Image analysis with subpixel precision - The Coverage model

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Models and methods for precise image analysis

Few words about the project:

- **Info**
  - http://imft.ftn.uns.ac.rs/∼natasa/CoverageModelCourse
  - Joakim and Nataša are sitting in Room 2144
- **Goals**
  - To initiate concrete research tracks which include practical application of the framework of the coverage model to already existing research projects at CBA.
  - To further develop the framework through cooperation.
  - Collaborations initiated will be intensively carried on during the visit, along with the course, which will serve as a discussion platform/idea incubator. The collaborating projects will be continued, with an aim to be brought to some results and conclusions (hopefully, some publications) during the expected additional visit the following year.

Models and methods for precise image analysis

- **Content**
  - Workshop (course) for initial exchange of ideas and experiences related to the topic, according to the schedule available from the course page (Sept 11–25)
  - Further work on project tasks for students interested in applying, or further developing, the described methods (Sept 25 – Oct 5);
  - Presentation (defence) of the project results. If supported by a moderately sized report, this work is assigned 3 ECTS credits (Oct 5)
  - Even further collaborative work on promising projects (Oct 5 – (Oct 25) ...).

Coverage model - what and why, briefly

- A particular way to represent shapes/objects in images - **Coverage representation** - will be presented.
- Main motivation to introduce it is to enable **increased precision** of image processing tasks, compared to the classical - crisp/binary - shape representation.
- Ways how to obtain coverage representation - **coverage segmentation algorithms** - will be presented.
- Performance of the model, esp. in terms of **feature extraction**, will be explored with intention to motivate its usage.
- Examples of **applications** of the model will be given.

Motivation and context

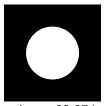
- The task of **Image Analysis** is to **extract relevant information from images**.
- **Numerical descriptors**, such as area, perimeter, and moments of objects are often of interest, for the tasks of shape analysis, classification, etc.

The standard image analysis task (and its solution)

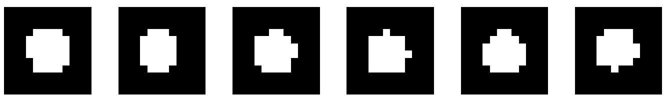
1. Sample preparation and Imaging
2. Pre-processing (optional)
3. Segmentation
   - Usually crisp
4. Feature extraction
   - Discrete representation problematic
5. Classification

Motivation - feature estimation

Crisp discrete object representations, especially at low spatial resolutions, put strong limitations to the precision of estimated features:



Area: 28.274
Perim: 18.850



Area: 31.000 Perim: 19.422 | Area: 26.000 Perim: 17.526 | Area: 28.000 Perim: 18.867 | Area: 27.000 Perim: 19.422 | Area: 28.000 Perim: 18.311 | Area: 27.000 Perim: 18.867

Motivation - feature extraction

Unrealistically low resolution?

- Low resolution will always be a challenge; the more powerful imaging devices, the smaller objects are of interest!
- Partial Volume Effect in high resolution 3D images; consistent small displacement of object boundaries leads to significant errors of feature estimates.

Approach based on fuzzy sets

A fuzzy set of a reference set is a set of ordered pairs

$$F = \{\langle x, \mu_F(x)\rangle \mid x \in X\},$$

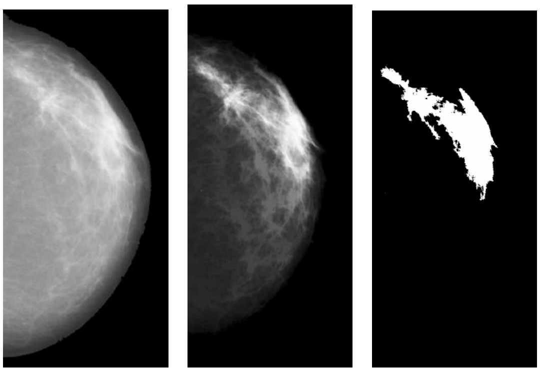
where the membership function $\mu_F : X \to [0, 1]$ indicates, for each element $x \in X$, to what extent it belongs to the fuzzy set F.

Observations:

- A fuzzy set is defined/identified by its membership function
- A crisp set is a special case of fuzzy set, where membership function takes only two values, 0 and 1.

An example of a fuzzy segmented image

Breast density, as measured from the **volume of dense tissue** in the breast is considered to indicate a risk factor for breast cancer.



A digitized X-ray mammogram, the fuzzy connectivity scene of a dense (fibroglandular) region (as opposed to fatty regions), and a corresponding crisp region.

The Fuzzy Approach

- **Do not throw away information by making crisp decisions.**
- The more nuanced view of a fuzzy approach allows **preservation of more information**.
- A representation based on **fuzzy sets** can provide numerical descriptors with **higher precision** than what can be achieved from a crisp representation.

The Fuzzy Approach

A standard image analysis task and its fuzzy solution

1. Sample preparation and Imaging
2. Pre-processing (optional)
3. **Segmentation**
   - Fuzzy segmentation (a lot of **freedom**)
4. **Feature extraction**
   - Fuzzy representation provides robustness and precision
   - May be **difficult to interpret** the results; different meanings of memberships
5. **Classification, statistical evaluation, . . .**

New entries in the “standard chain of tasks” required:

- New segmentation methods that result in fuzzy representations;
- New analysis methods that can be applied to fuzzy object representations;
- New ways of interpreting and understanding the obtained results.

The Fuzzy Approach
Important observations

- The original grey levels are, in general, **not directly used** as fuzzy representations.
- Appropriately designed fuzzy segmentation methods combine original grey levels together with several other types of information to define membership value of a pixel to an object.
- Meaning of grey levels in a fuzzy segmented image is always a consequence of many criteria, many of them only implicitly present.

- Instead of dependence on the properties of imaging devices, we are dependent on criteria used for fuzzy segmentation.
- Interpretation of results can become rather difficult.

The Coverage Model

- Keep good sides of fuzzy; stay close to the digital image, high information content, soft boundaries, robustness.
- Restrict to **one single meaning of memberships**; clear unique interpretation, enabling stronger theoretical results.

The Coverage Model

- **Let the (membership) value of an image element be equal to its relative coverage by the image object.**

- A representation close to the original (discrete) image data.
- Based on very weak assumptions about the imaged objects.
- Utilizing the coverage information, significant improvement in precision of extracted feature values can be reached.

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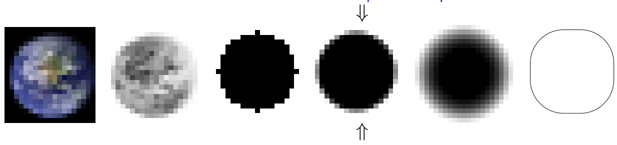
Introduction to the topic
The Coverage Model

The Coverage Model

Lots of possible representations. . .



So, what do we want, and what do we not want?

- First and second representations preserve quite some information, but are too dependent on imaging;
- Binary representation sacrifices too much information;
- We cannot complain on the forth one!
- Too flexible definition of membership values in a (general) fuzzy representation leads to difficult interpretations;
- Continuous models often impose too strong (and unrealistic) assumptions on the image objects.

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The Coverage Model

Pixel coverage digitization

Let a square grid in 2D be given. The Voronoi region associated to a grid point $(i,j) \in \mathbb{Z}^2$ is called pixel $p_{(i,j)}$.

Definition (non-quantized case)

For a given continuous object $S \subset \mathbb{R}^2$, inscribed into an integer grid with pixels $p_{(i,j)}$, the *pixel coverage digitization* of S is

$$\mathcal{D}(S) = \left\{ \left((i,j), \frac{A(p_{(i,j)} \cap S)}{A(p_{(i,j)})} \right) \middle| \, (i,j) \in \mathbb{Z}^2 \right\} ,$$

where $A(X)$ denotes the area of a set X.

Digital images $\rightarrow$ Quantized grey values

Definition (n-level quantized case)

$$\mathcal{D}^n(S) = \left\{ \left((i,j), \frac{1}{n} \left\lfloor n \frac{A(p_{(i,j)} \cap S)}{A(p_{(i,j)})} + \frac{1}{2} \right\rfloor \right) \middle| \, (i,j) \in \mathbb{Z}^2 \right\} ,$$

where $\lfloor x \rfloor$ denotes the largest integer not greater than x.

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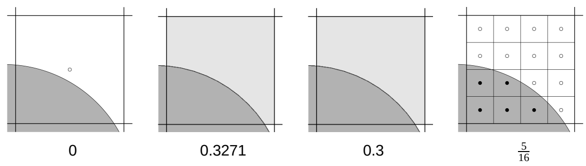
The Coverage Model

Pixel coverage digitization

The membership value of a pixel is
equal to its relative coverage by an imaged object (here a part of a disk).



- Gauss digitization assigns value 0 to the observed pixel.
- By coverage digitization assigned value is 0.3271.
- If 10-level quantized coverage digitization is used, assigned value is 0.3.
- If coverage is approximated by 4-sampled coverage digitization, assigned value is $\frac{5}{16}$.

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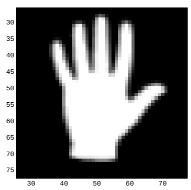
Properties of coverage representations



Intuitively, a coverage representation of a crisp object, with a well defined continuous border, is characterized by the presence of **homogeneous connected regions of "pure" pixels**, completely covered by either object or background **separated by thin layers of "mixed" pixels**, i.e., those partially covered by both object and background.

If a crisp continuous set has a reasonably smooth boundary and is represented at a high enough resolution, then **the fuzzy border** of its digital coverage representation **is not more than one pixel thick**.

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The Coverage Model

The image analysis task and its coverage solution

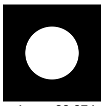
1. Sample preparation and Imaging
2. Pre-processing (optional)
3. **Segmentation**
   - Coverage segmentation (restricted freedom)
4. **Feature extraction**
   - Estimate features from the coverage representation
   - Easy to interpret the results, robustness and precision
5. Classification, statistical evaluation, . . .

New entries in the "standard chain of tasks" required:

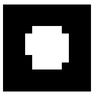
- New segmentation methods that result in coverage representations;
- New analysis methods that can be applied to coverage object representations.

However, there is no need for new ways of interpreting and understanding the obtained results.

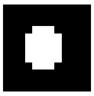
Feature extraction revisited



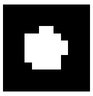
Area: 28.274
Perim: 18.850



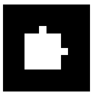
Area: 31.000
Perim: 19.422



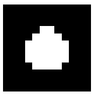
Area: 26.000
Perim: 17.526



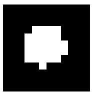
Area: 28.000
Perim: 18.867



Area: 27.000
Perim: 19.422



Area: 28.000
Perim: 18.311



Area: 27.000
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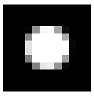

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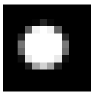

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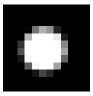

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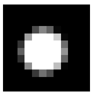

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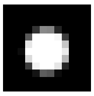
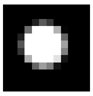


Area: 28.274
Perim: 18.654



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