SLOBODNI VEKTORI

1. Dati su vektori $\vec{a} = (4, -3, 1), \vec{b} = (1, 3, -1)$ i $\vec{c} = (-2, -4, 3)$. Odrediti:
   (a) intenzitet vektora $\vec{a} - \vec{b}$;
   (b) skalarni proizvod vektora $\vec{a}$ i $\vec{b}$;
   (c) ugao izmedju vektora $\vec{b}$ i $2\vec{b} + \vec{c}$;
   (d) projekciju vektora $\vec{a}$ na vektor $\vec{b}$;
   (e) vektorski proizvod vektora $\vec{a}$ i $\vec{c}$;
   (f) mešoviti proizvod vektora $\vec{a}, \vec{b}$ i $\vec{c}$.

2. Odrediti parametar α tako da za $\vec{a} = (1, 1, 1)$ i $\vec{b} = (0, 2, 0)$ vektori $\vec{p} = \alpha\vec{a} + 5\vec{b}$ i $\vec{q} = 3\vec{a} - \vec{b}$ budu: (a) paralelni; (b) ortogonalni.

3. Dati su vektori $\vec{a} = (2k - 1, 2, k + 2), \vec{b} = (3, k - 1, -1)$ i $\vec{c} = (p, 1, 3)$, gde je $k \in \mathbb{R}$ i $p \in \mathbb{R}^-$.
   (a) Odrediti vrednost parametara k i p tako da važi $\vec{a} \perp \vec{b}$ i $|\vec{c}| = \sqrt{26}$.
   (b) Pokazati da su vektori $\vec{a}$, $\vec{b}$ i $\vec{c}$ koplanarni, a zatim izraziti vektor $\vec{a}$ kao linearnu kombinaciju vektora $\vec{b}$ i $\vec{c}$.

4. Za koje vrednosti realnog parametra a su vektori $\vec{x} = (a, 1 - a, a), \vec{y} = (2a, 2a - 1, a + 2)$ i $\vec{z} = (-2a, a, -a)$ koplanarni?

5. Neka je $\vec{p} = \alpha\vec{a} + 2\vec{b}$ i $\vec{q} = 5\vec{a} - 4\vec{b}$, neka je $\vec{p} \perp \vec{q}$ i neka je $|\vec{a}| = |\vec{b}| = 1$.
   (a) Naći α ako se zna da je $\vec{a} \perp \vec{b}$.
   (b) Za $\alpha = 1$ naći $\angle(\vec{a}, \vec{b})$ i odrediti $|\vec{p}|$.

6. Za koju vrednost koeficijenta α će vektori $\vec{p} = 3\vec{a} - \alpha\vec{b}$ i $\vec{q} = \vec{a} + 4\vec{b}$ biti kolinearni, ako vektori $\vec{a}$ i $\vec{b}$ nisu kolinearni?

7. Dati su vektori $\vec{a} = \vec{m} - 2\vec{n}$ i $\vec{b} = 2\vec{m} + \vec{n}$, gde je $|\vec{m}| = 2$, $|\vec{n}| = 3$ i $\angle(\vec{m}, \vec{n}) = \frac{\pi}{3}$.
   (a) Odrediti projekciju vektora $\vec{b}$ na vektor $\vec{a}$.
   (b) Izračunati površinu trougla odredjenog vektorima $\vec{a}$ i $\vec{b}$.

8. Date su tačke $A(1, 0, 1), B(1, -1, 0), C(3, -2, 1)$. Odrediti tačku D tako da $ABCD$ bude paralelogram, a zatim izračunati ugao koji obrazuju dijagonale tog paralelograma.

9. Izračunati zapreminu i visinu paralelopipeda konstruisanog nad vektorima $\vec{a} = (1, 0, -2)$, $\vec{b} = (0, 1, -2)$ i $\vec{c} = (-1, 3, 5)$ ako je njegova osnova odredjena vektorima $\vec{a}$ i $\vec{b}$.

10. Izračunati površinu trougla ABC i zapreminu tetraedra $ABCD$ ako su $A(2, -3, 4), B(1, 2, -1)$, $C(3, -2, 1)$ i $D(3, 0, 5)$.

11. Neka su $\vec{m}, \vec{n}$ i $\vec{p}$ nekoplanarni vektori takvi da je $|\vec{m}| = 2, |\vec{n}| = 1, |\vec{p}| = 3$ i $\angle(\vec{m}, \vec{n}) = \frac{\pi}{4}$, $\angle(\vec{m}, \vec{p}) = \angle(\vec{n}, \vec{p}) = \frac{\pi}{2}$. Izračunati zapreminu paralelopipeda konstruisanog nad vektorima $\vec{m}, \vec{n}$ i $\vec{p}$.

12. (a) Dokazati da su nenula vektori $\vec{a}$, $\vec{b}$ i $(\vec{a} \times \vec{b}) \times \vec{c}$ koplanarni.
    (b) Ako važi $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0}$, pokazati da su nenula vektori $\vec{a}, \vec{b}$ i $\vec{c}$ koplanarni.

13. Data je jedinična kocka $ABCDA_1B_1C_1D_1$. Ako je tačka M središte strane $A_1B_1C_1D_1$, a tačka N središte strane BCC_1B_1, izračunati površinu trougla AMN.

Rešenja:

1. $\vec{a} = (4,-3,1)$
$\vec{b} = (1,3,-1)$
$\vec{c} = (-2,-4,3)$

(a) $\vec{a}-\vec{b} = (4,-3,1)-(1,3,-1) = (3,-6,2)$

$|\vec{a}-\vec{b}| = \sqrt{3^2+(-6)^2+2^2} = \sqrt{9+36+4} = \sqrt{49} = 7$

(b) $\vec{a}\cdot\vec{b} = (4,-3,1)\cdot(1,3,-1) = 4\cdot 1+(-3)\cdot 3+1\cdot(-1) = -6$

(c) $2\vec{b}+\vec{c} = (2,6,-2)+(-2,-4,3) = (0,2,1)$

$$\cos\sphericalangle(\vec{b},2\vec{b}+\vec{c}) = \frac{\vec{b}\cdot(2\vec{b}+\vec{c})}{|\vec{b}|\cdot|2\vec{b}+\vec{c}|} = \frac{(1,3,-1)\cdot(0,2,1)}{\sqrt{1^2+3^2+(-1)^2}\cdot\sqrt{0^2+2^2+1}} = \frac{0+6-1}{\sqrt{11}\cdot\sqrt{5}} = \sqrt{\frac{5}{11}}$$

$$\Rightarrow \sphericalangle(\vec{b},2\vec{b}+\vec{c}) = \arccos\sqrt{\frac{5}{11}}$$

(d) $$pr_{\vec{b}}\,\vec{a} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{-6}{\sqrt{11}}$$

(e) $$\vec{a}\times\vec{c} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -3 & 1 \\ -2 & -4 & 3 \end{vmatrix} = \vec{i}\begin{vmatrix} -3 & 1 \\ -4 & 3 \end{vmatrix} - \vec{j}\begin{vmatrix} 4 & 1 \\ -2 & 3 \end{vmatrix} + \vec{k}\begin{vmatrix} 4 & -3 \\ -2 & -4 \end{vmatrix} = -5\vec{i}-14\vec{j}-22\vec{k}$$
$$= (-5,-14,-22)$$

(f) $$(\vec{a}\times\vec{b})\cdot\vec{c} = \begin{vmatrix} 4 & -3 & 1 \\ 1 & 3 & -1 \\ -2 & -4 & 3 \end{vmatrix} 2+ = \begin{vmatrix} 4 & -3 & 1 \\ 5 & 0 & 0 \\ -2 & -4 & 3 \end{vmatrix} = -5\begin{vmatrix} -3 & 1 \\ -4 & 3 \end{vmatrix} = -5(-9+4) = 25$$

2. $\vec{p} = \alpha\vec{a}+5\vec{b} = \alpha\cdot(1,1,1)+5(0,2,0) = (\alpha,\alpha+10,\alpha)$

$\vec{q} = 3\vec{a}-\vec{b} = 3\cdot(1,1,1)-(0,2,0) = (3,1,3)$

(a) $\vec{p}\parallel\vec{q} \Leftrightarrow \vec{p}\times\vec{q} = 0$

$$\Leftrightarrow \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \alpha & \alpha+10 & \alpha \\ 3 & 1 & 3 \end{vmatrix} = \vec{i}\begin{vmatrix} \alpha+10 & \alpha \\ 1 & 3 \end{vmatrix} - \vec{j}\begin{vmatrix} \alpha & \alpha \\ 3 & 3 \end{vmatrix} + \vec{k}\begin{vmatrix} \alpha & \alpha+10 \\ 3 & 1 \end{vmatrix}$$
$$= \vec{i}(3\alpha+30-\alpha) - \vec{j}(3\alpha-3\alpha) + \vec{k}(\alpha-3\alpha-30)$$
$$= 2(\alpha+15)\vec{i} - 2(\alpha+15)\vec{k} = \vec{0}$$

$\Leftrightarrow \boxed{\alpha = -15}$

(b) $\vec{p}\perp\vec{q} \Leftrightarrow \vec{p}\cdot\vec{q} = 0$

$\Leftrightarrow (\alpha,\alpha+10,\alpha)\cdot(3,1,3) = 0$

$\Leftrightarrow 3\alpha+\alpha+10+3\alpha = 0$

$\Leftrightarrow 7\alpha+10 = 0$

$\Leftrightarrow \boxed{\alpha = -\frac{10}{7}}$

(3) $\vec{a} = (2k-1,\ 2,\ k+2)$

$\vec{b} = (3,\ k-1,\ -1)$

$\vec{c} = (p,\ 1,\ 3)$

$k \in \mathbb{R}$, $p \in \mathbb{R}^-$

(a) $\vec{a} \perp \vec{b} \Leftrightarrow \vec{a} \cdot \vec{b} = 0$

$\Leftrightarrow (2k-1, 2, k+2)(3, k-1, -1) = 0$

$\Leftrightarrow 6k-3+2k-2-k-2 = 0$

$\Leftrightarrow 7k-7 = 0$

$\Leftrightarrow \boxed{k=1} \rightarrow \vec{a} = (1,2,3)$, $\vec{b} = (3,0,-1)$

$|\vec{c}| = \sqrt{26} \Leftrightarrow \sqrt{p^2+1^2+3^2} = \sqrt{26}\ /^2$

$\Leftrightarrow p^2 + 10 = 26$

$\Leftrightarrow p^2 = 16$

$\overset{p\in\mathbb{R}^-}{\Leftrightarrow} p = -4 \rightarrow \vec{c} = (-4,1,3)$

(b) Vektori $\vec{a}, \vec{b}, \vec{c}$ su koplanarni akko $(\vec{a} \times \vec{b}) \cdot \vec{c} = 0$

$$(\vec{a}\times\vec{b})\cdot\vec{c} = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 0 & -1 \\ -4 & 1 & 3 \end{vmatrix} \cdot(-2) = \begin{vmatrix} 9 & 0 & -3 \\ 3 & 0 & -1 \\ -4 & 1 & 3 \end{vmatrix} = 0$$

$\beta \vec{b} + \gamma \vec{c} = \vec{a}$

$\beta(3,0,-1) + \gamma(-4,1,3) = (1,2,3)$

$3\beta - 4\gamma = 1$

$\gamma = 2$

$-\beta + 3\gamma = 3 \Rightarrow \beta = 3$

$\Rightarrow \boxed{\vec{a} = 3\vec{b} + 2\vec{c}}$

(4) $\vec{x} = (a,\ 1-a,\ a)$

$\vec{y} = (2a,\ 2a-1,\ a+2)$

$\vec{z} = (-2a,\ a,\ -a)$

$\vec{x}, \vec{y}, \vec{z}$ su koplanarni akko $(\vec{x} \times \vec{y}) \cdot \vec{z} = 0$

$$(\vec{x}\times\vec{y})\cdot\vec{z} = \begin{vmatrix} a & 1-a & a \\ 2a & 2a-1 & a+2 \\ -2a & a & -a \end{vmatrix} = a \begin{vmatrix} 1 & 1-a & a \\ 2 & 2a-1 & a+2 \\ -2 & a & -a \end{vmatrix} \begin{matrix} \cdot(-2)\ /\cdot 2 \\ + \\ + \end{matrix} = a \begin{vmatrix} 1 & 1-a & a \\ 0 & 4a-3 & 2-a \\ 0 & 2-a & a \end{vmatrix}$$

$$= a \begin{vmatrix} 4a-3 & 2-a \\ 2-a & a \end{vmatrix} = a\left(a(4a-3) - (2-a)^2\right)$$

$$= a(4a^2 - 3a - 4 + 4a - a^2) = a(3a^2 + a - 4)$$

$(\vec{x}\times\vec{y})\cdot\vec{z} = 0 \Leftrightarrow a = 0 \lor 3a^2 + a - 4 = 0$

$a_{1,2} = \dfrac{-1 \pm \sqrt{1+48}}{6} = \dfrac{-1 \pm 7}{6}$

$a_1 = 1$, $a_2 = -\dfrac{4}{3}$

$\Leftrightarrow a \in \{0, 1, -\frac{4}{3}\}$

(5) $\vec{p} = \alpha\vec{a} + 2\vec{b}$
$\vec{q} = 5\vec{a} - 4\vec{b}$
$\vec{p} \perp \vec{q}$
$|\vec{a}| = |\vec{b}| = 1$

(a) $\vec{p} \perp \vec{q} \Leftrightarrow \vec{p}\cdot\vec{q} = 0$

$\Leftrightarrow (\alpha\vec{a} + 2\vec{b})\cdot(5\vec{a} - 4\vec{b}) = 0$

$\Leftrightarrow 5\alpha\,\vec{a}\cdot\vec{a} - 4\alpha\underbrace{\vec{a}\cdot\vec{b}}_{=0} + 10\underbrace{\vec{b}\cdot\vec{a}}_{=0} - 8\,\vec{b}\cdot\vec{b} = 0$ $\quad \vec{a}\perp\vec{b}$

$\Rightarrow 5\alpha|\vec{a}|^2 - 8|\vec{b}|^2 = 0$

$\Leftrightarrow 5\alpha - 8 = 0 \Leftrightarrow \boxed{\alpha = \frac{8}{5}}$

(b) $\alpha = 1 \rightarrow \vec{p} = \vec{a} + 2\vec{b}$

$\vec{p} \perp \vec{q} \Rightarrow \vec{p}\cdot\vec{q} = 0$

$\Rightarrow (\vec{a} + 2\vec{b})\cdot(5\vec{a} - 4\vec{b}) = 5|\vec{a}|^2 + 6\vec{a}\cdot\vec{b} - 8|\vec{b}|^2$

$= 5 + 6\vec{a}\cdot\vec{b} - 8 = 6\vec{a}\cdot\vec{b} - 3 = 0$

$\Rightarrow \vec{a}\cdot\vec{b} = \frac{1}{2}$

$\cos\sphericalangle(\vec{a},\vec{b}) = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \frac{\frac{1}{2}}{1\cdot 1} = \frac{1}{2} \Rightarrow \boxed{\sphericalangle(\vec{a},\vec{b}) = \frac{\pi}{3}}$

$|\vec{p}|^2 = \vec{p}\cdot\vec{p} = (\vec{a} + 2\vec{b})\cdot(\vec{a} + 2\vec{b}) = |\vec{a}|^2 + 4\vec{a}\cdot\vec{b} + 4|\vec{b}|^2 = 1^2 + 4\cdot\frac{1}{2} + 4\cdot 1^2 = 7$

$\Rightarrow \boxed{|\vec{p}| = \sqrt{7}}$

(6) $\vec{p} = 3\vec{a} - \alpha\vec{b}$
$\vec{q} = \vec{a} + 4\vec{b}$

$\vec{a} \nparallel \vec{b} \Leftrightarrow \vec{a}\times\vec{b} \neq 0$

$\vec{p} \parallel \vec{q} \Rightarrow \vec{p}\times\vec{q} = 0$

$\Rightarrow (3\vec{a} - \alpha\vec{b})\times(\vec{a} + 4\vec{b}) = 3\vec{a}\times\vec{a} + 12\vec{a}\times\vec{b} - \alpha\vec{b}\times\vec{a} - 4\alpha\vec{b}\times\vec{b}$

$= 12\vec{a}\times\vec{b} + \alpha\vec{a}\times\vec{b}$

$= (12 + \alpha)\vec{a}\times\vec{b} = 0$

$\Rightarrow 12 + \alpha = 0$

$\Rightarrow \boxed{\alpha = -12}$

(7) $\vec{a} = \vec{m} - 2\vec{n}$
$\vec{b} = 2\vec{m} + \vec{n}$

$\left.\begin{array}{l}|\vec{m}| = 2,\ |\vec{n}| = 3 \\ \sphericalangle(\vec{m},\vec{n}) = \frac{\pi}{3}\end{array}\right\} \Rightarrow \vec{m}\cdot\vec{n} = |\vec{m}|\cdot|\vec{n}|\cos\sphericalangle(\vec{m},\vec{n})$

$= 2\cdot 3\cdot\frac{1}{2} = 3$

(a) $pr_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$

$\vec{a}\cdot\vec{b} = (\vec{m} - 2\vec{n})\cdot(2\vec{m} + \vec{n}) = 2\vec{m}\cdot\vec{m} + \vec{m}\cdot\vec{n} - 4\vec{n}\cdot\vec{m} - 2\vec{n}\cdot\vec{n}$

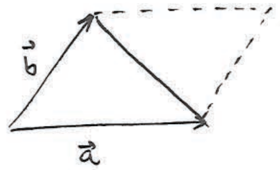
$= 2|\vec{m}|^2 - 3\vec{m}\cdot\vec{n} - 2\cdot|\vec{n}|^2 = 2\cdot 2^2 - 3\cdot 3 - 2\cdot 3^2 = -19$

$|\vec{a}|^2 = \vec{a}\cdot\vec{a} = (\vec{m} - 2\vec{n})\cdot(\vec{m} - 2\vec{n}) = |\vec{m}|^2 - 4\vec{m}\cdot\vec{n} + 4|\vec{n}|^2$

$= 2^2 - 4\cdot 3 + 4\cdot 3^2 = 28 \Rightarrow |\vec{a}| = \sqrt{28}$

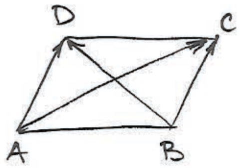
$pr_{\vec{a}}\vec{b} = \frac{-19}{\sqrt{28}}$

(b)



$$P_\Delta = \frac{1}{2}|\vec{a}\times\vec{b}| = \frac{1}{2}|(\vec{m}-2\vec{n})\times(2\vec{m}+\vec{n})|$$
$$= \frac{1}{2}|2\vec{m}\times\vec{m} + \vec{m}\times\vec{n} - 4\vec{n}\times\vec{m} - 2\vec{n}\times\vec{n}|$$
$$= \frac{1}{2}|5\vec{m}\times\vec{n}| = \frac{5}{2}|\vec{m}|\cdot|\vec{n}|\cdot\sin\angle(\vec{m},\vec{n})$$
$$= \frac{5}{\cancel{2}}\cdot\cancel{2}\cdot 3\cdot\sin\frac{\pi}{3} = 15\cdot\frac{\sqrt{3}}{2}$$
$$\boxed{P_\Delta = \frac{15\sqrt{3}}{2}}$$

(8)



A(1, 0, 1)
B(1, -1, 0)
C(3, -2, 1)

$$\vec{AD} = \vec{BC}$$
$$(x-1,\ y-0,\ z-1) = (3-1,\ -2+1,\ 1-0)$$
$x-1 = 2 \Rightarrow x = 3$
$y = -1$
$z-1 = 1 \Rightarrow z = 2$
D(3, -1, 2)

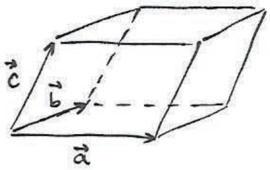
$$\vec{AC} = (2, -2, 0)$$
$$\vec{BD} = (2, 0, 2)$$
$$|\vec{AC}| = \sqrt{2^2+(-2)^2} = 2\sqrt{2}$$
$$|\vec{BD}| = \sqrt{2^2+2^2} = 2\sqrt{2}$$

$$\cos\angle(\vec{AC},\vec{BD}) = \frac{\vec{AC}\cdot\vec{BD}}{|\vec{AC}|\cdot|\vec{BD}|} = \frac{(2,-2,0)\cdot(2,0,2)}{2\sqrt{2}\cdot 2\sqrt{2}} = \frac{4+0+0}{8} = \frac{1}{2}$$

$$\Rightarrow \angle(\vec{AC},\vec{BD}) = \frac{\pi}{3}$$

(9)

$\vec{a} = (1, 0, -2)$
$\vec{b} = (0, 1, -2)$
$\vec{c} = (-1, 3, 5)$

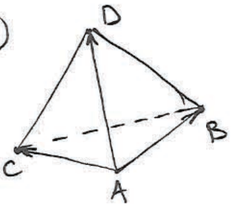


$$V = |(\vec{a}\times\vec{b})\cdot\vec{c}|$$
$$V = B\cdot H \Rightarrow H = \frac{V}{B}$$
$$B = |\vec{a}\times\vec{b}|$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -2 \\ 0 & 1 & -2 \end{vmatrix} = \vec{i}\begin{vmatrix} 0 & -2 \\ 1 & -2 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & -2 \\ 0 & -2 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 2\vec{i} + 2\vec{j} + \vec{k} = (2, 2, 1)$$

$$\left.\begin{aligned} &B = \sqrt{2^2+2^2+1^2} = \sqrt{9} = 3 \\ &(\vec{a}\times\vec{b})\cdot\vec{c} = (2,2,1)\cdot(-1,3,5) = -2+6+5 = 9 \Rightarrow \boxed{V = 9} \end{aligned}\right\} \Rightarrow \boxed{H = 3}$$

(10)



A(2, -3, 4)
B(1, 2, -1)
C(3, -2, 1)
D(3, 0, 5)

$$P_{ABC} = \frac{1}{2}|\vec{AB}\times\vec{AC}|$$
$$V_{ABCD} = \frac{1}{6}|(\vec{AB}\times\vec{AC})\cdot\vec{AD}|$$

$$\vec{AB} = (-1, 5, -5)$$
$$\vec{AC} = (1, 1, -3)$$
$$\vec{AD} = (1, 3, 1)$$

$$\vec{AB}\times\vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 5 & -5 \\ 1 & 1 & -3 \end{vmatrix} = \vec{i}\begin{vmatrix} 5 & -5 \\ 1 & -3 \end{vmatrix} - \vec{j}\begin{vmatrix} -1 & -5 \\ 1 & -3 \end{vmatrix} + \vec{k}\begin{vmatrix} -1 & 5 \\ 1 & 1 \end{vmatrix} = -10\vec{i} - 8\vec{j} - 6\vec{k}$$

$$(\vec{AB}\times\vec{AC})\cdot\vec{AD} = (-10,-8,-6)\cdot(1,3,1) = -10-24-6 = -40$$

$$P_{ABC} = \frac{1}{2}\sqrt{(-10)^2+(-8)^2+(-6)^2} = \frac{1}{2}\sqrt{100+64+36} = \frac{1}{2}\cdot 10\sqrt{2} \Rightarrow \boxed{P_{ABC} = 5\sqrt{2}}$$

$$V_{ABCD} = \frac{1}{6}\cdot|-40| \Rightarrow \boxed{V_{ABCD} = \frac{20}{3}}$$

(11) $\vec{m}, \vec{n}, \vec{p}$ - nekoplanarni vektori

$|\vec{m}|=2,\ |\vec{n}|=1,\ |\vec{p}|=3$

$\measuredangle(\vec{m},\vec{n})=\frac{\pi}{4},\ \measuredangle(\vec{m},\vec{p})=\measuredangle(\vec{n},\vec{p})=\frac{\pi}{2}$

$$\left.\begin{array}{l} \vec{m}\times\vec{n}\perp\vec{m},\ \vec{m}\times\vec{n}\perp\vec{n} \\ \vec{p}\perp\vec{m},\ \vec{p}\perp\vec{n} \end{array}\right\} \Rightarrow \vec{m}\times\vec{n} \parallel \vec{p}$$

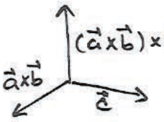
$$\Rightarrow \cos\measuredangle(\vec{m}\times\vec{n},\vec{p}) \in \{0,\pi\}$$

$$\Rightarrow |\cos\measuredangle(\vec{m}\times\vec{n},\vec{p})| = 1$$

$$V = |(\vec{m}\times\vec{n})\cdot\vec{p}| = |\vec{m}\times\vec{n}|\cdot|\vec{p}|\cdot|\cos\measuredangle(\vec{m}\times\vec{n}\ \ \vec{p})| = |\vec{m}|\cdot|\vec{n}|\cdot\sin\measuredangle(\vec{m},\vec{n})\cdot|\vec{p}|\cdot 1$$

$$= 2\cdot 1\cdot \sin\frac{\pi}{4}\cdot 3 = 6\cdot\frac{\sqrt{2}}{2} = \underline{3\sqrt{2}}$$

(12) (a)



$(\vec{a}\times\vec{b})\times\vec{c} \perp \vec{a}\times\vec{b} \Rightarrow (\underline{\vec{a}}\times\underline{\vec{b}})\cdot(\underline{(\vec{a}\times\vec{b})\times\vec{c}}) = 0$

Mešoviti proizvod vektora $\vec{a}, \vec{b}$ i $(\vec{a}\times\vec{b})\times\vec{c}$ jednak je 0, pa su oni koplanarni.

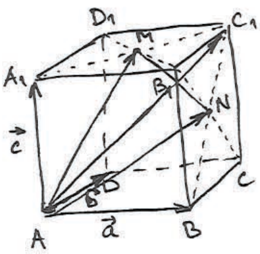
(b) $\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a} = \vec{0} \quad /\cdot\vec{c}$

$$(\vec{a}\times\vec{b})\cdot\vec{c} + \underbrace{(\vec{b}\times\vec{c})\cdot\vec{c}}_{\substack{=0 \\ \vec{b}\times\vec{c}\perp\vec{c}}} + \underbrace{(\vec{c}\times\vec{a})\cdot\vec{c}}_{\substack{=0 \\ \vec{c}\times\vec{a}\perp\vec{c}}} = \underbrace{\vec{0}\cdot\vec{c}}_{=0} \Rightarrow (\vec{a}\times\vec{b})\cdot\vec{c} = 0$$

$\Updownarrow$

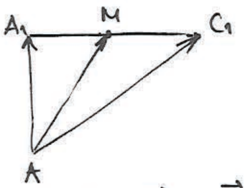
vektori $\vec{a}, \vec{b}$ i $\vec{c}$ su koplanarni

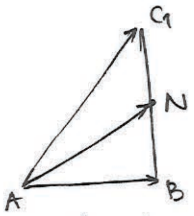
(13)



$P_{AMN} = \frac{1}{2}|\vec{AM}\times\vec{AN}|$

$\vec{AC_1} = \vec{a}+\vec{b}+\vec{c}$





$\vec{a}\times\vec{b}=\vec{c}$

$\vec{b}\times\vec{c}=\vec{a}$

$\vec{c}\times\vec{a}=\vec{b}$

$$\vec{AM} = \frac{1}{2}(\vec{AA_1}+\vec{AC_1}) = \frac{1}{2}(\vec{a}+\vec{b}+2\vec{c})$$

$$\vec{AN} = \frac{1}{2}(\vec{AB}+\vec{AC_1}) = \frac{1}{2}(2\vec{a}+\vec{b}+\vec{c})$$

$$\vec{AM}\times\vec{AN} = \frac{1}{4}(\vec{a}+\vec{b}+2\vec{c})\times(2\vec{a}+\vec{b}+\vec{c})$$

$$= \frac{1}{4}(2\vec{a}\times\vec{a} + \vec{a}\times\vec{b} + \vec{a}\times\vec{c} + 2\vec{b}\times\vec{a} + \vec{b}\times\vec{b} + \vec{b}\times\vec{c} + 4\vec{c}\times\vec{a} + 2\vec{c}\times\vec{b} + 2\vec{c}\times\vec{c})$$

$$= \frac{1}{4}(-\vec{a}\times\vec{b} - \vec{b}\times\vec{c} + 3\vec{c}\times\vec{a})$$

$$= \frac{1}{4}(-\vec{a}+3\vec{b}-\vec{c})$$

$$|\vec{AM}\times\vec{AN}| = \frac{1}{4}|-\vec{a}+3\vec{b}-\vec{c}|$$

$$|-\vec{a}+3\vec{b}-\vec{c}|^2 = (-\vec{a}+3\vec{b}-\vec{c})\cdot(-\vec{a}+3\vec{b}-\vec{c})$$

$$= |\vec{a}|^2 + 9|\vec{b}|^2 + |\vec{c}|^2 - 6\underbrace{\vec{a}\cdot\vec{b}}_{=0} + 2\underbrace{\vec{a}\cdot\vec{c}}_{=0} - 6\underbrace{\vec{b}\cdot\vec{c}}_{=0}$$

$$= 1^2 + 9\cdot 1^2 + 1^2 = 11$$

$$P_{AMN} = \frac{1}{2}\cdot\frac{1}{4}\cdot\sqrt{11} \Rightarrow \boxed{P_{AMN} = \frac{\sqrt{11}}{8}}$$