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PROPERTIES OF WEIGHTED MINIMUM AND WEIGHTED MAXIMUM

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Abstract. In this paper we present and prove some properties of weighted minimum and weighted maximum aggregation function.

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1. Introduction

This paper is continuations of research from [3, 4]. Aggregation functions are important tools in many scientific fields, and belong to wide class of fuzzy operations. As model of unprecise information fusion, aggregation functions have many applications in engineering, mathematical and even social scientific fields. We examine properties of weighted minimum and maximum aggregation functions which are important for construction of new measure-type functions.

2. Preliminaries and previous research

The definition and some other possible properties of aggregation function follows, see [2].

Definition 2.1 (Aggregation function)**.** For fixed $n \in \mathbb{N}$, an n-ary *aggregation function* is a function $A_{[n]} : [0,1]^n \to [0,1]$ with the following properties.

(a01) Boundary conditions $A_{[n]}(0,\dots,0) = 0$ and $A_{[n]}(1,\dots,1) = 1$ hold.

(a02) Function $A_{[n]}$ is monotonically non-decreasing in each component, i.e.,

$$\forall i \in \{1,\dots,n\},\ a_i \le b_i \quad \Rightarrow \quad A_{[n]}(a_1,\dots,a_n) \le A_{[n]}(b_1,\dots,b_n)$$

hold for arbitrary n-tuples $(a_1,\dots,a_n) \in [0,1]^n$, $(b_1,\dots,b_n) \in [0,1]^n$.

Additionaly, by definition is $A_{[1]}(x) = x$ for all $x \in [0,1]$. The following properties can be relevant for construction of new measure-type functions.

Definition 2.2. Let $n \in \mathbb{N}$, and let $A_{[n]} : [0,1]^n \to [0,1]$ be an n-ary aggregation function. It may have some of following properties.

(a03) $A_{[n]}$ is continuous function.

(a04) $A_{[n]}$ is symmetric, i.e., for each $(a_1,\dots,a_n) \in [0,1]^n$ and each permutation p of $\{1,\dots,n\}$ hold $A_{[n]}(a_1,\dots,a_n) = A_{[n]}(a_{p(1)},\dots,a_{p(n)})$.

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(a05) $A_{[n]}$ is idempotent, i.e., for all $(a,\ldots,a)\in[0,1]^n$ is $A_{[n]}(a,...,a)=a$.

(a06) $A_{[n]}$ is additive, i.e., for all $(a_1,...,a_n),(b_1,...,b_n)\in[0,1]^n$ that satisfy $(a_1+b_1,...,a_n+b_n)\in[0,1]^n$ hold
$A_{[n]}(a_1+b_1,\ldots,a_n+b_n)=A_{[n]}(a_1,\ldots,a_n)+A_{[n]}(b_1,\ldots,b_n)$.

(a07) $A_{[n]}$ is subadditive, i.e., for all $(a_1,\ldots,a_n),(b_1,\ldots,b_n)\in[0,1]^n$ that satisfy $(a_1+b_1,\ldots,a_n+b_n)\in[0,1]^n$ hold
$A_{[n]}(a_1+b_1,\ldots,a_n+b_n)\le A_{[n]}(a_1,\ldots,a_n)+A_{[n]}(b_1,\ldots,b_n)$.

(a08) $A_{[n]}$ is superadditive, i.e., for all $(a_1,\ldots,a_n),(b_1,\ldots,b_n)\in[0,1]^n$ that satisfy $(a_1+b_1,\ldots,a_n+b_n)\in[0,1]^n$ hold
$A_{[n]}(a_1+b_1,\ldots,a_n+b_n)\ge A_{[n]}(a_1,\ldots,a_n)+A_{[n]}(b_1,\ldots,b_n)$.

(a09) $A_{[n]}$ is positively homogeneous, i.e., for $t\ge 0$ and $(a_1,...,a_n)\in[0,1]^n$ such that $(ta_1,...,ta_n)\in[0,1]^n$ hold $A_{[n]}(ta_1,...,ta_n)=tA_{[n]}(a_1,...,a_n)$.

(a10) $A_{[n]}$ is positively subhomogeneous, i.e., for $t\ge 0$, $(a_1,...,a_n)\in[0,1]^n$ such that $(ta_1,...,ta_n)\in[0,1]^n$ hold $A_{[n]}(ta_1,...,ta_n)\le tA_{[n]}(a_1,...,a_n)$.

(a11) $A_{[n]}$ is positively superhomogeneous, i.e., for $t\ge 0$, $(a_1,...,a_n)\in[0,1]^n$ such that $(ta_1,...,ta_n)\in[0,1]^n$ hold $A_{[n]}(ta_1,...,ta_n)\ge tA_{[n]}(a_1,...,a_n)$.

(a12) $A_{[n]}(a_1,\ldots,a_n)=0 \;\Rightarrow\; \forall i\in\{1,\ldots,n\},\ a_i=0$.

(a13) $A_{[n]}(a_1,\ldots,a_n)=0 \;\Rightarrow\; \exists i\in\{1,\ldots,n\},\ a_i=0$.

(a14) $A_{[n]}(a_1,\ldots,a_n)<1 \;\Rightarrow\; \forall i\in\{1,\ldots,n\},\ a_i<1$.

3. Weighted minimum and maximum

In this section we analyze the properties from the Definition 2.2 of the weighted minimum and weighted maximum aggregation function, see [1].

Definition 3.1. For $n\in\mathbb{N}$, let $\omega_{[n]}=(\omega_{n,1},\ldots,\omega_{n,n})\in[0,1]^n$ be an n-tuple of nonnegative coefficient from interval $[0,1]$ such that $\max\limits_{i\in\{1,\ldots,n\}}\omega_{n,i}=1$, i.e., such that $\omega_{n,i_0}=1$ for some $i_0\in\{1,\ldots,n\}$. *Weighted minimum* is a function $\min_{[n]}^{\omega_{[n]}}:[0,1]^n\to[0,1]$ defined by

$$\min\nolimits_{[n]}^{\omega_{[n]}}(a_1,...,a_n)=\min_{i\in\{1,\ldots,n\}}\max(1-\omega_{n,i},a_i),\quad (a_1,...,a_n)\in[0,1]^n, \tag{3.1}$$

and *weighted maximum* is a function $\max_{[n]}^{\omega_{[n]}}:[0,1]^n\to[0,1]$ defined by

$$\max\nolimits_{[n]}^{\omega_{[n]}}(a_1,...,a_n)=\max_{i\in\{1,\ldots,n\}}\min(\omega_{n,i},a_i),\quad (a_1,...,a_n)\in[0,1]^n. \tag{3.2}$$

In the case when $\omega_{n,i}=1$ for all $i\in\{1,\ldots,n\}$, $\min_{[n]}^{\omega_{[n]}}$ and $\max_{[n]}^{\omega_{[n]}}$ reduce to ordinary min and max respectively. Let we analyze which of the properties from the Definition 2.2 hold for them. In proof that function $\max_{[n]}^{\omega_{[n]}}$ have the property of subadditivity, we will use the following proposition.

Proposition 3.2. *For arbitrary nonnegative numbers $\omega\ge 0$, $a\ge 0$ and $b\ge 0$ the* $\min(\omega,a+b)\le\min(\omega,a)+\min(\omega,b)$ *inequality holds.*

Proof: In the case $a+b\le\omega$ is $a\le\omega$ and $b\le\omega$, and then $\min(\omega,a+b)=a+b=\min(\omega,a)+\min(\omega,b)$. In the case $a+b>\omega$, we have the following.

- For $a<b<\omega$ (and the analogous case $b<a<\omega$) hold
  $\min(\omega,a+b)=\omega<a+b=\min(\omega,a)+\min(\omega,b)$.
- For $a<\omega<b$ (and the analogous case $b<\omega<a$) hold

$\min(\omega, a+b) = \omega < a + \omega = \min(\omega, a) + \min(\omega, b)$.

- For $\omega < a < b$ (and the analogous case $\omega < b < a$) hold
  $\min(\omega, a+b) = \omega < \omega + \omega = \min(\omega, a) + \min(\omega, b)$. □

Let we discus the considered properties of $\min_{[n]}^{\omega_{[n]}}$ and $\max_{[n]}^{\omega_{[n]}}$.

(a06) Functions $\min_{[n]}^{\omega_{[n]}}$ and $\max_{[n]}^{\omega_{[n]}}$ are not additive, see (a07) and (a08).

(a07) We will show that aggregation function $\max_{[n]}^{\omega_{[n]}}$ is subadditive, and that $\min_{[n]}^{\omega_{[n]}}$ is not subadditive.

$\min_{[n]}^{\omega_{[n]}}$: For $n = 2$, $\omega_{[2]} = (0.55, 1)$, $(a_1, a_2) = (0.2, 0.6)$, $(b_1, b_2) = (0.8, 0.1)$,

$$\min_{[2]}^{\omega_{[2]}}(a_1 + b_1, a_2 + b_2) = \min_{[2]}^{\omega_{[2]}}(1, 0.7)$$
$$= \min(\max(1 - 0.55, 1), \max(1 - 1, 0.7)) = \min(1, 0.7) = 0.7,$$
$$\min_{[2]}^{\omega_{[2]}}(a_1, a_2) + \min_{[2]}^{\omega_{[2]}}(b_1, b_2)$$
$$= \min(\max(1 - 0.55, 0.2), \max(1 - 1, 0.6)) +$$
$$+ \min(\max(1 - 0.55, 0.8), \max(1 - 1, 0.1))$$
$$= \min(0.45, 0.6) + \min(0.8, 0.1) = 0.55 < \min_{[2]}^{\omega_{[2]}}(a_1 + b_1, a_2 + b_2).$$

$\max_{[n]}^{\omega_{[n]}}$: Let $\omega_{[n]} = (\omega_{n,1}, .., \omega_{n,n}) \in [0,1]^n$ and $(a_1, .., a_n), (b_1, .., b_n) \in [0,1]^n$ such that $(a_1 + b_1, ..., a_n + b_n) \in [0,1]^n$, we have the following. Let

$$\max_{[n]}^{\omega_{[n]}}(a_1, \ldots, a_n) = \max_{i \in \{1,\ldots,n\}} \min(\omega_{n,i}, a_i) = \min(\omega_{n,p}, a_p),$$
$$\max_{[n]}^{\omega_{[n]}}(b_1, \ldots, b_n) = \max_{i \in \{1,\ldots,n\}} \min(\omega_{n,i}, b_i) = \min(\omega_{n,q}, b_q)$$

for some $p, q \in \{1, \ldots, n\}$. From the Proposition 3.2 follows that

$$\forall i \in \{1, \ldots, n\}, \quad \min(\omega_{n,i}, a_i + b_i) \le \min(\omega_{n,i}, a_i) + \min(\omega_{n,i}, b_i)$$

and then

$$\max_{[n]}^{\omega_{[n]}}(a_1 + b_1, \ldots, a_n + b_n) = \max_{i \in \{1,\ldots,n\}} \min(\omega_{n,i}, a_i + b_i)$$
$$\le \max_{i \in \{1,\ldots,n\}} (\min(\omega_{n,i}, a_i) + \min(\omega_{n,i}, b_i)),$$

and using subadditivity of ordinary max aggregation function, see [3], we finally obtain

$$\max_{[n]}^{\omega_{[n]}}(a_1 + b_1, ..., a_n + b_n) \le \max_{i \in \{1,\ldots,n\}} (\min(\omega_{n,i}, a_i) + \min(\omega_{n,i}, b_i))$$
$$\le \max_{i \in \{1,\ldots,n\}} \min(\omega_{n,i}, a_i) + \max_{i \in \{1,\ldots,n\}} \min(\omega_{n,i}, b_i)$$
$$= \max_{[n]}^{\omega_{[n]}}(a_1, \ldots, a_n) + \max_{[n]}^{\omega_{[n]}}(b_1, \ldots, b_n).$$

(a08) Next examples show that $\min_{[n]}^{\omega_{[n]}}$ and $\max_{[n]}^{\omega_{[n]}}$ are not superadditive.

$\min_{[n]}^{\omega_{[n]}}$: For $n = 2$, $\omega_{[2]} = (0.3, 1)$, $(a_1, a_2) = (0.7, 0.4)$, $(b_1, b_2) = (0.25, 0.6)$,

$$\min_{[2]}^{\omega_{[2]}}(a_1 + b_1, a_2 + b_2) = \min_{[2]}^{\omega_{[2]}}(0.95, 1)$$
$$= \min(\max(1 - 0.3, 0.95), \max(1 - 1, 1)) = \min(0.95, 1) = 0.95,$$
$$\min_{[2]}^{\omega_{[2]}}(a_1, a_2) + \min_{[2]}^{\omega_{[2]}}(b_1, b_2)$$
$$= \min(\max(1 - 0.3, 0.7), \max(1 - 1, 0.4)) +$$
$$+ \min(\max(1 - 0.3, 0.25), \max(1 - 1, 0.6))$$
$$= \min(0.7, 0.4) + \min(0.7, 0.6) = 1 > \min_{[2]}^{\omega_{[2]}}(a_1 + b_1, a_2 + b_2).$$

$\max_{[n]}^{\omega_{[n]}}$: For $n = 2$, $\omega_{[2]} = (1, 0.5)$, $(a_1, a_2) = (0.4, 0.9)$, $(b_1, b_2) = (0.05, 0.1)$,

$$\begin{aligned}
&\max_{[2]}^{\omega_{[2]}}(a_1 + b_1, a_2 + b_2) = \max_{[2]}^{\omega_{[2]}}(0.45, 1) \\
&\quad = \max(\min(1, 0.45), \min(0.5, 1)) = \max(0.45, 0.5) = 0.5, \\
&\max_{[2]}^{\omega_{[2]}}(a_1, a_2) + \max_{[2]}^{\omega_{[2]}}(b_1, b_2) \\
&= \max(\min(1, 0.4), \min(0.5, 0.9)) + \max(\min(1, 0.05), \min(0.5, 0.1)) \\
&= \max(0.4, 0.5) + \max(0.05, 0.1) = 0.6 > \max_{[2]}^{\omega_{[2]}}(a_1 + b_1, a_2 + b_2).
\end{aligned}$$

(a09) As we will see, $\min_{[n]}^{\omega_{[n]}}$ and $\max_{[n]}^{\omega_{[n]}}$ are not positively subhomogeneous nor positively superhomogeneous. Hence, they are not positively homogeneous functions. Both are homogeneous just in special case when $\omega_{n,i} = 1$ for all $i \in \{1, \ldots, n\}$, i.e., when they reduce to ordinary min and max.

(a10) The following examples illustrate that functions $\min_{[n]}^{\omega_{[n]}}$ and $\max_{[n]}^{\omega_{[n]}}$ are not positively subhomogeneous.

$\min_{[n]}^{\omega_{[n]}}$: For $n = 2$, $\omega_{[n]} = (0.7, 1)$, $t = 0.2$ and $(a_1, a_2) = (0.1, 0.9)$ hold

$$\begin{aligned}
&\min_{[2]}^{\omega_{[2]}}(t \cdot a_1, t \cdot a_2) = \min_{[2]}^{\omega_{[2]}}(0.02, 0.18) \\
&= \min(\max(1-0.7, 0.02), \max(1-1, 0.18)) = \min(0.3, 0.18) = 0.18, \\
&t \cdot \min_{[2]}^{\omega_{[2]}}(a_1, a_2) = 0.2 \cdot \min(\max(1-0.7, 0.1), \max(1-1, 0.9)) \\
&= 0.2 \cdot \min(0.3, 0.9) = 0.2 \cdot 0.3 = 0.06 < \min_{[2]}^{\omega_{[2]}}(t \cdot a_1, t \cdot a_2).
\end{aligned}$$

$\max_{[n]}^{\omega_{[n]}}$: For $n = 2$, $\omega_{[n]} = (1, 0.4)$, $t = 0.5$ and $(a_1, a_2) = (0.4, 0.8)$ hold

$$\begin{aligned}
&\max_{[2]}^{\omega_{[2]}}(t \cdot a_1, t \cdot a_2) = \max_{[2]}^{\omega_{[2]}}(0.2, 0.4) \\
&\qquad = \max(\min(1, 0.2), \min(0.4, 0.4)) = \max(0.2, 0.4) = 0.4, \\
&t \cdot \max_{[2]}^{\omega_{[2]}}(a_1, a_2) = 0.5 \cdot \max(\min(1, 0.4), \min(0.4, 0.8)) \\
&\qquad = 0.5 \cdot \max(0.4, 0.4) = 0.5 \cdot 0.4 = 0.2 < \max_{[2]}^{\omega_{[2]}}(t \cdot a_1, t \cdot a_2).
\end{aligned}$$

(a11) Next examples show that $\min_{[n]}^{\omega_{[n]}}$ and $\max_{[n]}^{\omega_{[n]}}$ have not property (a11).

$\min_{[n]}^{\omega_{[n]}}$: For $n = 2$, $\omega_{[n]} = (0.3, 1)$, $t = 1.6$ and $(a_1, a_2) = (0.4, 0.6)$ hold

$$\begin{aligned}
&\min_{[2]}^{\omega_{[2]}}(t \cdot a_1, t \cdot a_2) = \min_{[2]}^{\omega_{[2]}}(0.64, 0.96) \\
&= \min(\max(1-0.3, 0.64), \max(1-1, 0.96)) = \min(0.7, 0.96) = 0.7, \\
&t \cdot \min_{[2]}^{\omega_{[2]}}(a_1, a_2) = 1.6 \cdot \min(\max(1-0.3, 0.4), \max(1-1, 0.6)) \\
&= 1.6 \cdot \min(0.7, 0.6) = 1.6 \cdot 0.6 = 0.96 > \min_{[2]}^{\omega_{[2]}}(t \cdot a_1, t \cdot a_2).
\end{aligned}$$

$\max_{[n]}^{\omega_{[n]}}$: For $n = 2$, $\omega_{[n]} = (1, 0.07)$, $t = 2$ and $(a_1, a_2) = (0.05, 0.1)$ hold

$$\begin{aligned}
&\max_{[2]}^{\omega_{[2]}}(t \cdot a_1, t \cdot a_2) = \max_{[2]}^{\omega_{[2]}}(0.1, 0.2) \\
&\quad = \max(\min(1, 0.1), \min(0.07, 0.2)) = \max(0.1, 0.07) = 0.1, \\
&t \cdot \max_{[2]}^{\omega_{[2]}}(a_1, a_2) = 2 \cdot \max(\min(1, 0.05), \min(0.07, 0.1)) \\
&\quad = 2 \cdot \max(0.05, 0.07) = 2 \cdot 0.07 = 0.14 > \max_{[2]}^{\omega_{[2]}}(t \cdot a_1, t \cdot a_2).
\end{aligned}$$

(a12) In general, functions $\min_{[n]}^{\omega_{[n]}}$ and $\max_{[n]}^{\omega_{[n]}}$ have not the property (a12). For example, for $n=2$, $\omega_{[2]}=(0,1)$ and $(a_1,a_2)=(0.5,0)$ hold

$$\min_{[2]}^{\omega_{[2]}}(a_1,a_2)=\min\left(\max(1-0,0.5),\max(1-1,0)\right)=\min\left(1,0\right)=0,$$

$$\max_{[2]}^{\omega_{[2]}}(a_1,a_2)=\max\left(\min(0,0.5),\min(1,0)\right)=\max\left(0,0\right)=0.$$

We will additionaly prove that weighted maximum function $\max_{[n]}^{\omega_{[n]}}$ have the property (a12) if and only if $\omega_{n,i}>0$ for all $i\in\{1,\ldots,n\}$.

($\Rightarrow$) In the case when the condition $\omega_{n,i}>0$, $i\in\{1,\ldots,n\}$ is satisfied, then

$$\max_{[n]}^{\omega_{[n]}}(a_1,\ldots,a_n)=\max_{i\in\{1,\ldots,n\}}\min\left(\omega_{n,i},a_i\right)=0$$

implies that $\min\left(\omega_{n,i},a_i\right)=0$ for all $i\in\{1,\ldots,n\}$, and because of $\omega_{n,i}>0$, $i\in\{1,\ldots,n\}$ follows that $a_i=0$ hold for all $i\in\{1,\ldots,n\}$.

($\Leftarrow$) On the other hand, if $\omega_{n,i_0}=0$ for some $i_0\in\{1,\ldots,n\}$, then for some $a_{n,i_0}>0$ is $\min\left(\omega_{n,i_0},a_{i_0}\right)=0$. For $a_i=0$, $i\neq i_0$ then follows that $\min\left(\omega_{n,i},a_i\right)=0$ also for $i\neq i_0$, so we have $a_{n,i_0}>0$ and

$$\max_{[n]}^{\omega_{[n]}}(a_1,\ldots,a_n)=\max_{i\in\{1,\ldots,n\}}\min\left(\omega_{n,i},a_i\right)=0.$$

(a13) We will prove that functions $\min_{[n]}^{\omega_{[n]}}$ and $\max_{[n]}^{\omega_{[n]}}$ have the property (a13).

$\min_{[n]}^{\omega_{[n]}}$: If $\min_{[n]}^{\omega_{[n]}}(a_1,\ldots,a_n)=\min_{i\in\{1,\ldots,n\}}\max\left(1-\omega_{n,i},a_i\right)=0$ hold, then $\max\left(1-\omega_{n,i_0},a_{i_0}\right)=0$ for some $i_0\in\{1,\ldots,n\}$, and consequently $a_{i_0}=0$ for some $i_0\in\{1,\ldots,n\}$.

$\max_{[n]}^{\omega_{[n]}}$: If $\max_{[n]}^{\omega_{[n]}}(a_1,\ldots,a_n)=\max_{i\in\{1,\ldots,n\}}\min\left(\omega_{n,i},a_i\right)=0$, then we have $\min\left(\omega_{n,i},a_i\right)=0$ for all $i\in\{1,\ldots,n\}$, and also for $i_0\in\{1,\ldots,n\}$ for which is $\omega_{n,i_0}=1$, and therefore is then $a_{i_0}=0$.

(a14) The following examples show that $\min_{[n]}^{\omega_{[n]}}$ and $\max_{[n]}^{\omega_{[n]}}$ have not the property (a14). For $n=2$, $\omega_{[2]}=(0.4,1)$ and $(a_1,a_2)=(1,0.5)$ hold

$$\min_{[2]}^{\omega_{[2]}}(a_1,a_2)=\min\left(\max\left(1-0.4,1\right),\max\left(1-1,0.5\right)\right)=0.5<1,$$

$$\max_{[2]}^{\omega_{[2]}}(a_1,a_2)=\max\left(\min\left(0.4,1\right),\min\left(1,0.5\right)\right)=0.5<1.$$

Notice that function $\max_{[n]}^{\omega_{[n]}}$ have the property (a14) if and only if is $\omega_{n,i}=1$ for all $i\in\{1,\ldots,n\}$, i.e., when $\max_{[n]}^{\omega_{[n]}}$ is reduced on ordinary max. In fact, max aggregation function obviously satisfies the property (a14). On the other hand, when $\omega_{n,i}=1$ for all $i\in\{1,\ldots,n\}$, we have that $\max_{[n]}^{\omega_{[n]}}(a_1,\ldots,a_n)=\max_{i\in\{1,\ldots,n\}}\min\left(\omega_{n,i},a_i\right)<1$ only if $\min\left(\omega_{n,i},a_i\right)<1$ i.e. $\min\left(1,a_i\right)<1$ for all $i\in\{1,\ldots,n\}$, and that is only if $a_i<1$ for all $i\in\{1,\ldots,n\}$.

Considered properties of $\min_{[n]}^{\omega_{[n]}}$ and $\max_{[n]}^{\omega_{[n]}}$ aggregation functions, in general case, are summarized in the Table 1.

	(a03)	(a04)	(a05)	(a06)	(a07)	(a08)	(a09)	(a10)	(a11)	(a12)	(a13)	(a14)
$\min_{[n]}^{\omega_{[n]}}$	YES	no	YES	no	no	no	no	no	no	no	YES	no
$\max_{[n]}^{\omega_{[n]}}$	YES	no	YES	no	YES	no	no	no	no	no	YES	no

Table 1: Properties of $\min_{[n]}^{\omega_{[n]}}$ i $\max_{[n]}^{\omega_{[n]}}$ aggregation functions.

4. Conclusions

As generalization of min and max, aggregation functions *weighted minimum* and *weighted minimum* have less of considered properties then min and max, but they can be suitable for information aggregation in some applications.

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