sub>)  $\max\{d_1(A_1), d_2(A_1), d_3(A_1), d_4(A_1)\} = \max\{d_1(-3, 1), d_2(-3, 1), d_3(-3, 1), d_4(-3, 1)\}$   
 $= \max\{11, 15, -21, 0\} = 15 = d_2(A_1) \Rightarrow A_1 \in \omega_2.$

(A<sub>2</sub>)  $\max\{d_1(A_2), d_2(A_2), d_3(A_2), d_4(A_2)\} = \max\{d_1(5, -3), d_2(5, -3), d_3(5, -3), d_4(5, -3)\}$   
 $= \max\{19, -37, 31, -28\} = 31 = d_3(A_2) \Rightarrow A_2 \in \omega_3.$

(A<sub>3</sub>)  $\max\{d_1(A_3), d_2(A_3), d_3(A_3), d_4(A_3)\} = \max\{d_1(-2, -4), d_2(-2, -4), d_3(-2, -4), d_4(-2, -4)\}$   
 $= \max\{39, -23, 8, -26\} = 39 = d_1(A_3) \Rightarrow A_3 \in \omega_1.$

(A<sub>4</sub>)  $\max\{d_1(A_4), d_2(A_4), d_3(A_4), d_4(A_4)\} = \max\{d_1(3, 2), d_2(3, 2), d_3(3, 2), d_4(3, 2)\}$   
 $= \max\{-7, 4, -2, -1\} = 4 = d_2(A_4) \Rightarrow A_4 \in \omega_2.$

(A<sub>5</sub>)  $\max\{d_1(A_5), d_2(A_5), d_3(A_5), d_4(A_5)\} = \max\{d_1(4, -2), d_2(4, -2), d_3(4, -2), d_4(4, -2)\}$   
 $= \max\{15, -27, 22, -22\} = 22 = d_3(A_5) \Rightarrow A_5 \in \omega_3.$

Klasifikacija je za rezultat dala klase

$$\omega_1 = \{A_3\}, \quad \omega_2 = \{A_1, A_4\}, \quad \omega_3 = \{A_2, A_5\}, \quad \omega_4 = \emptyset.$$

## REŠENJA - KOLOKVIJUM 2

1. Svakoje od klasa  $\omega_1$ ,  $\omega_2$  i  $\omega_3$  pridružujemo redom odgovarajuće rešavajuće funkcije

$$d_1(x, y) = (x, y) \cdot z_1 - \frac{1}{2}|z_1|^2 = (x, y) \cdot (-3, -2) - \frac{1}{2}|(-3, -2)|^2 = -3x - 2y - \frac{1}{2}((-3)^2 + (-2)^2) = -3x - 2y - \frac{13}{2},$$

$$d_2(x, y) = (x, y) \cdot z_2 - \frac{1}{2}|z_2|^2 = (x, y) \cdot (4, -1) - \frac{1}{2}|(4, -1)|^2 = 4x - y - \frac{1}{2}(4^2 + (-1)^2) = 4x - y - \frac{17}{2},$$

$$d_3(x, y) = (x, y) \cdot z_3 - \frac{1}{2}|z_3|^2 = (x, y) \cdot (1, -6) - \frac{1}{2}|(1, -6)|^2 = x - 6y - \frac{1}{2}(1^2 + (-6)^2) = x - 6y - \frac{37}{2}.$$

Njihovom primenom dobijamo sledeću klasifikaciju.

$$\max\{d_1(t_1), d_2(t_1), d_3(t_1)\} = \max\{d_1(-1, 1), d_2(-1, 1), d_3(-1, 1)\} = \max\left\{-\frac{11}{2}, -\frac{27}{2}, -\frac{51}{2}\right\} = -\frac{11}{2} = d_1(t_1)$$

$$\Rightarrow t_1 \in \omega_1,$$

$$\max\{d_1(t_2), d_2(t_2), d_3(t_2)\} = \max\{d_1(-3, 4), d_2(-3, 4), d_3(-3, 4)\} = \max\left\{-\frac{11}{2}, -\frac{49}{2}, -\frac{91}{2}\right\} = -\frac{11}{2} = d_1(t_2)$$

$$\Rightarrow t_2 \in \omega_1,$$

$$\max\{d_1(t_3), d_2(t_3), d_3(t_3)\} = \max\{d_1(1, 2), d_2(1, 2), d_3(1, 2)\} = \max\left\{-\frac{27}{2}, -\frac{13}{2}, -\frac{59}{2}\right\} = -\frac{13}{2} = d_2(t_3)$$

$$\Rightarrow t_3 \in \omega_2,$$

$$\max\{d_1(t_4), d_2(t_4), d_3(t_4)\} = \max\{d_1(4, -1), d_2(4, -1), d_3(4, -1)\} = \max\left\{-\frac{33}{2}, \frac{17}{2}, -\frac{17}{2}\right\} = \frac{17}{2} = d_2(t_4)$$

$$\Rightarrow t_4 \in \omega_2,$$

Klasifikacija je za rezultat dala klase:  $\omega_1 = \{t_1, t_2\}$ ,  $\omega_2 = \{t_3, t_4\}$ ,  $\omega_3 = \emptyset$ .

Deobena prava klasa  $\omega_1$  i  $\omega_2$  je

$$H_{1,2} = \{(x, y) \in \mathbb{R}^3 \mid d_1(x, y) = d_2(x, y)\} = \left\{(x, y) \in \mathbb{R}^3 \mid -3x - 2y - \frac{13}{2} = 4x - y - \frac{17}{2}\right\}$$

$$= \{(x, y) \in \mathbb{R}^3 \mid y = -7x + 2\}.$$

Deobena prava klasa  $\omega_1$  i  $\omega_3$  je

$$H_{1,3} = \{(x, y) \in \mathbb{R}^3 \mid d_1(x, y) = d_3(x, y)\} = \left\{(x, y) \in \mathbb{R}^3 \mid -3x - 2y - \frac{13}{2} = x - 6y - \frac{37}{2}\right\}$$

$$= \{(x, y) \in \mathbb{R}^3 \mid y = x - 3\}.$$

Deobena prava klasa  $\omega_2$  i  $\omega_3$  je

$$H_{2,3} = \{(x, y) \in \mathbb{R}^3 \mid d_2(x, y) = d_3(x, y)\} = \left\{(x, y) \in \mathbb{R}^3 \mid 4x - y - \frac{17}{2} = x - 6y - \frac{37}{2}\right\}$$

$$= \left\{(x, y) \in \mathbb{R}^3 \mid y = -\frac{3}{5}x - \frac{10}{5}\right\}.$$

2. Uslovne verovatnoće klasifikovanja prirodnih brojeva 5 i 10 u klase  $\omega_i$ ,  $i \in \{1, 2, 3\}$  su

$$p(5 \mid \omega_1) = \frac{p(\omega_1)}{5+1} = \frac{0.2}{5} = \frac{1}{25} = 0.04,$$

$$p(5 \mid \omega_2) = \frac{p(\omega_2)}{5+2} = \frac{0.3}{6} = \frac{1}{20} = 0.05,$$

$$p(5 \mid \omega_3) = \frac{p(\omega_3)}{5+3} = \frac{0.5}{7} = \frac{1}{14} = 0.0714,$$

$$p(10 \mid \omega_1) = \frac{p(\omega_1)}{10+1} = \frac{0.2}{10} = \frac{1}{50} = 0.02,$$

$$p(10 \mid \omega_2) = \frac{p(\omega_2)}{10+2} = \frac{0.3}{11} = \frac{3}{110} = 0.0273,$$

$$p(10 \mid \omega_3) = \frac{p(\omega_3)}{10+3} = \frac{0.5}{123} = \frac{1}{24} = 0.0417.$$

Pragovi za rešavajuće funkcije  $\ell_{ij}(n) = \frac{p(n \mid \omega_i)}{p(n \mid \omega_j)}$ ,  $i, j \in \{1, 2, 3\}$ ,  $i \neq j$  su

$$\theta_{ij} = \frac{(L_{ji} - L_{jj})p(\omega_j)}{(L_{ij} - L_{ii})p(\omega_i)}, \quad i, j \in \{1, 2, 3\}, \quad i \neq j,$$

dakle

$$\theta_{12} = \frac{(L_{21} - L_{22})p(\omega_2)}{(L_{12} - L_{11})p(\omega_1)} = \frac{(5 - 0.2) \cdot 0.3}{(3 - 0.1) \cdot 0.2} = \frac{72}{29} = 2.4828,$$

$$\theta_{13} = \frac{(L_{31} - L_{33})p(\omega_3)}{(L_{13} - L_{11})p(\omega_1)} = \frac{(4 - 0.2) \cdot 0.5}{(2 - 0.1) \cdot 0.2} = 5,$$

$$\theta_{23} = \frac{(L_{32} - L_{33})p(\omega_3)}{(L_{23} - L_{22})p(\omega_2)} = \frac{(8 - 0.2) \cdot 0.5}{(6 - 0.2) \cdot 0.3} = \frac{65}{29} = 2.2414,$$

$$\theta_{21} = \frac{1}{\theta_{12}} = \frac{29}{72} = 0.4028,$$

$$\theta_{31} = \frac{1}{\theta_{13}} = \frac{1}{5} = 0.2,$$

$$\theta_{32} = \frac{1}{\theta_{23}} = \frac{29}{65} = 0.4462.$$

[n5] Vrednosti rešavajucih funkcija  $\ell_{ij}(n) = \frac{p(n|\omega_i)}{p(n|\omega_j)}$ ,  $i, j \in \{1, 2, 3\}$ ,  $i \neq j$  za  $n = 5$  su

$$\ell_{12}(5) = \frac{p(5|\omega_1)}{p(5|\omega_2)} = \frac{4}{5} = 0.8,$$

$$\ell_{13}(5) = \frac{p(5|\omega_1)}{p(5|\omega_3)} = \frac{14}{25} = 0.56,$$

$$\ell_{23}(5) = \frac{p(5|\omega_2)}{p(5|\omega_3)} = \frac{7}{10} = 0.7,$$

$$\ell_{21}(5) = \frac{p(5|\omega_2)}{p(5|\omega_1)} = \frac{1}{\ell_{12}(5)} = \frac{5}{4} = 1.25,$$

$$\ell_{31}(5) = \frac{p(5|\omega_3)}{p(5|\omega_1)} = \frac{1}{\ell_{13}(5)} = \frac{25}{14} = 1.7857,$$

$$\ell_{32}(5) = \frac{p(5|\omega_3)}{p(5|\omega_2)} = \frac{1}{\ell_{23}(5)} = \frac{10}{7} = 1.4286.$$

(n5.1) Iz  $\ell_{12}(5) = 0.8 < \theta_{12} = 2.48276$  sledi  $5 \notin \omega_1$ .

(n5.2) Iz  $\ell_{21}(5) = 1.25 > \theta_{21} = 0.4028$  i  $\ell_{23}(5) = 0.7 < \theta_{23} = 2.2414$  sledi  $5 \notin \omega_2$ .

(n5.2) Iz  $\ell_{31}(5) = 1.7857 > \theta_{31} = 0.2$  i  $\ell_{32}(5) = 1.4286 > \theta_{32} = 0.4462$  sledi  $5 \in \omega_3$ .

[n10] Vrednosti rešavajucih funkcija  $\ell_{ij}(n) = \frac{p(n|\omega_i)}{p(n|\omega_j)}$ ,  $i, j \in \{1, 2, 3\}$ ,  $i \neq j$  za  $n = 10$  su

$$\ell_{12}(10) = \frac{p(10|\omega_1)}{p(10|\omega_2)} = \frac{11}{15} = 0.7333,$$

$$\ell_{13}(10) = \frac{p(10|\omega_1)}{p(10|\omega_3)} = \frac{12}{25} = 0.48,$$

$$\ell_{23}(10) = \frac{p(10|\omega_2)}{p(10|\omega_3)} = \frac{36}{55} = 0.6545,$$

$$\ell_{21}(10) = \frac{p(10|\omega_2)}{p(10|\omega_1)} = \frac{1}{\ell_{12}(10)} = \frac{15}{11} = 1.3636,$$

$$\ell_{31}(10) = \frac{p(10|\omega_3)}{p(10|\omega_1)} = \frac{1}{\ell_{13}(10)} = \frac{25}{12} = 2.0833,$$

$$\ell_{32}(10) = \frac{p(10|\omega_3)}{p(10|\omega_2)} = \frac{1}{\ell_{23}(10)} = \frac{55}{36} = 1.5278.$$

(n5.1) Iz  $\ell_{12}(10) = 0.7333 < \theta_{12} = 2.4828$  sledi  $10 \notin \omega_1$ .

(n5.2) Iz  $\ell_{21}(10) = 1.3636 > \theta_{21} = 0.4028$  i  $\ell_{23}(10) = 0.6545 < \theta_{23} = 2.2414$  sledi  $10 \notin \omega_2$ .

(n5.2) Iz  $\ell_{31}(10) = 2.0833 > \theta_{31} = 0.2$  i  $\ell_{32}(10) = 1.5278 > \theta_{32} = 0.4462$  sledi  $10 \in \omega_3$ .

Dakle,  $5 \in \omega_3$  i  $10 \in \omega_3$ .

3. Označimo redom sa  $S_i$ ,  $i = 1, 2, \dots$  i  $z_i$ ,  $i = 1, 2, \dots$  tekuće klase i njihove centre. Inicijalno je  $S_i = \emptyset$ ,  $i = 1, 2, \dots$ . Najpre inicijalizujemo klase i njihove centre.

$bc := 1$ ;  $X := \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ ;

(1)  $z_1 := x_1 := (5, -2)$ ;  $S_1 := \{z_1\} := \{(5, -2)\}$ ;  
 $X := \{x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ ;

(2)  $d(x_2, z_1) = d((-1, -1), (5, -2)) = |-1 - 5| + |-1 - (-2)| = 7 < FP$ ;  
sledi

$bc := 1$ ;  $X := \{x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ ;

- (3)  $d(x_3, z_1) = d((-5, -4), (5, -2)) = |-5 - 5| + |-4 - (-2)| = 12 > \text{FP}$ ;  
sledi  
 $bc := 2$ ;  $z_2 := x_3 := (-5, -4)$ ;  $S_2 := \{z_2\} := \{(-5, -4)\}$ ;  $X := \{x_2, x_4, x_5, x_6, x_7, x_8\}$ ;
- (4)  $d(x_4, z_1) = d((3, 4), (5, -2)) = |3 - 5| + |4 - (-2)| = 8 \leq \text{FP}$ ;  
sledi  
 $bc := 2$ ;  $X := \{x_2, x_4, x_5, x_6, x_7, x_8\}$ ;
- (5)  $d(x_5, z_1) = d((-1, 4), (5, -2)) = |-1 - 5| + |4 - (-2)| = 12 > \text{FP}$ ;  
 $d(x_5, z_2) = d((-1, 4), (-5, -4)) = |-1 - (-5)| + |4 - (-4)| = 12 > \text{FP}$ ;  
sledi  
 $bc := 3$ ;  $z_3 := x_5 := (-1, 4)$ ;  $S_3 := \{z_3\} := \{(-1, 4)\}$ ;  $X := \{x_2, x_4, x_6, x_7, x_8\}$ ;
- (6)  $d(x_6, z_1) = d((1, 3), (5, -2)) = |1 - 5| + |3 - (-2)| = 9 > \text{FP}$ ;  
 $d(x_6, z_2) = d((1, 3), (-5, -4)) = |1 - (-5)| + |3 - (-4)| = 13 > \text{FP}$ ;  
 $d(x_6, z_3) = d((1, 3), (-1, 4)) = |1 - (-1)| + |3 - 4| = 3 < \text{FP}$ ;  
sledi  
 $bc := 3$ ;  $X := \{x_2, x_4, x_6, x_7, x_8\}$ ;
- (7)  $d(x_7, z_1) = d((-1, 2), (5, -2)) = |-1 - 5| + |2 - (-2)| = 10 > \text{FP}$ ;  
 $d(x_7, z_2) = d((-1, 2), (-5, -4)) = |-1 - (-5)| + |2 - (-4)| = 10 > \text{FP}$ ;  
 $d(x_7, z_3) = d((-1, 2), (-1, 4)) = |-1 - (-1)| + |2 - 4| = 2 < \text{FP}$ ;  
sledi  
 $bc := 3$ ;  $X := \{x_2, x_4, x_6, x_7, x_8\}$ ;
- (8)  $d(x_8, z_1) = d((3, 5), (5, -2)) = |3 - 5| + |5 - (-2)| = 9 > \text{FP}$ ;  
 $d(x_8, z_2) = d((3, 5), (-5, -4)) = |3 - (-5)| + |5 - (-4)| = 17 > \text{FP}$ ;  
 $d(x_8, z_3) = d((3, 5), (-1, 4)) = |3 - (-1)| + |5 - 4| = 5 < \text{FP}$ ;  
sledi  
 $bc := 3$ ;  $X := \{x_2, x_4, x_6, x_7, x_8\}$ ;

Sada preostale elemente skupa  $X = \{x_2, x_4, x_6, x_7, x_8\}$  raspoređujemo u klase  $S_i$ ,  $i \in \{1, 2, 3\}$  sa njihovim centrima redom  $z_1 = (5, -2)$ ,  $z_2 = (-5, -4)$  i  $z_3 = (-1, 4)$ .

- (2)  $d(x_2, z_1) = d((-1, -1), (5, -2)) = |-1 - 5| + |-1 - (-2)| = 7$ ,  
 $d(x_2, z_2) = d((-1, -1), (-5, -4)) = |-1 - (-5)| + |-1 - (-4)| = 7$ ,  
 $d(x_2, z_3) = d((-1, -1), (-1, 4)) = |-1 - (-1)| + |-1 - 4| = 5$ ,  
te iz  $d(x_2, z_3) = 5 = \min\{d(x_2, z_1), d(x_2, z_2), d(x_2, z_3)\} = \min\{7, 7, 5\}$  sledi  $x_2 \in S_3$ .
- (4)  $d(x_4, z_1) = d((3, 4), (5, -2)) = |3 - 5| + |4 - (-2)| = 8$ ,  
 $d(x_4, z_2) = d((3, 4), (-5, -4)) = |3 - (-5)| + |4 - (-4)| = 16$ ,  
 $d(x_4, z_3) = d((3, 4), (-1, 4)) = |3 - (-1)| + |4 - 4| = 4$ ,  
te iz  $d(x_4, z_3) = 4 = \min\{d(x_4, z_1), d(x_4, z_2), d(x_4, z_3)\} = \min\{8, 16, 4\}$  sledi  $x_4 \in S_3$ .
- (6)  $d(x_6, z_1) = d((1, 3), (5, -2)) = |1 - 5| + |3 - (-2)| = 9$ ,  
 $d(x_6, z_2) = d((1, 3), (-5, -4)) = |1 - (-5)| + |3 - (-4)| = 13$ ,  
 $d(x_6, z_3) = d((1, 3), (-1, 4)) = |1 - (-1)| + |3 - 4| = 3$ ,  
te iz  $d(x_6, z_3) = 3 = \min\{d(x_6, z_1), d(x_6, z_2), d(x_6, z_3)\} = \min\{9, 13, 3\}$  sledi  $x_6 \in S_3$ .
- (7)  $d(x_7, z_1) = d((-1, 2), (5, -2)) = |-1 - 5| + |2 - (-2)| = 10$ ,  
 $d(x_7, z_2) = d((-1, 2), (-5, -4)) = |-1 - (-5)| + |2 - (-4)| = 10$ ,  
 $d(x_7, z_3) = d((-1, 2), (-1, 4)) = |-1 - (-1)| + |2 - 4| = 2$ ,  
te iz  $d(x_7, z_3) = 2 = \min\{d(x_7, z_1), d(x_7, z_2), d(x_7, z_3)\} = \min\{10, 10, 2\}$  sledi  $x_7 \in S_3$ .
- (8)  $d(x_8, z_1) = d((3, 5), (5, -2)) = |3 - 5| + |5 - (-2)| = 9$ ,  
 $d(x_8, z_2) = d((3, 5), (-5, -4)) = |3 - (-5)| + |5 - (-4)| = 17$ ,  
 $d(x_8, z_3) = d((3, 5), (-1, 4)) = |3 - (-1)| + |5 - 4| = 5$ ,  
te iz  $d(x_8, z_3) = 5 = \min\{d(x_8, z_1), d(x_8, z_2), d(x_8, z_3)\} = \min\{9, 17, 5\}$  sledi  $x_8 \in S_3$ .

Dakle,  $S_1 = \{x_1\}$ ,  $S_2 = \{x_3\}$ ,  $S_3 = \{x_5, x_2, x_4, x_6, x_7, x_8\}$ .