ANALITIČKA GEOMETRIJA

1. Ispitati medjusobni položaj pravih p i q. Ako se seku, naći tačku preseka.
   (a) $p : \vec{r} = (2t, 1+t, 6t),\ t \in \mathbb{R}, \quad q : \vec{r} = (2+4s, 2+2s, 6+12s),\ s \in \mathbb{R}$
   (b) $p : \frac{x}{2} = \frac{y-1}{1} = \frac{z}{6}, \quad q : \frac{x-2}{-4} = \frac{y-3}{-2} = \frac{z+15}{-12}$
   (c) $p : \frac{x-2}{1} = \frac{y-2}{3} = \frac{z-3}{1}, \quad q : \frac{x-2}{1} = \frac{y-3}{4} = \frac{z-4}{2}$
   (d) $p : \vec{r} = (3+4t, 3+t, -1-t),\ t \in \mathbb{R}, \quad q : \vec{r} = (2s, 0, -2+s),\ s \in \mathbb{R}$

2. Ispitati medjusobni položaj ravni $\alpha : 2x - y + z - 6 = 0$ i prave p, ako je
   (a) $p : \frac{x-1}{-1} = \frac{y+1}{-1} = \frac{z-4}{1}$
   (b) $p : \vec{r} = (0,0,6) + (1,1,-1)t,\ t \in \mathbb{R}$
   (c) $p : \vec{r} = (2t, 1+3t, -1+t),\ t \in \mathbb{R}$
   (d) $p : \frac{x+2}{-2} = \frac{y-1}{1} = \frac{z+1}{-1}$

3. Date su tačke $A(0,6,-2), B(3,-3,4)$ i $P(1,-1,1)$.
   (a) Napisati jednačinu prave p koja sadrži tačke A i B.
   (b) Napisati jednačinu ravni α koja sadrži tačku P i normalna je na pravu p.
   (c) Naći tačku P_1 koja je presek prave p i ravni α.
   (d) Izračunati dužinu duži PP_1.

4. Naći jednačinu presečne prave p ravni $\alpha : 4x - y + 3z - 1 = 0$ i $\beta : 2x + y + 6z + 1 = 0$.

5. Za koju vrednost parametra $a \in \mathbb{R}$ će prava $p : \frac{x-1}{-2} = \frac{y}{a} = \frac{z+1}{1}$ biti paralelna ravni α odredjenoj tačkama $A(1,0,1), B(1,-1,0), C(3,-2,1)$?

6. Date su prave $p : \frac{x}{1} = \frac{y-2}{-1} = \frac{z+1}{0}$, $q : \frac{x+2}{3} = \frac{y-2}{-1} = \frac{z}{-1}$ i $r : \frac{x+1}{-4} = \frac{y-1}{0} = \frac{z-3}{5}$.
   (a) Naći jednačinu ravni α odredjene pravama p i q.
   (b) Odrediti projekciju prave r na ravan α.
   (c) Izračunati ugao koji prava r zaklapa sa ravni α.

7. Date su ravni $\alpha : x + y - 2z - 1 = 0$, $\beta : 2x - y + 3z + 4 = 0$ i $\gamma : -x + 2y + z - 5 = 0$.
   (a) Naći zajedničku tačku P ravni α, β i γ.
   (b) Odrediti tačku R simetričnu tački $Q(2,-1,3)$ u odnosu na ravan α.

8. Date su prave $p : \vec{r} = (0,1,-2) + (2,a,1)t,\ t \in \mathbb{R}$ i $q : \vec{r} = (-1,3,-2) + (b,2,-2)s,\ s \in \mathbb{R}$.
   (a) Odrediti vrednosti realnih parametara a i b tako da prave p i q budu paralelne.
   (b) Naći rastojanje pravih p i q.
   (c) Odrediti jednačinu ravni α odredjene pravama p i q.

9. Kroz tačku $Q(2,-3,0)$ postaviti pravu q koja je paralelna ravni $\alpha : 2x + y - z + 5 = 0$ i koja seče pravu $p : \vec{r} = (t, -4+2t, 5-t),\ t \in \mathbb{R}$.

10. Date su tačka $A(1,2,3)$ i prava $a : \vec{r} = (t, 2-t, -1),\ t \in \mathbb{R}$.
    (a) Odrediti jednačinu ravni α koja sadrži tačku A i pravu a.
    (b) Odrediti jednačinu ravni β koja sadrži pravu a i normalna je na ravan α.

11. Tačka $A(0,0,-5)$ je jedno teme pravougaonika, dok se preostala temena nalaze na pravama $p : \frac{x-3}{2} = \frac{y+1}{1} = \frac{z-2}{1}$ i $q : \frac{x}{1} = \frac{y+5}{3} = \frac{z-6}{-5}$. Odrediti temena B, C i D traženog pravougaonika.

12. Date su tačka $A(1,2,3)$, prave $p : \frac{x}{2} = \frac{y+1}{3} = \frac{z-2}{-1}$ i $q : \frac{x+3}{-5} = \frac{y}{-2} = \frac{z-4}{3}$, i ravan $\alpha : 2x - y - 3z = 5$. Ako je tačka B presek pravih p i q, a tačka C projekcija tačke A na ravan α, odrediti tačku D tako da $ABCD$ bude paralelogram i izračunati ugao izmedju dijagonala tog paralelograma.

13. Napisati parametrizaciju vektora od A do B, ako je $A(2,-1,5)$ i $B(4,2,1)$.

14. Skicirati geometrijsko mesto tačaka $r(t) = (-1+3t, 2+t, -2t),\ t \in [-1,2]$.

15. Date su tačke A, B i prava $\vec{r} = \vec{r}_A + t\overrightarrow{AB}$. Skicirati datu pravu i podebljati tačke za koje je $-\frac{1}{2} \le t \le \frac{4}{3}$.

16. Date su tačke $A(1,2,5), B(-1,0,3)$ i $C(1,1,2)$.
    (a) Odrediti parametarske jednačine ravni odredjene tačkama A, B i C.
    (b) Napisati parametrizaciju paralelograma odredjenog tačkama A, B i C.

17. Ako je ravan data parametarskim jednačinama: $x = 1 + u - v,\ y = 2 - 2u + v,\ z = 2$, skicirati geometrijsko mesto tačaka za koje je $u \in [1,3]$ i $v \in [-2,1]$.

Rešenja:

(1) $p: \vec{r} = \vec{r}_P + t\vec{p}\ , \quad t \in \mathbb{R}$

$q: \vec{r} = \vec{r}_Q + s\vec{q}\ , \quad s \in \mathbb{R}$

- p i q se poklapaju ($p \equiv q$) $\Leftrightarrow \vec{p} \parallel \vec{q} \wedge P \in q$
- p i q su paralelne ($p \parallel q$) $\Leftrightarrow \vec{p} \parallel \vec{q} \wedge P \notin q$
- p i q se seku $\Leftrightarrow \vec{p} \nparallel \vec{q} \wedge (\vec{p} \times \vec{q}) \cdot \vec{PQ} = 0$
- p i q su mimoilazne $\Leftrightarrow \vec{p} \nparallel \vec{q} \wedge (\vec{p} \times \vec{q}) \cdot \vec{PQ} \neq 0$

(a) $p: \vec{p} = (2,1,6)\ ,\ P(0,1,0)$

$q: \vec{q} = (4,2,12)\ ,\ Q(2,2,6)$

$\vec{q} = 2\vec{p} \Rightarrow \vec{p} \parallel \vec{q}$ (1)

$P \overset{?}{\in} q$:
$2+4s = 0 \Rightarrow s = -\frac{1}{2}$
$2+2s = 2-1 = 1$ ✓
$6+12s = 6-6 = 0$ ✓
$\Rightarrow P \in q$ (2)

(1) + (2) $\Rightarrow p \equiv q$

(b) $p: \vec{p} = (2,1,6)\ ,\ P(0,1,0)$

$q: \vec{q} = (-4,-2,-12)\ ,\ Q(2,3,-15)$

$\vec{q} = -2\vec{p} \Rightarrow \vec{p} \parallel \vec{q}$ (1)

$P \overset{?}{\in} q$: $\frac{0-2}{-4} = \frac{1}{2}\ ,\ \frac{1-3}{-2} = 1$

$\frac{1}{2} \neq 1 \Rightarrow P \notin q$ (2)

(1) + (2) $\Rightarrow p \parallel q$

(c) $p: \vec{p} = (1,3,1)\ ,\ P(2,2,3)$

$q: \vec{q} = (1,4,2)\ ,\ Q(2,3,4)$

$\vec{p} \neq k\vec{q},\ k \in \mathbb{Z} \Rightarrow \vec{p} \nparallel \vec{q}$ (1)

$$(\vec{p} \times \vec{q}) \cdot \vec{PQ} = \begin{vmatrix} 1 & 3 & 1 \\ 1 & 4 & 2 \\ 0 & 1 & 1 \end{vmatrix} \cdot(-1) = \begin{vmatrix} 1 & 3 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 0 \quad (2)$$

(1) + (2) $\Rightarrow$ p i q se seku

$p \cap q = \{S\}$

$p: x = t+2$, $y = 3t+2$, $z = t+3$

$q: x = s+2$, $y = 4s+3$, $z = 2s+4$

$t+2 = s+2$
$3t+2 = 4s+3$
$t+3 = 2s+4$

$t-s = 0 \Rightarrow t = s$
$3t-4s = 1 \Rightarrow s = -1 = t$
$t-2s = 1$

$S(1,-1,2)$

(d) $p: \vec{p} = (4,1,-1)\ ,\ P(3,3,-1)$

$q: \vec{q} = (2,0,1)\ ,\ Q(0,0,-2)$

$\vec{p} \neq k\vec{q},\ k \in \mathbb{Z} \Rightarrow \vec{p} \nparallel \vec{q}$ (1)

$$(\vec{p} \times \vec{q}) \cdot \vec{PQ} = \begin{vmatrix} 4 & 1 & -1 \\ 2 & 0 & 1 \\ -3 & -3 & -1 \end{vmatrix} \cdot 3 = \begin{vmatrix} 4 & 1 & -1 \\ 2 & 0 & 1 \\ 9 & 0 & -4 \end{vmatrix} = -\begin{vmatrix} 2 & 1 \\ 9 & -4 \end{vmatrix}$$

$$= -(-8-9) = 17 \neq 0 \quad (2)$$

(1) + (2) $\Rightarrow$ p i q su mimoilazne prave

② $\alpha: (\vec{r}-\vec{r}_A)\cdot\vec{n}_\alpha = 0$

$p: \vec{r} = \vec{r}_p + t\vec{p}$, $t\in\mathbb{R}$

- $p\subseteq\alpha \iff \vec{p}\perp\vec{n}_\alpha \wedge P\in\alpha$
- $p\parallel\alpha \iff \vec{p}\perp\vec{n}_\alpha \wedge P\notin\alpha$
- p prodire $\alpha \iff \vec{p}\not\perp\vec{n}_\alpha$ (Specijalno, $p\perp\alpha \iff \vec{p}\parallel\vec{n}_\alpha$)

$\alpha: 2x-y+z-6=0 \Rightarrow \vec{n}_\alpha=(2,-1,1)$

(a) $p: \vec{p}=(-1,-1,1)$, $P(1,-1,4)$

$\vec{p}\cdot\vec{n}_\alpha=(-1,-1,1)\cdot(2,-1,1)=-2+1+1=0 \Rightarrow \vec{p}\perp\vec{n}_\alpha$ (1)

$P\overset{?}{\in}\alpha$: $2\cdot1-(-1)+4-6=1\neq0 \Rightarrow P\notin\alpha$ (2)

(1)+(2) $\Rightarrow p\parallel\alpha$

(b) $p: \vec{p}=(1,1,-1)$, $P(0,0,6)$

$\vec{p}\cdot\vec{n}_\alpha=(1,1,-1)\cdot(2,-1,1)=2-1-1=0 \Rightarrow \vec{p}\perp\vec{n}_\alpha$ (1)

$P\overset{?}{\in}\alpha$: $2\cdot0-0+6-6=0 \Rightarrow P\in\alpha$ (2)

(1)+(2) $\Rightarrow p\subseteq\alpha$

(c) $p: \vec{p}=(2,3,1)$, $P(0,1,-1)$

$\vec{p}\cdot\vec{n}_\alpha=(2,3,1)\cdot(2,-1,1)=4-3+1=2\neq0 \Rightarrow \vec{p}\not\perp\vec{n}_\alpha \Rightarrow p$ prodire α

(d) $p: \vec{p}=(-2,1,-1)$, $P(-2,1,-1)$

$\vec{p}=-\vec{n}_\alpha \Rightarrow \vec{p}\parallel\vec{n}_\alpha \Rightarrow p\perp\alpha$

③ $A(0,6,-2)$, $B(3,-3,4)$, $P(1,-1,1)$

(a) $p=p(A,B)$

$\vec{p}\parallel\vec{AB}=(3,-9,6)=3(1,-3,2)$

$$\left.\begin{array}{l}\vec{p}=(1,-3,2)\\ A(0,6,-2)\in p\end{array}\right\} \Rightarrow p: \vec{r}=(0,6,-2)+(1,-3,2)t=(t,6-3t,-2+2t),\ t\in\mathbb{R}$$

ili $p: \frac{x}{1}=\frac{y-6}{-3}=\frac{z+2}{2}$

(b) α: $P\in\alpha$, $\alpha\perp p$

$$\left.\begin{array}{l}\vec{n}_\alpha=\vec{p}=(1,-3,2)\\ P(1,-1,1)\in\alpha\end{array}\right\} \Rightarrow \alpha: 1(x-1)-3\cdot(y+1)+2\cdot(z-1)=0 \Rightarrow \alpha: x-3y+2z-6=0$$

(c) $p: \begin{array}{l}x=t\\ y=6-3t\\ z=-2+2t\end{array}$ $\{P_1\}=p\cap\alpha \Rightarrow t-3\cdot(6-3t)+2\cdot(-2+2t)-6=0$

$14t-28=0$

$t=2 \Rightarrow P_1(2,0,2)$

(d) $d(P,P_1)=|\vec{PP_1}|=|(1,1,1)|=\sqrt{1^2+1^2+1^2}=\sqrt{3}$

④ $\alpha: 4x - y + 3z - 1 = 0 \Rightarrow \vec{n}_\alpha = (4,-1,3)$

$\beta: 2x + y + 6z + 1 = 0 \Rightarrow \vec{n}_\beta = (2,1,6)$

$p = \alpha \cap \beta$

$$\vec{p} \parallel \vec{n}_\alpha \times \vec{n}_\beta = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -1 & 3 \\ 2 & 1 & 6 \end{vmatrix} = -9\vec{i} - 18\vec{j} + 6\vec{k} = (-9,-18,6) = -3(3,6,-2)$$

$\vec{p} = (3,6,-2)$

$P \in \alpha \cap \beta \leadsto x = 0 \Rightarrow \left.\begin{matrix} -y + 3z = 1 \\ y + 6z = -1 \end{matrix}\right) \Rightarrow P(0,-1,0) \in p$

$p: \vec{r} = (0,-1,0) + (3,6,-2)t = (3t, -1+6t, -2t),\ t \in \mathbb{R}$

ili $p: \frac{x}{3} = \frac{y+1}{6} = \frac{z}{-2}$

⑤ $A(1,0,1)$, $B(1,-1,0)$, $C(3,-2,1)$

$$\alpha = r(A,B,C) \Rightarrow \vec{n}_\alpha \parallel \vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -1 & -1 \\ 2 & -2 & 0 \end{vmatrix} = -2\vec{i} - 2\vec{j} + 2\vec{k} = -2(1,1,-1)$$

$\vec{n}_\alpha = (1,1,-1)$

$p: \frac{x-1}{-2} = \frac{y}{a} = \frac{z+1}{1} \Rightarrow \vec{p} = (-2,a,1)$

$p \parallel \alpha \Leftrightarrow \vec{p} \perp \vec{n}_\alpha \Leftrightarrow \vec{p} \cdot \vec{n}_\alpha = 0$

$\vec{p} \cdot \vec{n}_\alpha = (-2,a,1) \cdot (1,1,-1) = -2 + a - 1 = a - 3 = 0 \Leftrightarrow a = 3$

⑥ $p: \vec{p} = (1,-1,0)$, $P(0,2,-1)$

$q: \vec{q} = (3,-1,-1)$, $Q(-2,2,0)$

$r: \vec{r} = (-4,0,5)$, $R(-1,1,3)$

(a) $\vec{p} \nparallel \vec{q}$

$$(\vec{p} \times \vec{q}) \cdot \vec{PQ} = \begin{vmatrix} 1 & -1 & 0 \\ 3 & -1 & -1 \\ -2 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 0 \\ 1 & -1 & 0 \\ -2 & 0 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ p i q se seku

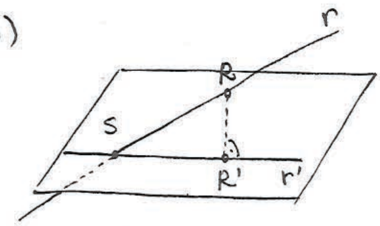
$\alpha = r(p,q)$

$$\vec{n}_\alpha \parallel \vec{p} \times \vec{q} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 0 \\ 3 & -1 & -1 \end{vmatrix} = \vec{i} + \vec{j} + 2\vec{k} = (1,1,2)$$

$\left.\begin{matrix} \vec{n}_\alpha = (1,1,2) \\ P(0,2,-1) \in \alpha \end{matrix}\right\} \Rightarrow \alpha: 1\cdot(x-0) + 1\cdot(y-2) + 2\cdot(z+1) = 0$

$\alpha: x + y + 2z = 0$

(b)



$$\left.\begin{array}{l}\{S\} = r \cap \alpha \\ R' = pr_\alpha(R)\end{array}\right\} \Rightarrow r' = pr_\alpha(r) = p(S, R')$$

- $\underline{\{S\} = r \cap \alpha}$

$r: \; x = -4t - 1$
$\quad y = 1$
$\quad z = 5t + 3$

$\alpha: x + y + 2z = 0$
$-4t - \cancel{1} + \cancel{1} + 10t + 6 = 0$
$6t + 6 = 0$
$t = -1 \quad \Rightarrow \quad S(3, 1, -2)$

- $\underline{R' = pr_\alpha(R)}$

$n: R \in n, \; n \perp \alpha$

$$\left.\begin{array}{l}\vec{n} = \vec{n}_\alpha = (1, 1, 2) \\ R(-1, 1, 3) \in n\end{array}\right\} \Rightarrow n: \vec{r} = (-1, 1, 3) + (1, 1, 2)t = (-1 + t, 1 + t, 3 + 2t), \; t \in \mathbb{R}$$

$\{R'\} = n \cap \alpha \Rightarrow -\cancel{1} + t + \cancel{1} + t + 6 + 4t = 0$
$6t + 6 = 0$
$t = -1 \quad \Rightarrow \quad R'(-2, 0, 1)$

- $\underline{r' = p(S, R')}$

$$\left.\begin{array}{l}\vec{r'} = \overrightarrow{R'S} = (5, 1, -3) \\ R'(-2, 0, 1) \in r'\end{array}\right\} \Rightarrow r': \frac{x+2}{5} = \frac{y}{1} = \frac{z-1}{-3}$$

(c) $\measuredangle(r, \alpha) = \measuredangle(r, r_1') = \measuredangle(\vec{r}, \vec{r'})$

$$\cos \measuredangle(\vec{r}, \vec{r'}) = \frac{\vec{r} \cdot \vec{r'}}{|\vec{r}| \cdot |\vec{r'}|} = \frac{(-4, 0, 5) \cdot (5, 1, -3)}{\sqrt{(-4)^2 + 5^2} \cdot \sqrt{5^2 + 1^2 + (-3)^2}} = \frac{-20 - 15}{\sqrt{41} \cdot \sqrt{35}} = \frac{-35}{\sqrt{41} \cdot \sqrt{35}} = -\sqrt{\frac{35}{41}}$$

7 (a) $\alpha: x + y - 2z = 1$
$\beta: 2x - y + 3z = -4$
$\gamma: -x + 2y + z = 5$

$x + y - 2z = 1$
$-3y + 7z = -6$
$3y - z = 6$

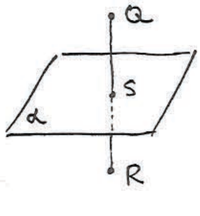
$x + y - 2z = 1$
$3y - z = 6$
$6z = 0$

$z = 0$
$3y = 6 \Rightarrow y = 2$
$x = -y + 2z + 1$
$\Rightarrow x = -1$

$P(-1, 2, 0)$

$\{P\} = \alpha \cap \beta \cap \gamma$

(b) $\alpha: x + y - 2z = 1$
$Q(2, -1, 3)$



$S = pr_\alpha(Q) = sr(QR)$

$n: Q \in n, \; n \perp \alpha$
$\vec{n} = \vec{n}_\alpha = (1, 1, -2)$
$Q(2, -1, 3) \in n$
$\Rightarrow n: \vec{r} = (2, -1, 3) + (1, 1, -2)t$
$= (2 + t, -1 + t, 3 - 2t)$

$S = n \cap \alpha$
$2 + t + (-1 + t) - 2 \cdot (3 - 2t) - 1 = 0$
$6t - 6 = 0$
$t = 1 \quad \Rightarrow \quad S(3, 0, 1)$

$\vec{r}_S = \frac{1}{2}(\vec{r}_Q + \vec{r}_R) \Rightarrow \vec{r}_R = 2\vec{r}_S - \vec{r}_Q$
$= (6, 0, 2) - (2, -1, 3)$
$= (4, 1, -1)$

$R(4, 1, -1)$

⑧ $p: \vec{p}=(2,a,1)$, $P(0,1,-2)$

$q: \vec{q}=(b,2,-2)$, $Q(-1,3,-2)$

(a) $p \parallel q \iff \vec{p} \parallel \vec{q} \iff \vec{p}\times\vec{q}=\vec{0}$

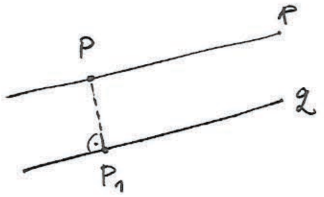
$$\vec{p}\times\vec{q} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & a & 1 \\ b & 2 & -2 \end{vmatrix} = (-2a-2)\vec{i} + (4+b)\vec{j} + (4-ab)\vec{k} = \vec{0}$$

$-2a-2=0 \Rightarrow a=-1 \Rightarrow \vec{p}=(2,-1,1)$

$4+b=0 \Rightarrow b=-4 \Rightarrow \vec{q}=(-4,2,-2)$

$4-ab=0$ ✓

(b)



$d(p,q)=d(P,P_1)$, $P_1=pr_q(P)$

$\pi: P\in\pi,\ \pi\perp q \Rightarrow \vec{n}_\pi \parallel \vec{q} = (-4,2,-2) = -2(2,-1,1)$

$\left.\begin{array}{l}\vec{n}_\pi=(2,-1,1)\\ P(0,1,-2)\in\pi\end{array}\right\} \Rightarrow \pi: 2\cdot(x-0)-1\cdot(y-1)+1\cdot(z+2)=0$

$2x-y+z+3=0$

$\{P_1\}=q\cap\alpha$

$q: x=-4s-1$, $y=2s+3$, $z=-2s-2$

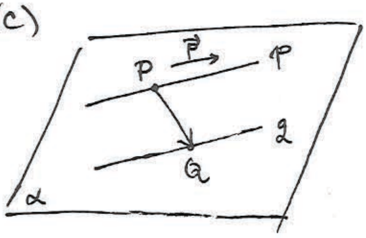
$2(-4s-1)-(2s+3)+(-2s-2)+3=0$

$-12s-4=0$

$s=-\frac{1}{3} \Rightarrow P_1\left(\frac{1}{3},\frac{7}{3},-\frac{4}{3}\right)$

$$d(p,q)=|\vec{PP_1}| = \left|\left(\tfrac{1}{3},\tfrac{4}{3},\tfrac{2}{3}\right)\right| = \sqrt{\left(\tfrac{1}{3}\right)^2+\left(\tfrac{4}{3}\right)^2+\left(\tfrac{2}{3}\right)^2} = \sqrt{\tfrac{21}{9}} = \sqrt{\tfrac{7}{3}}$$

(c)



$\alpha = r(p,q)$

$$\vec{n}_\alpha \parallel \vec{p}\times\vec{PQ} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 1 \\ -1 & 2 & 0 \end{vmatrix} = -2\vec{i}-\vec{j}+3\vec{k} = -(2,1,-3)$$

$\left.\begin{array}{l}\vec{n}_\alpha=(2,1,-3)\\ P(0,1,-2)\in\alpha\end{array}\right\} \Rightarrow \alpha: 2\cdot(x-0)+1\cdot(y-1)-3\cdot(z+2)=0$

$\alpha: 2x+y-3z-7=0$

⑨ $Q(2,-3,0)$

$\alpha: 2x+y-z+5=0 \Rightarrow \vec{n}_\alpha=(2,1,-1)$

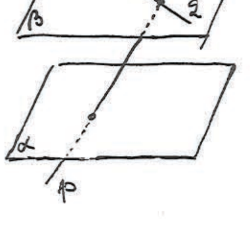
$p: \vec{p}=(1,2,-1)$, $P(0,-4,5)$

$\vec{n}_\alpha\cdot\vec{p}=(2,1,-1)\cdot(1,2,-1)=2+2+1=5\neq 0 \Rightarrow p$ prodire α

$Q\overset{?}{\in}\alpha: 2\cdot 2-3-0+5=6\neq 0 \Rightarrow Q\notin\alpha$

$Q\overset{?}{\in}p:\ t=2$

$-4+2t=-4+4=0\neq -3 \Rightarrow Q\notin p$



$\beta: Q\in\beta,\ \beta\parallel\alpha$

$\left.\begin{array}{l}\vec{n}_\beta=\vec{n}_\alpha=(2,1,-1)\\ Q(2,-3,0)\in\beta\end{array}\right\} \Rightarrow \beta: 2\cdot(x-2)+1\cdot(y+3)-1\cdot(z-0)=0$

$2x+y-z-1=0$

$\{S\}=p\cap\beta$

$p: x=t$, $y=-4+2t$, $z=5-t$

$2t-4+2t-5+t-1=0$

$5t-10=0$

$t=2 \Rightarrow S(2,0,3)$

→

$q = p(Q, S)$

$\vec{q} \parallel \overrightarrow{QS} = (0,3,3) = 3(0,1,1)$

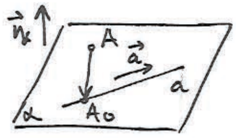
$\left.\begin{array}{l} \vec{q} = (0,1,1) \\ Q(2,-3,0) \in q \end{array}\right\} \Rightarrow \quad q: \vec{r} = (2,-3,0) + (0,1,1)t = (2, -3+t, t), \; t \in \mathbb{R}$

ili $\quad q: \dfrac{x-2}{0} = \dfrac{y+3}{1} = \dfrac{z}{1}$

(10) $A(1,2,3)$

$a: \vec{a} = (1,-1,0), \quad A_0(0,2,-1)$

(a) $\alpha = r(A, a)$

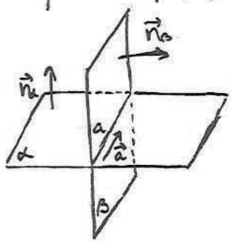


$\vec{n}_\alpha \parallel \vec{a} \times \overrightarrow{AA_0} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 0 \\ -1 & 0 & -4 \end{vmatrix} = 4\vec{i} + 4\vec{j} - \vec{k} = (4,4,-1)$

$\left.\begin{array}{l} \vec{n}_\alpha = (4,4,-1) \\ A(1,2,3) \in \alpha \end{array}\right\} \Rightarrow \quad \alpha: 4\cdot(x-1) + 4(y-2) - 1\cdot(z-3) = 0$

$\alpha: 4x + 4y - z - 9 = 0$

(b) $\beta: a \subseteq \beta, \; \beta \perp \alpha$



$\vec{n}_\beta \parallel \vec{n}_\alpha \times \vec{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 4 & -1 \\ 1 & -1 & 0 \end{vmatrix} = -\vec{i} - \vec{j} - 8\vec{k} = -(1,1,8)$

$\left.\begin{array}{l} \vec{n}_\beta = (1,1,8) \\ A_0(0,2,-1) \end{array}\right\} \Rightarrow \quad \beta: 1\cdot(x-0) + 1\cdot(y-2) + 8(z+1) = 0$

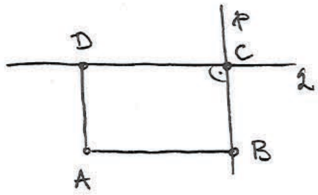
$\beta: x + y + 8z + 6 = 0$

(11) $A(0,0,-5)$

$p: \vec{p} = (2,1,1), \; P(3,-1,2)$

$q: \vec{q} = (1,3,-5), \; Q(0,-5,6)$

$\left.\begin{array}{l} \vec{p}\cdot\vec{q} = (2,1,1)\cdot(1,3,-5) = 2+3-5 = 0 \Rightarrow \vec{p} \perp \vec{q} \\ (\vec{p}\times\vec{q})\cdot\overrightarrow{PQ} = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 3 & -5 \\ -3 & -4 & 4 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 1 \\ -2 & -1 & -1 \\ -3 & -4 & 4 \end{vmatrix} = 0 \Rightarrow p \text{ i } q \text{ se sijeku} \end{array}\right\} \Rightarrow \quad p \perp q$



$\{C\} = p \cap q$

$B = pr_p(A)$

$D: \overrightarrow{AD} = \overrightarrow{BC}$

- $\{C\} = p \cap q$

$p: \begin{array}{l} x = 2t+3 \\ y = t-1 \\ z = t+2 \end{array} \qquad q: \begin{array}{l} x = s \\ y = 3s-5 \\ z = -5s+6 \end{array} \qquad \begin{array}{l} 2t+3 = s \\ t-1 = 3s-5 \\ t+2 = -5s+6 \end{array} \Leftrightarrow \begin{array}{l} 2t-s = -3 \\ t-3s = -4 \,/\cdot(-1) \\ t+5s = 4 \end{array} \Leftrightarrow \begin{array}{l} 8s = 8 \Rightarrow s = 1 \\ t = 3s-4 \Rightarrow t = -1 \end{array}$

$C(1,-2,1)$

- $B = pr_p(A)$

$\alpha: A \in \alpha, \; \alpha \perp p$

$\left.\begin{array}{l} \vec{n}_\alpha = \vec{p} = (2,1,1) \\ A(0,0,-5) \in \alpha \end{array}\right\} \Rightarrow \alpha: 2\cdot(x-0) + 1\cdot(y-0) + 1\cdot(z+5) = 0$

$2x + y + z + 5 = 0$

$\{B\} = \alpha \cap p$

$2\cdot(2t+3) + t - 1 + t + 2 + 5 = 0$

$6t + 12 = 0$

$t = -2$

$B(-1,-3,0)$

- $D: \overrightarrow{AD} = \overrightarrow{BC}$

$(x, y, z+5) = (2,1,1) \Rightarrow D(2,1,-4)$

(12) $A(1,2,3)$

$p: \vec{p} = (2,3,-1)$, $P(0,-1,2)$

$q: \vec{q} = (-5,-2,3)$, $Q(-3,0,4)$

$\alpha: 2x - y - 3z = 5 \Rightarrow \vec{n}_\alpha = (2,-1,-3)$

• $\underline{B = p \cap q}$

$p: \begin{array}{l} x = 2t \\ y = 3t-1 \\ z = -t+2 \end{array} \qquad q: \begin{array}{l} x = -5s-3 \\ y = -2s \\ z = 3s+4 \end{array} \qquad \begin{array}{l} 2t = -5s-3 \\ 3t-1 = -2s \\ -t+2 = 3s+4 \end{array} \Leftrightarrow \begin{array}{l} 2t+5s = -3 \\ 3t+2s = 1 \\ -t-3s = 2 \,/\cdot 2 \end{array} \Leftrightarrow \begin{array}{l} -t-3s = 2 \\ -s = 1 \\ \hline s = -1 \\ t = 1 \end{array}$

$B(2,2,1)$

• $\underline{C = pr_\alpha(A)}$

$a: A \in a,\ a \perp \alpha$

$\left.\begin{array}{l} \vec{a} = \vec{n}_\alpha = (2,-1,-3) \\ A(1,2,3) \in a \end{array}\right\} \Rightarrow a: \vec{r} = (1,2,3) + (2,-1,-3)t$
$= (1+2t, 2-t, 3-3t)$, $t \in \mathbb{R}$

$\{C\} = a \cap \alpha \Rightarrow 2\cdot(1+2t) - (2-t) - 3(3-3t) - 5 = 0$

$14t - 14 = 0$

$t = 1 \Rightarrow C(3,1,0)$

• $\underline{D : \vec{AD} = \vec{BC}}$

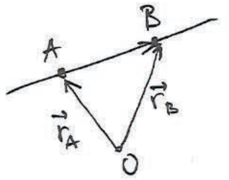
$(x-1, y-2, z-3) = (1,-1,-1) \Rightarrow D(2,1,2)$

$\vec{AC} = (2,-1,-3)$

$\vec{BD} = (0,-1,1)$

$$\cos\angle(\vec{AC},\vec{BD}) = \frac{\vec{AC}\cdot\vec{BD}}{|\vec{AC}|\cdot|\vec{BD}|} = \frac{(2,-1,-3)\cdot(0,-1,1)}{\sqrt{2^2+(-1)^2+(-3)^2}\cdot\sqrt{(-1)^2+1^2}} = \frac{0+1-3}{\sqrt{14}\cdot\sqrt{2}} = -\frac{1}{\sqrt{7}}$$

(13)



$\vec{r} = \vec{r}_A + t\vec{AB} = (2,-1,5) + t(2,3,-4)$

$\vec{AB}: \begin{array}{l} x = 2+2t \\ y = -1+3t \\ z = 5-4t \end{array} \quad t \in [0,1] \quad \left\{\begin{array}{l} t=0 \rightarrow \vec{r} = \vec{r}_A \\ t=1 \rightarrow \vec{r} = \vec{r}_A + \vec{AB} = \vec{r}_B \end{array}\right\}$

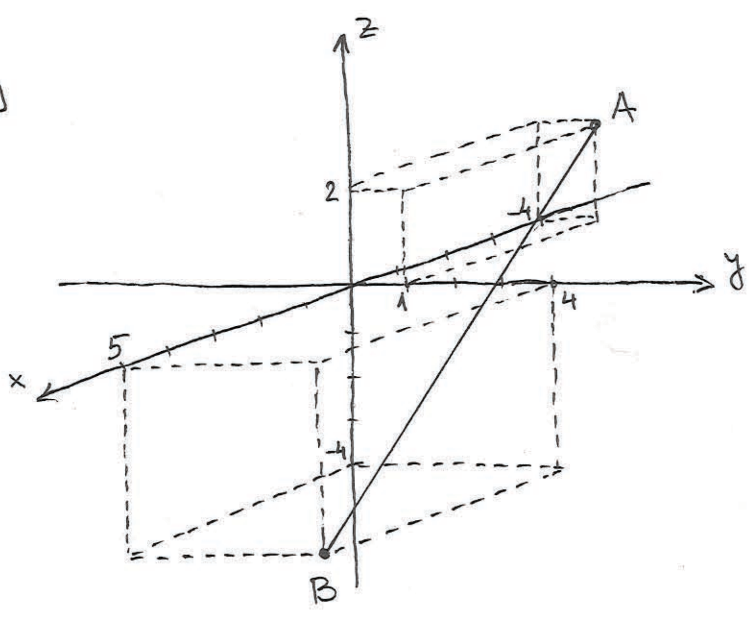
(14) Geometrijsko mesto tačaka

$r(t) = (-1+3t, 2+t, -2t)$, $t \in [-1,2]$

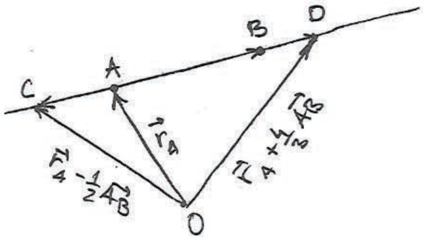
je duž AB gde je

$\vec{r}_A = r(-1) = (-4,1,2)$

$\vec{r}_B = r(2) = (5,4,-4)$



(15)



$\vec{AC} = -\frac{1}{2}\vec{AB}$

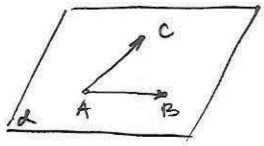
$\vec{BD} = \frac{1}{3}\vec{AB} \Rightarrow \vec{AD} = \frac{4}{3}\vec{AB}$

$\left.\begin{array}{l}\vec r_C = \vec r_A - \frac{1}{2}\vec{AB} \\ \vec r_D = \vec r_A + \frac{4}{3}\vec{AB}\end{array}\right\} \Rightarrow$ CD je tražena duž

$\vec r = \vec r_A + t\,\vec{AB}\,, \quad t \in [-\frac{1}{2}, \frac{4}{3}]$

(16) $A(1,2,5)$, $B(-1,0,3)$, $C(1,1,2)$

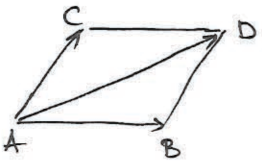
(a)



$\alpha: \vec r = \vec r_A + u\vec{AB} + v\vec{AC} = (1,2,5) + u(-2,-2,-2) + v(0,-1,-3)$

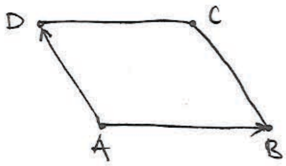
$\alpha: \begin{array}{l} x = 1 - 2u \\ y = 2 - 2u - v \\ z = 5 - 2u - 3v \end{array} \quad u, v \in \mathbb{R}$

(b) • D : $\vec{AD} = \vec{AB} + \vec{AC}$



ABCD : $\begin{array}{l} x = 1 - 2u \\ y = 2 - 2u - v \\ z = 5 - 2u - 3v \end{array} \quad \begin{array}{l} u \in [0,1] \\ v \in [0,1] \end{array}$

• D : $\vec{AD} = \vec{BC}$



$(x-1, y-2, z-5) = (2, 1, -1)$

$D(3,3,4)$

ABCD : $\vec r = \vec r_A + u\vec{AB} + v\vec{AD}$
$= (1,2,5) + u(-2,-2,-2) + v(2,1,-1)$

ABCD : $\begin{array}{l} x = 1 - 2u + 2v \\ y = 2 - 2u + v \\ z = 5 - 2u - v \end{array} \quad \begin{array}{l} u \in [0,1] \\ v \in [0,1] \end{array}$

(17) $\alpha: \begin{array}{l} x = 1 + u - v \\ y = 2 - 2u + v \\ z = 2 \end{array} \quad \begin{array}{l} u \in [1,3] \\ v \in [-2,1] \end{array} \Rightarrow$ Geometrijsko mesto tačaka je paralelogram u ravni $z=2$

$u=1 \wedge v=-2 \Rightarrow A(4,-2,2)$

$u=1 \wedge v=1 \Rightarrow B(1,1,2)$

$u=3 \wedge v=1 \Rightarrow C(3,-3,2)$

$u=3 \wedge v=-2 \Rightarrow D(6,-6,2)$