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GEOMETRICAL MOMENT AND SHAPE DESCRIPTORS

Tamara Kopanja [1] and Tibor Lukić [2]

Abstract. This paper gives a short introduction to the shape descriptors. Shape descriptors are widely used in image processing problems such as identifacation, classification or matching. They represent measurment for specific feature of object.

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1. Introduction

The topic of this article are shape descriptors and their application to binary images which we represent as binary matrices. There are different descriptors which can be classified in two groups: area and boundary based. The area based use information of whole object, while boundary based use just pixels at the edge of the object. In this paper we represent descriptors belonging to the both groups. Shape descriptors are useful for solving different image processing problems, for example recognition and classification.

One of the vast field of application of descriptors is tomography image reconstruction ([5]). In this case descriptors are used to involve a priori information about the solution of the reconstruction process. The main benefit of a priori information in the reconstruction process is the reduction of amount of radiation which is particulary important in human radiology, see [2, 3, 4].

2. Geometrical moment

We can understand shape descriptors as measures for some specific property of image. They are quite useful in recognition characteristics in images. To form descriptors we use geometrical moments $m_{p,q}$ of image $u : \Omega \to \mathbb{R}$, $\Omega \subseteq \mathbb{R}^2$.

Definition 2.1. For a given planar shape $u : \Omega \to \mathbb{R}$ geometrical moment $m_{p,q}$ of order $p + q$ is defined as

$$m_{p,q}(u) = \iint_\Omega u(x, y)x^p y^q dxdy. \tag{2.1}$$

[1]Department of Fundamental Sciences, Faculty of Technical Sciences, University of Novi Sad, e-mail: kopanja.v11.2019@.uns.ac.rs

[2]Department of Fundamental Sciences, Faculty of Technical Sciences, University of Novi Sad, e-mail: tibor@uns.ac.rs

For a digital image (discrete and bounded) $u : \Omega \to \mathbb{R}$, $\Omega \subseteq \mathbb{Z}^2$ the descrete geometric moment $m_{p,q}$ is defined as

$$m_{p,q}(u) = \sum_{i=1}^{n}\sum_{j=1}^{m} u(i,j) i^p j^q. \tag{2.2}$$

It is easy to see that $m_{0,0}(u)$ represents the area of an object, it is sum of all ones in the image, that is

$$A(u) = m_{0,0}(u) = \sum_{i=1}^{n}\sum_{j=1}^{m} u(i,j). \tag{2.3}$$

The centroid (center of gravity) of an image (shape) can be determined by geometrical moments.

Definition 2.2. For a given planar shape u its centroid is

$$(C_x(u), C_y(u)) = \left(\frac{m_{1,0}(u)}{m_{0,0}(u)}, \frac{m_{0,1}(u)}{m_{0,0}(u)}\right). \tag{2.4}$$

Since the geometric moment is not translation invariant, the central moment $\overline{m}_{p,q}(u)$ is introduced, which is translation invariant.

Definition 2.3. Central moment $\overline{m}_{p,q}$ for given image $u : \Omega \to \mathbb{R}$, where $\Omega \subseteq \mathbb{R}^2$ is

$$\overline{m}_{p,q}(u) = \iint_{\Omega} u(x,y)\,(x - c_x)^p\,(y - c_y)^q dxdy. \tag{2.5}$$

In descrete case, central moment $\overline{m}_{p,q}(u)$ is

$$\overline{m}_{p,q}(u) = \sum_{i=1}^{n}\sum_{j=1}^{m} u(i,j)(i - c_x)^p (j - c_y)^q. \tag{2.6}$$

However, since central moment is not scaling invariant, the normalised moment $\mu_{p,q}$ is introduced.

Definition 2.4. For a given image u normalised moment $\mu_{p,q}(u)$ is defined as

$$\mu_{p,q}(u) = \frac{\overline{m}_{p,q}(u)}{A(u)^{1+\frac{p+q}{2}}}. \tag{2.7}$$

where $A(u)$ is already mentioned area of shape (2.3).

Using normalised moments which are translation and scaling invariat, Hu proposed set of algebraic invariants[3], also well-known as Hu moment invariants.

[3] They were introduced in Hu's work "Visual Pattern Recognition by Moment Invariants", 1962.

Definition 2.5. Hu moment invariants are defined as following

$I_1 = \mu_{2,0} + \mu_{0,2}$

$I_2 = (\mu_{2,0} - \mu_{0,2})^2 + 4 \cdot (\mu_{1,1})^2$

$I_3 = (\mu_{3,0} - 3 \cdot \mu_{1,2})^2 + (3\mu_{2,1} - \mu_{0,3})^2$

$I_4 = (\mu_{3,0} + \mu_{1,2})^2 + (\mu_{2,1} + \mu_{0,3})^2$

$I_5 = (\mu_{3,0} - 3 \cdot \mu_{1,2}) \cdot (\mu_{3,0} + \mu_{1,2}) \cdot \big((\mu_{3,0} + \mu_{1,2})^2 - \ 3 \cdot (\mu_{2,1} + \mu_{0,3})^2\big) + (3 \cdot \mu_{2,1} - \mu_{0,3}) \cdot (\mu_{2,1} + \mu_{0,3}) \cdot \ \big(3 \cdot (\mu_{3,0} + \mu_{1,2})^2 - (\mu_{2,1} + \mu_{0,3})^2\big)$

$I_6 = (\mu_{2,0} - \mu_{0,2}) \cdot \big((\mu_{3,0} + \mu_{1,2})^2 - (\mu_{2,1} + \mu_{3,0})^2\big) + 4\mu_{1,1} \cdot (\mu_{3,0} + \mu_{1,2}) \cdot (\mu_{2,1} + \mu_{0,3})$

$I_7 = (3\mu_{2,1} - \mu_{0,3}) \cdot (\mu_{3,0} + \mu_{1,2}) \cdot \big((\mu_{3,0} + \mu_{1,2})^2 - \ -3(\mu_{2,1} + \mu 0,3)^2\big) + (\mu_{3,0} - 3\mu_{1,2}) \cdot \ \cdot (\mu_{2,1} + \mu_{0,3}) \cdot (\ 3(\mu_{3,0} + \mu_{1,2})^2 \ -(\mu_{2,1} + \mu_{0,3})^2\big)$

The important property of Hu moments is that there are rotation invatiant too.

3. Shape descriptors

In this section we represent some of the most important shape descriptors.

3.1. Shape convexity

Shape convexity is widely used shape descriptor. The definition of convexity follows.

Definition 3.1. A shape α is convex if and only if for each two points A and $B \in \alpha$ the line segment (AB) completely belongs to the shape α.

There are many algorithms for making convex hull for observed object. If object is stil convex then its convex hull is itself. For measurment how much some object is close to convex we use descriptor of convexity $conv(u)$.

Definition 3.2. Let $\tilde{u}$ is convex hull of given shape u. Measure of convexity is defined as

$$conv(u) = \frac{A(u)}{A(\tilde{u})} \tag{3.1}$$

where $A(u)$ is the area of a shape represented by the function u.

3.2. Perimeter of object

Earlier, we calculated the area of object, but it is also interesting and useful to know perimeter of object. Perimeter can be realized as a measure of the lenght of an object. Boundary is found using gradient which is direction of the fastest growth, finding pixels surrounded by zeros on one side and ones on the other.

Definition 3.3. For a given shape u its perimeter $P(u)$ is

$$P(u) = \sum_{i,j} \left(|u(i,j) - u(i,j+1)| + |u(i,j) - u(i+1,j)| \right). \tag{3.2}$$

3.3. Shape circularity

Shape circularity is well-known and useful shape desciptor. It gives us information about how much is object "similar" to circle. The circularity measure for shape is between 0 and 1. It's equal to 1 just for a circle.

Definition 3.4. The shape circularity of a given shape u is defined as

$$C(u) = \frac{1}{2\pi\left(\mu_{2,0}(u) + \mu_{0,2}(u)\right)} = \frac{1}{2\pi \cdot I_1(u)} \tag{3.3}$$

where $\mu_{p,q}(u)$ is normalised moment and $I_1(u)$ is first Hu's moment.

3.4. Shape compactness

The shape compactness is quite similar to circularity. One well-used method for definition is

$$C_{st}(u) = \frac{4\pi \cdot A(u)}{\left(P(u)\right)^2}$$

where $A(u)$ is area and $P(u)$ is perimeter of object u. This one is not geometrical moment based because perimeter does not use geometrical moment.

3.5. Shape orientation

Shape orientation of given image u which defines shape S is defined by the angle α which represents the slope of the axis of the second moment of inertia of the given shape. The axis of the least second moment of inertia is the line which minimizes the integral of the squared distances of the points to the line

$$I(\alpha, S, \rho) = \iint_S r^2(x, y, \alpha, \rho)dxdy \tag{3.4}$$

where $r(x, y, \alpha, \rho)$ is the closest distance from the point $(x, y) \in S$ to the line $X \cdot \sin\alpha - Y\cos\alpha = \rho$.

Considered integral reaches extreme value where its first derivative of a variable α is equal to 0. By simple transformation it implies that angle α which defines the orientation of u satisfies the equation:

$$\mathrm{tg}(2\alpha) = \frac{2\overline{m}_{1,1}(S)}{\overline{m}_{2,0}(S) - \overline{m}_{0,2}(S)}. \tag{3.5}$$

3.6. Shape elongation

When the maximum and minimum values of the integral 3.4 are quite distinct, then it is suitable consider their quotient. This quotient is defined as a shape elongation.

Definition 3.5. The shape elongation of a given shape u is defined as

$$E(u) = \frac{\overline{m}_{2,0}(u) + \overline{m}_{0,2}(u) + \sqrt{4 \cdot \left(\overline{m}_{1,1}(u)\right)^2 + \left(\overline{m}_{2,0}(u) - \overline{m}_{0,2}(u)\right)^2}}{\overline{m}_{2,0}(u) + \overline{m}_{0,2}(u) - \sqrt{4 \cdot \left(\overline{m}_{1,1}(u)\right)^2 + \left(\overline{m}_{2,0}(u) - \overline{m}_{0,2}(u)\right)^2}} \tag{3.6}$$

Actually, the denominator of this fraction is maximum value of integral $I(\alpha, u, \rho)$, and nominator of this fraction is minimal value of function $I(\alpha, u, \rho)$. Of course, these extreme values are reached for angles which sastisfied equation 3.5. The shape elongation reaches the minimal possible value of 1 for a circle, i.e. circle has the lowest possible elongation that we have expected.

4. Conclusion

Shape descriptors are useful tools in image processing tasks. They are applicable in different fields such as medicine, security, astronomy, biology, industry, etc. There are many different descriptors, but here we represented some most popular and useful descriptor which could be foundation for further work. We mentioned two different types of shape descriptors, area and boundary based one. Actually, there are descriptors which could not classified in any of these groups, these use the both approaches. For specific set of images it can combine existing descriptors or make some new which well describes those images.

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