Image analysis with subpixel precision - The Coverage model

Part 3 - Distances between sets

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Outline

1 Distances between sets
2 Distances between fuzzy sets
3 Concluding remarks

Motivation

Task:
Characterize a pair of sets by a single number
- the distance between them -
reflecting size of a displacement and/or difference in some other way.

- Interest is both in
  - theoretical aspects of the problem i.e., different properties of different measure defined,
  - practical issues, i.e. performance of distance measures in applications.
- Our choice of distance measures to explore:
  - applicable to image registration and pattern/shape matching;
  - of linear computational complexity;
  - applicable to crisp sets and applicable to fuzzy sets.

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Our results are summarized in the following papers:

- J. Lindblad, V. Ćurić, and N. Sladoje. On set distances and their application to image registration. In Proceedings of the 6th International Symposium on Image and Signal Processing and Analysis (ISPA),Salzburg, Austria. IEEE, pp. 449-454, 2009.
- V. Ćurić, J. Lindblad, N. Sladoje, H. Sarve, and G. Borgefors. A new set distance and its application to shape registration. Accepted for Pattern Analysis and Applications, 2012.
- V. Ćurić, J. Lindblad, and N. Sladoje. Distance measures between digital fuzzy objects and their applicability in image processing. In Proceedings of the 14th International Workshop on Combinatorial Image Analysis (IWCIA2011), Madrid, Spain. Lecture Notes in Computer Science, Vol. 6636, pp. 385-395 , 2011.

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- J. Lindblad, and N. Sladoje. Distance measures between digital fuzzy objects - cutting vertically vs. cutting horizontally. In preparation.

Distance

Most generally, **distance** is any mapping $d : X \times X \to \mathbb{R}^+ \cup \{0\}$. A list of desirable properties of a distance contains the following:

- Non-negativity: $d(A,B) \geq 0$.
- Separability: $d(A,B) = 0$ if and only if $A = B$.
- Symmetry: $d(A,B) = d(B,A)$.
- Subadditivity (triangle inequality): $d(A,C) \leq d(A,B) + d(B,C)$.

Distances between objects

Distances

- between two points in a set
- between a point and a set
- between two sets

are of interest.

Point-to-point:
Distance d between two points $a, b \in X$, is $d(a, b) = \|a - b\|_2$.

Point-to-set:
The distance between a point $a \in X$ and a non-empty set $B \subseteq X$ is

$$d(a, B) = \inf_{b \in B} d(a, b) \ .$$

The distance **between sets** often incorporates information on point-to-set distances, for some selection of the points involved.

Distances between crisp objects

- Hausdorff distance
$$d_H(A, B) = \max\left(\sup_{a \in A} d(a, B), \sup_{b \in B} d(b, A)\right)$$
- Modified Hausdorff distance
$$d_{MH}(A, B) = \max\left(\frac{1}{|A|}\sum_{a \in A} d(a, B), \frac{1}{|B|}\sum_{b \in B} d(b, A)\right)$$
- Metric by Symmetric difference
$$d_{SD}(A, B) = |(A \setminus B) \cup (B \setminus A)|$$
- Chamfer matching distance
$$d_{CH}(A, B) = \sum_{a \in \partial A} d(a, \partial B)$$
- The Sum of minimal distances
$$d_{SMD}(A, B) = \frac{1}{2}\left(\sum_{a \in A} d(a, B) + \sum_{b \in B} d(b, A)\right)$$

Main properties of the set distances

- Hausdorff distance is a metric, however very sensitive to noise.
- Modified Hausdorff distance is not a metric, but it is less sensitive to noise.
- Metric by Symmetric Difference is a metric, but it is not sensitive to spatial displacements of non-overlapping sets.
- Chamfer Matching distance is not a metric. It can be sensitive to boundary noise and to an exchange of foreground and background.
- Sum of Minimal Distances is not a metric. Exhibits reasonably good properties.

The Chamfer matching distance does not distinguish between points of the object and points of the background. A: Reference image I_r, B: Observed image I_o, C: Observed image (Red) and Registered image (Green) superimposed.

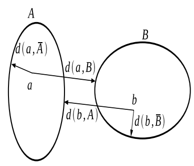
A B C

Complement Weighted Sum of Minimal Distances

Newly proposed set distance

Intention:

To assign a higher importance to the points of A deep inside the set than to points closer to the boundary of the set, and by that further improve The Sum of Minimal Distances.



$$d_{CW}(A, B) = \frac{1}{2}\left(\frac{\sum_{a \in A} d(a, B) d(a, \overline{A})}{\sum_{a \in A} d(a, \overline{A})} + \frac{\sum_{b \in B} d(b, A) d(b, \overline{B})}{\sum_{b \in B} d(b, \overline{B})}\right) .$$

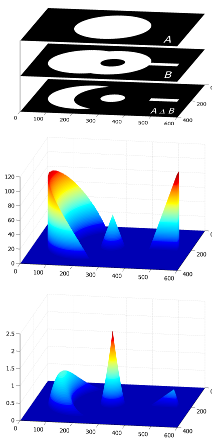
Properties:

non-negativity, separability and symmetry.

Complement Weighted Sum of Minimal Distances

Comparison of weighting performed - an illustration

Two binary shapes A and B and their symmetric difference $A \triangle B$.
Values assigned to individual points of sets A and B for d_{SMD} and for d_{CW}.



Set distances between crisp sets

An overview of the results

- Newly proposed set distance measure, CWSMD, is a semimetric, and is of a linear computational complexity.
- CWSMD is a weighted version of the Sum of Minimal Distances, SMD.
- An improved performance (regarding monotonicity under translation and rotation, and noise sensitivity), compared to SMD (and even more to other observed set distances) is evident, even if not dramatic.
- Applicability of CWSMD in image registration is confirmed on synthetic and real tasks.
- Applicability of CWSMD to the distance based handwritten characters recognition task is also shown to be high.

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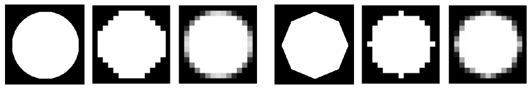
Set distances between fuzzy sets

Motivation - an illustration

A crisp discrete representation (B) of a disk (A) looks like a discrete representation of an octagon, while a crisp discrete representation (E) of an octagon (D) appears more similar to a discretized disk.

A fuzzy discrete representation of a disk (C) looks more like a disk, analogous observation can be made for (D) and (F).



(A) (B) (C) (D) (E) (F)

An appropriately defined distance measure should be able to utilize the information contained in fuzzy representations and, e.g., classify (C) and (F) correctly, even in the cases when the crisp measures fail at classifying (B) and (D).

Distances between fuzzy sets

Motivation

- Well defined and thoughtfully adjusted image processing tools can utilize, or pass to further processing steps, a great deal of information preserved in fuzzy representations.
- Extension of the existing set distances so that they can be applied to fuzzy sets, with meaningful interpretations and applications in image processing, is one of important tasks, considering range of applications of set distances.
- It is expected that distances between fuzzy representations of objects provide better discriminatory power than distances between crisp representations at the same spatial resolution.

Set distances between fuzzy sets

Extension principle based on α-cuts

A **"horizontal"** approach is a common fuzzification approach based on integration over $\alpha-$cuts.

A distance measure between two fuzzy sets $\mathcal{A}$ and $\mathcal{B}$ defined that way is

$$d^{\alpha}(\mathcal{A},\mathcal{B}) = \int_0^1 d({}^{\alpha}\!A, {}^{\alpha}\!B)\, d\alpha,$$

where d is some crisp set distance.

An important requirement of this approach:
The heights of the two observed fuzzy sets, $h(\mathcal{A})$ and $h(\mathcal{B})$, must be equal for a distance between the sets to be finite.

The **height** of a fuzzy set $\mathcal{S}$ is $\quad h(\mathcal{S}) = \max_{x\in X} \mu_S(x),$

Set distances between fuzzy sets

A "horizontal" approach

- The Hausdorff distance

$$d_H^{\alpha}(\mathcal{A},\mathcal{B}) = \int_0^1 d_H({}^{\alpha}\!A, {}^{\alpha}\!B)\, d\alpha\ .$$

- Sum of minimal distances

$$d_{SMD}^{\alpha}(\mathcal{A},\mathcal{B}) = \int_0^1 d_{SMD}({}^{\alpha}\!A, {}^{\alpha}\!B)\, d\alpha\ .$$

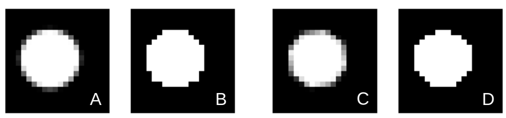
- Complement weighted sum of minimal distances

$$d_{CW}^{\alpha}(\mathcal{A},\mathcal{B}) = \int_0^1 d_{CW}({}^{\alpha}\!A, {}^{\alpha}\!B)\, d\alpha\ .$$

Set distances between fuzzy sets

Performance analysis - shape matching

- Classification performance on fuzzy and crisp discrete representations of disks and octagons at the same resolution is compared.
- Distances d_{CW} and d_{CW}^{α} are used, showing best performances regarding monotonicity. Distance minimization by greedy search is utilized for object alignment, as well as in the nearest neighbour rule based assignment of the object to a class.
- 5 disks and 5 octagons are used as template objects; 1000 disks and 1000 octagons are created as observed objects.

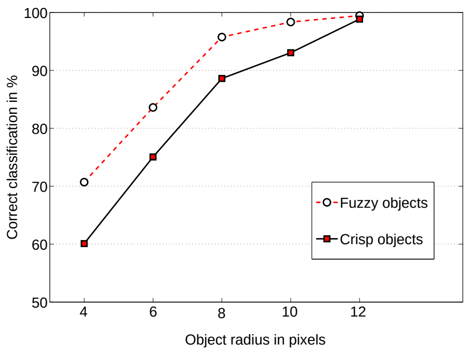


An example: Fuzzy representations of a disk (A) and an octagon (C), as well as the crisp representation of an octagon (D), are correctly classified, whereas the crisp disk (B) is incorrectly classified as an octagon.

Set distances between fuzzy sets

Performance analysis - Shape matching results

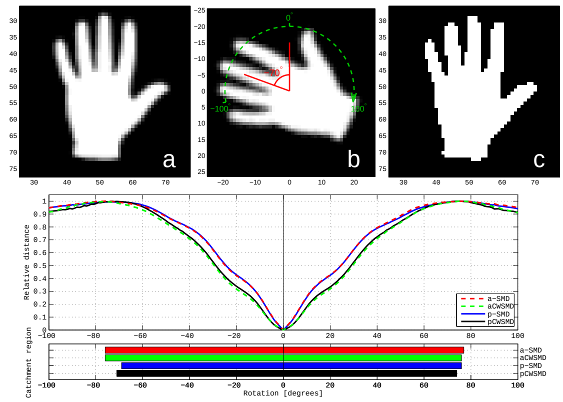
Correct classification ratios for distance based classification of objects of different radii, based on fuzzy and crisp discrete object representations:



Fuzzy set distances

Performance comparison - “crisp” vs. “fuzzy”

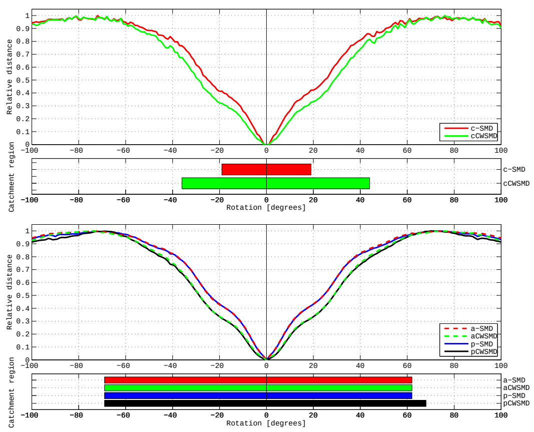
Top Relative distance between R and $\mathcal{T}(F)$ plotted against rotation angle for the set in (a) when bilinear interpolation is used. **Bottom** The catchment region of the different distance measures.



Fuzzy set distances

Performance comparison - crisp vs. fuzzy

Relative distance as a function of rotation angle for the *crisp* set in (c) when (top) nearest neighbour interpolation and (bottom) bilinear interpolation is used.



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Distances between fuzzy sets

Concluding remarks

- Fuzzy object representations and appropriately adjusted image analysis tools are, one more time, shown to provide image processing with improved performance.
- Most common fuzzification approach of α-cutting provides distance measures that perform very well in many situations and significantly outperform their corresponding crisp counterparts.
- An important observation is that, even in analysis performed on crisp objects, utilization of distance measures defined for fuzzy sets in intermediate steps and intermediate fuzzy object representations, lead to significant improvements in terms of precision. They, here as well, enable analysis at sub-pixel precision.

Set distances between fuzzy sets

Possible step further – When (and how) to cut?

- Distance between sets is, in crisp case, based on distance between a point and a set.
- Fuzzification can be performed earlier than at a “set distance level”.
- What about fuzzifying a point-to-set distance?
- What about point-to-point distance in a fuzzy set?

Set distances between fuzzy sets

Fuzzification of a point-to-set distance

Definitions of so far used set distances can be adjusted to fuzzy sets also as:

- Point-to-set based Hausdorff distance:

$$d_H^{ps}(\mathcal{A},\mathcal{B}) = \max\left(\sup_{a\in Supp(\mathcal{A})} d(a,\mathcal{B}), \sup_{b\in Supp(\mathcal{B})} d(b,\mathcal{A})\right);$$

- Point-to-set based Sum of Minimal Distances:

$$d_{SMD}^{ps}(\mathcal{A},\mathcal{B}) = \frac{1}{2}\left(\sum_{a\in Supp(\mathcal{A})} d(a,\mathcal{B}) + \sum_{b\in Supp(\mathcal{B})} d(b,\mathcal{A})\right);$$

- Point-to-set based Complement Weighted Sum of Minimal Distances:

$$d_{CW}^{ps}(\mathcal{A},\mathcal{B}) = \frac{1}{2}\left(\frac{\sum\limits_{a\in Supp(\mathcal{A})} d(a,\mathcal{B})\cdot d(a,\overline{\mathcal{A}})}{\sum\limits_{a\in Supp(\mathcal{A})} d(a,\overline{\mathcal{A}})} + \frac{\sum\limits_{b\in Supp(\mathcal{B})} d(b,\mathcal{A})\cdot d(b,\overline{\mathcal{B}})}{\sum\limits_{b\in Supp(\mathcal{B})} d(b,\overline{\mathcal{B}})}\right).$$

Distances between fuzzy sets – Future work

"Vertical approach" - Motivation

Main issue:

How to define $d(a,\mathcal{B})$, for a (fuzzy) point a and a fuzzy set $\mathcal{B}$.

- "Horizontal approach" is possible here too:

$$d(a,\mathcal{B}) = \int_0^1 d(a,{}^{\alpha}\mathcal{B})\,d\alpha = \int_0^1 \min_{b\in{}^{\alpha}\mathcal{B}} d(a,b)\,d\alpha.$$

- Intuitively unappealing property:
  Every α-cut is observed independently of the others; distance from a point to a set may follow different paths at different α-levels, depending on a shape of a membership function.

Idea:

To define a **path-based distance** between a point and a set
seems rather promising and will be addressed in our future work!