The 5th Conference on Mathematics in Engineering: Theory and Applications
Novi Sad, May 9-10th 2020

# NEW PIXEL DESCRIPTORS BASED ON NEIGHBOR SIMILARITY

**Ljubo Nedović [1] and Marija Delić [2]**

**Abstract.** In this paper, some new gray-scale and color pixel descriptors are introduced. Presented descriptors can be interpreted as a measure of the similarity between an observed pixel and its neighbors. The few approaches presented in this paper are motivated by the well-known family of Local Binary Patterns (LBP), and introduced descriptors could be classified as a special part of the mentioned class. These descriptors can be used in various image processing tasks as image segmentation, image denoising, object and image recognition and classification, image texture analysis, etc.

*AMS Mathematics Subject Classification* (2010): 62H35, 65D18, 68U10, 94A08

*Key words and phrases:* Image analysis, Local binary pattern, Pixel descriptor

## 1. Introduction

Pixels descriptors are widely used in different image processing tasks, see [4, 5, 6, 8, 12]. All of the observed descriptors represent one class of pixels descriptors that are developed and used primarily for image texture analysis and classification problems, see [1, 2, 7, 9, 10]. Because of its success in texture classification, during the years the Local Binary Patterns (LBP) class has evolved and expanded significantly, so it finds many other applications, such as object recognition, face analysis, biometrics, and many medical image analysis applications.

In publications [3, 11], one LBP motivated pixel descriptor is introduced and achieved notable results in image segmentation tasks. The mentioned descriptor will be presented and described in detail in Section 2, because the other descriptors which we introduce in this paper, Section 3, are the descriptors of a similar type and represent some extension of the mentioned one. All of the observed descriptors represents gray-scale or color similarity between an observed pixel and its neighboring pixels.

[1]Department of Fundamental Sciences, Faculty of Technical Sciences, University of Novi Sad, e-mail: nljubo@uns.ac.rs

[2]Department of Fundamental Sciences, Faculty of Technical Sciences, University of Novi Sad, e-mail: marijadelic@uns.ac.rs

## 2. Preliminaries and previous research

In gray-scale images, each pixel is represented with its gray-scale value from the set $\{0,\ldots,255\}$, while in color images, the pixel can have $\ell$ color components, and each of them take the value from the set $\{0,\ldots,255\}$ of color component codes. For example, when image is written in RGB technique, it means that each pixel is represented with red R, green G and blue B color component, where $\ell = 3$ is the number of color components.

Let $p_{i,j}$ be pixel in $i$-th row and $j$-th column of $n \times m$ format pixel-matrix. For any number $\ell$ of color components, pixel can be represented as $\ell$-tuple $p_{i,j} = \left(c_{i,j}^{(1)},\ldots,c_{i,j}^{(\ell)}\right)$ of its color components, where each $c_{i,j}^{(k)} \in \{0,\ldots,255\}$, and $k \in \{1,\ldots,\ell\}$ is the $k$-th color component code.

In this paper, we will observe pixel $p_{i,j}$ and its 8 nearest neighbors, if $p_{i,j}$ is in interior of the image, or its 3 or 5 nearest neighbors when $p_{i,j}$ is in the corner or on the edge of the image, see figures 1, 2 and 3.

| | | |
|---|---|---|
| $p_{i-1,j-1}$ | $p_{i-1,j}$ | $p_{i-1,j+1}$ |
| $p_{i,j-1}$ | $p_{i,j}$ | $p_{i,j+1}$ |
| $p_{i+1,j-1}$ | $p_{i+1,j}$ | $p_{i+1,j+1}$ |

Figure 1: Pixel 8 neighbors.

| | |
|---|---|
| $p_{1,1}$ | $p_{1,2}$ |
| $p_{2,1}$ | $p_{2,2}$ |

| | |
|---|---|
| $p_{1,m-1}$ | $p_{1,m}$ |
| $p_{2,m-1}$ | $p_{2,m}$ |

| | |
|---|---|
| $p_{n-1,1}$ | $p_{n-1,2}$ |
| $p_{n,1}$ | $p_{n,2}$ |

| | |
|---|---|
| $p_{n-1,m-1}$ | $p_{n-1,m}$ |
| $p_{n,m-1}$ | $p_{n,m}$ |

Figure 2: Pixel 3 neighbors.

| | | |
|---|---|---|
| $p_{1,j-1}$ | $p_{1,j}$ | $p_{1,j+1}$ |
| $p_{2,j-1}$ | $p_{2,j}$ | $p_{2,j+1}$ |

| | | |
|---|---|---|
| $p_{m-1,j-1}$ | $p_{m-1,j}$ | $p_{m-1,j+1}$ |
| $p_{m,j-1}$ | $p_{m,j}$ | $p_{m,j+1}$ |

| | |
|---|---|
| $p_{i-1,1}$ | $p_{i-1,2}$ |
| $p_{i,1}$ | $p_{i,2}$ |
| $p_{i+1,1}$ | $p_{i+1,2}$ |

| | |
|---|---|
| $p_{i-1,m-1}$ | $p_{i-1,m}$ |
| $p_{i,m-1}$ | $p_{i,m}$ |
| $p_{i+1,m-1}$ | $p_{i+1,m}$ |

Figure 3: Pixel 5 neighbors.

For the aim of comparing pixel $p_{i,j}$ with each of neighbor pixels, for each color component and the threshold level $\alpha$ we define a function which values 0 and 1 indicates the level of "$\alpha$-similarity" between observed pixel $p_{i,j}$ and each of its neighbor pixels. The following pixel descriptor is introduced in [3].

**Definition 2.1.** For the pixel $p_{i,j}$, given threshold value $\alpha \in \{0,\dots,255\}$, and for each neighbor pixel $n_m$, $m \in \{1,\dots,8\}$, if exists, *$\alpha$-similarity indicator* $I^{(k)}_{p_{i,j};\alpha}(m) \in \{0,1\}$ of color-component $c^{(k)}_{i,j} \in \{0,\dots,255\}$, $k \in \{1,\dots,\ell\}$ of the pixel $p_{i,j}$ and the same color-component $n^{(k)}_m \in \{0,\dots,255\}$ of the neighbor pixel $n_m$ is defined with

$$I^{(k)}_{p_{i,j};\alpha}(m) = \begin{cases} 1 \quad , & \left|c^{(k)}_{i,j} - n^{(k)}_m\right| \le \alpha \\ 0 \quad , & \left|c^{(k)}_{i,j} - n^{(k)}_m\right| > \alpha \end{cases} . \tag{2.1}$$

For the pixel $p_{i,j}$ which is not in the interior of the image and because of that does not have some of the neighbors $n_m$, we define $I^{(k)}_{p_{i,j};\alpha}(m) = 0$.

*Remark* 2.2.

(i) For the neighbor pixel $n_m$ of the observed pixel $p_{i,j}$, $\alpha$-similarity indicator $I^{(k)}_{p_{i,j};\alpha}(m)$ takes value 1 when the difference of $k$-th color code is less or equal from the given limit $\alpha$, and in that sense represent signal of $\alpha$-similarity between pixel $p_{i,j}$ and neighboring pixel $n_m$, if $n_m$ exists. Notice that for each of the color component $k \in \{1,\dots,\ell\}$ we have a sequence of 8 indicator values (0 or 1) that represent $\alpha$-similarity of observed pixel with its 8 neighbors.

(ii) For pixel $p_{i,j}$ which is on the edge or in the corners of the image and because of that does not have some of the neighbors $n_m$, it is more natural to define $I^{(k)}_{p_{i,j};\alpha}(m) = 0$ instead of $I^{(k)}_{p_{i,j};\alpha}(m) = 1$, because indicator value 1 will represent similarity between $p_{i,j}$ and one non-existing pixel $n_m$.

The following pixel descriptor, also introduced in [3], counts the number of 1 occurrences in $I^{(k)}_{p_{i,j};\alpha}(m)$, $m \in \{1,\dots,8\}$, and represents the number of neighbor pixels that are $\alpha$-similar with the observed central pixel $p_{i,j}$. This descriptor counts the neighboring pixels that are $\alpha$-similar to the central one.

**Definition 2.3.** For given threshold value $\alpha \in \{0,\dots,255\}$, *$\alpha$-indicator counter* of $k$-th color component of the central pixel $p_{i,j}$ is the value in the range $0,\dots,8$ that is defined with the following equation:

$$\mathsf{IC}^{(k)}_\alpha(p_{i,j}) = \sum_{m=1}^{8} I^{(k)}_{p_{i,j};\alpha}(m), \tag{2.2}$$

where $I^{(k)}_{p_{i,j};\alpha}$ is the descriptor from Definition 2.1.

For each color component $k \in \{1,\dots,\ell\}$, descriptor $\mathsf{IC}^{(k)}_\alpha$ counts the number of $\alpha$-similar neighbor pixels for the central pixel $p_{i,j}$. From the definition of $\alpha$-indicator counter function follows that for all $\alpha$ and $\beta$ holds

$$\begin{aligned} \alpha \le \beta \quad &\Rightarrow \quad \forall n \in \{1,\dots,8\},\ I^{(k)}_{p_{i,j};\alpha}(n) \le I^{(k)}_{p_{i,j};\beta}(n) \\ &\Rightarrow \quad \mathsf{IC}^{(k)}_\alpha(p_{i,j}) \le \mathsf{IC}^{(k)}_\beta(p_{i,j}). \end{aligned}$$

## 3. New proposed neighbors-similarity pixel descriptors

In this section, we introduce four new pixel descriptors, based on $\alpha$-similarity indicator from the Definition 2.1. First of them is $\mathsf{SSLBP}_\alpha^{(k)}$, which is 1-byte, i.e. 8-bit value that represents a sequence of $\alpha$-indicators for 8 neighbor pixels.

**Definition 3.1.** Let $\alpha \in \{0, \dots, 255\}$. *Symmetric Shift $\alpha$-Local Binary Pattern* of $k$-th color component of the pixel $p_{i,j}$, denoted by $\mathsf{SSLBP}_\alpha^{(k)}(p_{i,j})$, is $0, \dots, 255$ ranged value (1 byte) defined by

$$\mathsf{SSLBP}_\alpha^{(k)}(p_{i,j}) = \sum_{m=1}^{8} I_{p_{i,j};\alpha}^{(k)}(m) \cdot 2^{m-1}. \tag{3.1}$$

Descriptor $\mathsf{SSLBP}_\alpha^{(k)}$ is not rotation invariant, i.e., for rotated neighborhood we obtain different sequence of 0 and 1. The definition below presents the rotation invariant modification of $\mathsf{SSLBP}_\alpha^{(k)}$ descriptor.

**Definition 3.2.** Let $\alpha \in \{0, \dots, 255\}$. *Rotation Invariant Symmetric Shift $\alpha$-Local Binary Pattern* of $k$-th color component of the pixel $p_{i,j}$, denoted by $\mathsf{RISSLBP}_\alpha^{(k)}(p_{i,j})$, is $0, \dots, 255$ ranged value defined by

$$\mathsf{RISSLBP}_\alpha^{(k)}(p_{i,j}) = \min\left\{\mathsf{ROT}\left(\mathsf{SSLBP}_\alpha^{(k)}(p_{i,j}), s\right) \mid s = 1, \dots, 8\right\}, \tag{3.2}$$

where $\mathsf{ROT}(uc, s)$ is operator which rotate values 0 and 1 in 8-bits register for $s$ place right (or left equivalently).

It is obvious that $\mathsf{RISSLBP}_\alpha^{(k)}$ is rotation invariant pixel descriptor.

When we want to compare central pixel $p_{i,j}$ with its 8 neighbors in the sense of $\mathsf{SSLBP}_\alpha^{(k)}$ or $\mathsf{RISSLBP}_\alpha^{(k)}$ descriptor for $k$-th color component, it can be desirable to see at once $\alpha$-similarity for more than one threshold value $\alpha \in \{0, \dots, 255\}$.

**Definition 3.3.** Let $M \in \mathbb{N}$ be some given number of levels $\Lambda = (\alpha_1, \dots, \alpha_M)$ of nondecreasing threshold values $\alpha_m \in \{0, \dots, 255\}$, i.e. such that $\alpha_m \leq \alpha_n$ for all $m < n$.

(a) *$M$-Level Symmetric Shift $\alpha$-Local Binary Pattern* of pixel $p_{i,j}$ is $M$-byte wide, i.e. $0, \dots, 2^{8M} - 1$ ranged number defined by

$$M\mathsf{LSSLBP}_\Lambda^{(k)}(p_{i,j}) = \sum_{m=1}^{M} \mathsf{SSLBP}_{\alpha_m}^{(k)}(p_{i,j}) \cdot 2^{8(m-1)}. \tag{3.3}$$

(b) *$M$-Level Rotation Invariant Symmetric Shift $\alpha$-Local Binary Pattern* is $M$-byte wide, i.e. $0, \dots, 2^{8M} - 1$ ranged number defined by

$$M\mathsf{LRISSLBP}_\Lambda^{(k)}(p_{i,j}) = \sum_{m=1}^{M} \mathsf{RISSLBP}_{\alpha_m}^{(k)}(p_{i,j}) \cdot 2^{8(m-1)}. \tag{3.4}$$

It is obvious that $M\mathsf{LSSLBP}_{\Lambda}^{(k)}$ descriptor is not rotation invariant, and $M\mathsf{LRISSLBP}_{\Lambda}^{(k)}$ is rotation invariant.

In the case of multi-color components, i.e., in the case $\ell \geq 2$, it is of interest to create one unique pixel similarity descriptor $\mathsf{IC}_\alpha$ by using particular color-component similarity descriptors $\mathsf{IC}_\alpha^{(k)}$, $k \in \{1, \dots, \ell\}$. We propose the creation of an 8-bit value using suitable function $f$, as

$$\mathsf{IC}_\alpha^f (p_{i,j}) = f\left(\mathsf{IC}_\alpha^{(1)}(p_{i,j}), \dots, \mathsf{IC}_\alpha^{(\ell)}(p_{i,j})\right). \tag{3.5}$$

An $\ell$-ary aggregation function $A_{[\ell]} : [0,1]^\ell \to [0,1]$ is a natural choice for function $f$, with the usage of value $\frac{1}{8}\mathsf{IC}_\alpha^{(k)}(p_{i,j})$, $k \in \{1, \dots, \ell\}$ instead of $\mathsf{IC}_\alpha^{(k)}(p_{i,j})$, $k \in \{1, \dots, \ell\}$ as arguments, because the values $\mathsf{IC}_\alpha^{(k)}(p_{i,j}) \in \{1, \dots, 8\}$, $k \in \{1, \dots, \ell\}$. So, we propose the creation of unique similarity descriptor $\mathsf{IC}_\alpha$ by using the following equation

$$\mathsf{IC}_\alpha^{A_{[\ell]}}(p_{i,j}) = A_{[\ell]}\left(\frac{1}{8}\mathsf{IC}_\alpha^{(1)}(p_{i,j}), \dots, \frac{1}{8}\mathsf{IC}_\alpha^{(\ell)}(p_{i,j})\right) \tag{3.6}$$

and an appropriate choice of aggregation function $A_{[\ell]}$. This proposal, with the usage of aggregation function, could be used in any other creation of a pixel descriptor where we want to unify few color-component in one pixel characteristic. The choice of aggregation function $A_{[\ell]}$ is a question of modelling the influence of a particular color-component in the unique descriptor value. For this purpose, we propose several generally accepted and frequently used aggregation functions that are listed bellow.

(a) Minimum and maximum functions

$$A_{[\ell]}(x_1, \dots, x_\ell) = \min(x_1, \dots, x_\ell), \quad A_{[\ell]}(x_1, \dots, x_\ell) = \max(x_1, \dots, x_\ell).$$

(b) Arithmetic mean and weighted arithmetic mean

$$A_{[\ell]}(x_1, \dots, x_\ell) = \frac{1}{\ell}\sum_{i=1}^{\ell} x_i, \qquad A_{[\ell]}(x_1, \dots, x_\ell) = \sum_{i=1}^{\ell} \lambda_{\ell,i} x_i,$$

for weight-coefficients $\lambda_{\ell,i} \geq 0$, $i \in \{1, \dots, \ell\}$, $\sum_{i=1}^{\ell} \lambda_{\ell,i} = 1$.

(c) Generalized means (for some suitable $p > 0$)

$$A_{[\ell]}(x_1, \dots, x_\ell) = \left(\frac{a_1^p + \dots + a_\ell^p}{\ell}\right)^{\frac{1}{p}}.$$

## 4. Conclusions

In this paper, we presented several pixel descriptors that can be useful in many image processing tasks. The Indicator Counter descriptor, [3], from the definition 2.3 showed excellent performances in image segmentation tasks. We suppose that Indicator Counter and Symmetric Shift Local Binary Pattern from the definition 3.1 can also contribute in image segmentation and image denoising, and that $M$-Level Symmetric Shift Local Binary Pattern from the definition 3.3 can contribute in image texture analysis. Future research will be focused on examining the usage of the presented descriptors in various digital image processing tasks.

## Acknowledgement

The authors acknowledge the financial support of Department of Fundamentals Sciences, Faculty of Technical Sciences, University of Novi Sad, in the frame of Project "NAUČNI I PEDAGOŠKI RAD NA DOKTORSKIM STUDIJAMA".

## References

[1] Sheryl Brahnam, Lakhmi C. Jain, Loris Nanni, and Alessandra Lumini. *Local binary patterns: new variants and applications*, volume 506. Springer, Berlin, Heidelberg, 09 2014.

[2] Marija Delić, Joakim Lindblad, and Nataša Sladoje. $\alpha$LBP - a novel member of the local binary pattern family based on $\alpha$-cutting. In *9th international symposium on image and signal processing and analysis, ISPA 2015, Zagreb, Croatia, september 7-9, 2015*, pages 13–18, 2015.

[3] Marija Delić, Ljubo Nedović, and Endre Pap. Extended power-based aggregation of distance functions and application in image segmentation. *Information sciences*, 494:155–173, August 2019.

[4] Richard O. Duda and Peter E. Hart. *Pattern classification and scene analysis*. Wiley-Interscience, New York, 1973.

[5] Rafael C. Gonzalez and Richard E. Woods. *Digital image processing (3rd edition)*. Prentice-Hall, Inc., Upper Saddle River, NJ, USA, 2006.

[6] Ulf Grenander. *Lectures in pattern theory, Volume 2: Pattern analysis*. Springer-Verlag, Berlin, 1978.

[7] Gustaf Kylberg and Ida-Maria Sintorn. Evaluation of noise robustness for local binary pattern descriptors in texture classification. *EURASIP journal on image and video processing*, 2013(1):1–20, April 2013.

[8] Rastislav Lukac and Konstantinos N. Plataniotis. *Color image processing: methods and applications*. Image processing series. Taylor & Francis, 2006.

[9] Loris Nanni, Alessandra Lumini, and Sheryl Brahnam. Survey on LBP based texture descriptors for image classification. *Expert systems with applications*, 39(3):3634–3641, 2012.

[10] Timo Ojala, Matti Pietikäinen, and Topi Mäenpää. Multiresolution gray-scale and rotation invariant texture classification with local binary patterns. *IEEE transactions on pattern analysis and machine intelligence*, 24(7):971–987, Jul 2002.

[11] Endre Pap, Marija Delić, and Ljubo Nedović. Information fusion by extended power-based aggregation in image segmentation. In *International scientific conference on information technology and data related research, SINTEZA 2019*, pages 162–167, 2019.

[12] Julius T. Tou and Rafael C. Gonzalez. *Pattern recognition principles.* Addison-Wesley, 1974.