DETERMINANTE

1. Razvijanjem po prvoj vrsti i drugoj koloni izračunati vrednost determinante $\begin{vmatrix} 2 & 1 & 3 \\ 4 & -2 & 1 \\ -1 & 0 & -3 \end{vmatrix}$.

2. Koristeći osobine determinanti izračunati: (a) $D_1 = \begin{vmatrix} 1 & 5 & 1 \\ 4 & 0 & -2 \\ -3 & 4 & 2 \end{vmatrix}$; (b) $D_2 = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{vmatrix}$.

3. Izračunati vrednost determinante:

   (a) $\begin{vmatrix} 1 & 1 & 1 & a \\ 1 & 1 & a & 1 \\ 1 & a & 1 & 1 \\ a & 1 & 1 & 1 \end{vmatrix}$; (b) $\begin{vmatrix} 1 & a & a^2 & a^3 \\ a^3 & 1 & a & a^2 \\ a^2 & a^3 & 1 & a \\ a & a^2 & a^3 & 1 \end{vmatrix}$; (c) $\begin{vmatrix} 1 & a & a^2 & a^3 \\ 1 & b & b^2 & b^3 \\ 1 & c & c^2 & c^3 \\ 1 & d & d^2 & d^3 \end{vmatrix}$.

4. Izračunati kompleksan broj z ako je $Re(z) > 0$ i $\begin{vmatrix} z & 1 & i \\ 0 & -1 & \overline{z} \\ 1 & 2 & 0 \end{vmatrix} = -8 + 2i$.

5. Primenom Kramerovog pravila rešiti sistem: $\begin{array}{rcrcrcr} x & + & y & + & z & = & 4 \\ x & - & 2y & + & 2z & = & 3 \\ 2x & - & y & + & 5z & = & 11 \end{array}$.

6. Primenom Kramerovog pravila u zavisnosti od realnog parametra a diskutovati i rešiti sistem jednačina:

$$\begin{array}{rcccrcr} x & + & y & + & z & = & a \\ x & + & (a+1)y & + & z & = & 2a \\ x & + & y & + & az & = & -a \end{array}$$

7. U zavisnosti od realnih parametara a i b diskutovati sistem jednačina:

$$\begin{array}{rcccrcr} 2x & + & (a-2)y & - & 3az & = & 0 \\ -x & & & + & az & = & b \\ 3x & + & a^2y & - & 2az & = & b^2 \end{array}$$

8. Odrediti inverznu matricu matrice $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ -1 & -1 & 1 \end{bmatrix}$ pomoću adjungovane matrice.

9. Odrediti rang matrice: (a) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$; (b) $B = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 4 & 1 & 3 & 2 \\ -2 & 0 & -2 & -4 \\ 33 & 11 & 22 & 0 \end{bmatrix}$.

10. U zavisnosti od realnog parametra a diskutovati rang matrice $A = \begin{bmatrix} 3+a & 0 & 3+a \\ a & 1+a & 6+3a \\ 3 & 2 & -3-2a \end{bmatrix}$.

11. Odrediti vrednost realnog parametra λ tako da rang matrice $A = \begin{bmatrix} 1 & \lambda & -1 & 2 \\ 2 & -1 & \lambda & 5 \\ 1 & 10 & -6 & 1 \end{bmatrix}$ bude 2.

12. Koristeći Kroneker-Kapelijevu teoremu, u zavisnosti od realnog parametra a, diskutovati sistem jednačina:

$$\begin{array}{rcrcrcr} ax & - & a^2y & + & 9z & = & a \\ ax & + & 3ay & - & 3az & = & -3 \\ a^2x & - & 9ay & + & a^3z & = & a^2 \end{array}.$$

Rešenja:

(1) • razvijanje po prvoj vrsti:

$$\begin{vmatrix} 2 & 1 & 3 \\ 4 & -2 & 1 \\ -1 & 0 & -3 \end{vmatrix} = 2 \cdot \begin{vmatrix} -2 & 1 \\ 0 & -3 \end{vmatrix} - 1 \cdot \begin{vmatrix} 4 & 1 \\ -1 & -3 \end{vmatrix} + 3 \begin{vmatrix} 4 & -2 \\ -1 & 0 \end{vmatrix} = 2 \cdot (6-0) - 1 \cdot (-12+1) + 3 \cdot (0-2) = 17$$

• razvijanje po drugoj koloni:

$$\begin{vmatrix} 2 & 1 & 3 \\ 4 & -2 & 1 \\ -1 & 0 & -3 \end{vmatrix} = -1 \cdot \begin{vmatrix} 4 & 1 \\ -1 & -3 \end{vmatrix} + (-2) \cdot \begin{vmatrix} 2 & 3 \\ -1 & -3 \end{vmatrix} = -1 \cdot (-12+1) - 2 \cdot (-6+3) = 17$$

(2) (a)

$$D_1 = \begin{vmatrix} 1 & 5 & 1 \\ 4 & 0 & -2 \\ -3 & 4 & 2 \end{vmatrix} = \begin{vmatrix} 3 & 5 & 1 \\ 0 & 0 & -2 \\ 1 & 4 & 2 \end{vmatrix} = 2 \cdot \begin{vmatrix} 3 & 5 \\ 1 & 4 \end{vmatrix} = 2 \cdot (12-5) = 14$$

(b)

$$D_2 = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{vmatrix} = \begin{vmatrix} 1 & 1+1 & 3 \\ 2 & 2+1 & 4 \\ 3 & 3+1 & 6 \end{vmatrix} = \underbrace{\begin{vmatrix} 1 & 1 & 3 \\ 2 & 2 & 4 \\ 3 & 3 & 6 \end{vmatrix}}_{=0} + \begin{vmatrix} 1 & 1 & 3 \\ 2 & 1 & 4 \\ 3 & 1 & 6 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 3 \\ 2 & 1 & 4 \\ 3 & 1 & 6 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 3 \\ 1 & 0 & 1 \\ 2 & 0 & 3 \end{vmatrix} = - \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = -(3-2) = -1$$

(3) (a)

$$\begin{vmatrix} 1 & 1 & 1 & a \\ 1 & 1 & a & 1 \\ 1 & a & 1 & 1 \\ a & 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} a+3 & 1 & 1 & a \\ a+3 & 1 & a & 1 \\ a+3 & a & 1 & 1 \\ a+3 & 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 & a-1 \\ 0 & 0 & a-1 & 0 \\ 0 & a-1 & 0 & 0 \\ a+3 & 1 & 1 & 1 \end{vmatrix}$$

$$= -(a+3) \begin{vmatrix} 0 & 0 & a-1 \\ 0 & a-1 & 0 \\ a-1 & 0 & 0 \end{vmatrix} = -(a+3)(a-1) \begin{vmatrix} 0 & a-1 \\ a-1 & 0 \end{vmatrix} = (a+3)(a-1)^3$$

(b)

$$\begin{vmatrix} 1 & a & a^2 & a^3 \\ a^3 & 1 & a & a^2 \\ a^2 & a^3 & 1 & a \\ a & a^2 & a^3 & 1 \end{vmatrix} = \begin{vmatrix} 1-a^4 & 0 & 0 & 0 \\ 0 & 1-a^4 & 0 & 0 \\ 0 & 0 & 1-a^4 & 0 \\ a & a^2 & a^3 & 1 \end{vmatrix} = (1-a^4)^3$$

(c)

$$\begin{vmatrix} 1 & a & a^2 & a^3 \\ 1 & b & b^2 & b^3 \\ 1 & c & c^2 & c^3 \\ 1 & d & d^2 & d^3 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & b-a & b(b-a) & b^2(b-a) \\ 1 & c-a & c(c-a) & c^2(c-a) \\ 1 & d-a & d(d-a) & d^2(d-a) \end{vmatrix} = (b-a)(c-a)(d-a) \begin{vmatrix} 1 & b & b^2 \\ 1 & c & c^2 \\ 1 & d & d^2 \end{vmatrix}$$

$$= (b-a)(c-a)(d-a) \begin{vmatrix} 1 & 0 & 0 \\ 1 & c-b & c(c-b) \\ 1 & d-b & d(d-b) \end{vmatrix} = (b-a)(c-a)(d-a)(c-b)(d-b) \begin{vmatrix} 1 & c \\ 1 & d \end{vmatrix}$$

$$= (b-a)(c-a)(d-a)(c-b)(d-b)(d-c)$$

(4)
$$\begin{vmatrix} z & 1 & i \\ 0 & -1 & \bar{z} \\ 1 & 2 & 0 \end{vmatrix} = \begin{vmatrix} z & 1-2z & i \\ 0 & -1 & \bar{z} \\ 1 & 0 & 0 \end{vmatrix} = \begin{vmatrix} 1-2z & i \\ -1 & \bar{z} \end{vmatrix} = \bar{z}(1-2z) + i = (x-yi)(1-2x-2yi)+i$$

$\cdot(-2)$, $+$

$$= x - 2x^2 - \cancel{2xyi} - yi + \cancel{2xyi} - 2y^2 + i = (x-2x^2-2y^2) + (1-y)i = -8+2i$$

$x - 2x^2 - 2y^2 = -8 \Rightarrow x - 2x^2 - 2 = -8 \Rightarrow x - 2x^2 + 6 = 0$

$1 - y = 2 \Rightarrow \boxed{y=-1}$

$2x^2 - x - 6 = 0 \Leftrightarrow \boxed{x=2} \vee \cancel{x = -\frac{3}{2}}$

$\Rightarrow \boxed{z = 2 - i}$

(5)
$$\begin{aligned} x + y + z &= 4 \\ x - 2y + 2z &= 3 \\ 2x - y + 5z &= 11 \end{aligned}$$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & 2 \\ 2 & -1 & 5 \end{vmatrix} \begin{matrix} \cdot(-1) / \cdot(-2) \\ + \\ + \end{matrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & -3 & 1 \\ 0 & -3 & 3 \end{vmatrix} \begin{matrix} \\ \cdot(-1) \\ + \end{matrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & -3 & 1 \\ 0 & 0 & 2 \end{vmatrix} = -6$$

$$D_x = \begin{vmatrix} 4 & 1 & 1 \\ 3 & -2 & 2 \\ 11 & -1 & 5 \end{vmatrix} \begin{matrix} \\ \cdot 2 \\ + \end{matrix} = \begin{vmatrix} 4 & 1 & 1 \\ 11 & 0 & 4 \\ 15 & 0 & 6 \end{vmatrix} = - \begin{vmatrix} 11 & 4 \\ 15 & 6 \end{vmatrix} = -(66-60) = -6 \Rightarrow x = \frac{D_x}{D} = \frac{-6}{-6} = 1$$

$$D_y = \begin{vmatrix} 1 & 4 & 1 \\ 1 & 3 & 2 \\ 2 & 11 & 5 \end{vmatrix} \begin{matrix} \cdot(-1) / \cdot(-2) \\ + \\ + \end{matrix} = \begin{vmatrix} 1 & 4 & 1 \\ 0 & -1 & 1 \\ 0 & 3 & 3 \end{vmatrix} = \begin{vmatrix} -1 & 1 \\ 3 & 3 \end{vmatrix} = -3-3 = -6 \Rightarrow y = \frac{D_y}{D} = \frac{-6}{-6} = 1$$

$$D_z = \begin{vmatrix} 1 & 1 & 4 \\ 1 & -2 & 3 \\ 2 & -1 & 11 \end{vmatrix} \begin{matrix} \cdot(-1) / \cdot(-2) \\ + \\ + \end{matrix} = \begin{vmatrix} 1 & 1 & 4 \\ 0 & -3 & -1 \\ 0 & -3 & 3 \end{vmatrix} = \begin{vmatrix} -3 & -1 \\ -3 & 3 \end{vmatrix} = -9-3 = -12 \Rightarrow z = \frac{D_z}{D} = \frac{-12}{-6} = 2$$

rešenje: $(x,y,z) = (1,1,2)$

(6)
$$\begin{aligned} x + y + z &= a \\ x + (a+1)y + z &= 2a \\ x + y + az &= -a \end{aligned}$$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a+1 & 1 \\ 1 & 1 & a \end{vmatrix} \begin{matrix} \cdot(-1) \\ + \\ + \end{matrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & a & 0 \\ 0 & 0 & a-1 \end{vmatrix} = a(a-1)$$

- $D \neq 0 \Leftrightarrow a \neq 0 \wedge a \neq 1 \Rightarrow$ sistem je određen

$$D_x = \begin{vmatrix} a & 1 & 1 \\ 2a & a+1 & 1 \\ -a & 1 & a \end{vmatrix} = a \begin{vmatrix} 1 & 1 & 1 \\ 2 & a+1 & 1 \\ -1 & 1 & a \end{vmatrix} \begin{matrix} \cdot(-2) \\ + \\ \end{matrix} = a \begin{vmatrix} 1 & 1 & 1 \\ 0 & a-1 & -1 \\ 0 & 2 & a+1 \end{vmatrix} = a \begin{vmatrix} a-1 & -1 \\ 2 & a+1 \end{vmatrix} = a\big((a-1)(a+1)+2\big)$$

$$= a(a^2-1+2) = a(a^2+1) \Rightarrow x = \frac{D_x}{D} = \frac{a(a^2+1)}{a(a-1)} = \frac{a^2+1}{a-1}$$

$$D_y = \begin{vmatrix} 1 & a & 1 \\ 1 & 2a & 1 \\ 1 & -a & a \end{vmatrix} \begin{matrix} \cdot(-1) \\ + \\ \end{matrix} = \begin{vmatrix} 1 & a & 1 \\ 0 & a & 0 \\ 1 & -a & a \end{vmatrix} = a \begin{vmatrix} 1 & 1 \\ 1 & a \end{vmatrix} = a(a-1) \Rightarrow y = \frac{D_y}{D} = \frac{a(a-1)}{a(a-1)} = 1$$

$$D_z = \begin{vmatrix} 1 & 1 & a \\ 1 & a+1 & 2a \\ 1 & 1 & -a \end{vmatrix} = \begin{vmatrix} 1 & 0 & a \\ 1 & a & 2a \\ 1 & 0 & -a \end{vmatrix} = a \begin{vmatrix} 1 & a \\ 1 & -a \end{vmatrix} = a^2 \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -2a^2$$

$\cdot(-1)$, $+$

$$\Rightarrow z = \frac{D_z}{D} = \frac{-2a^2}{a(a-1)} = \frac{2a}{1-a}$$

rešenje: $(x,y,z) = \left(\frac{a^2+1}{a-1}, 1, \frac{2a}{1-a}\right)$, $a \neq 0 \wedge a \neq 1$

$\rightarrow$

• $\boxed{a=0}$

$D_x = D_y = D_z = 0$

$$\begin{aligned} x+y+z&=0\\ x+y+z&=0 \\ x+y\quad &=0 \quad /\cdot(-1) \end{aligned} \qquad \begin{aligned} x+y&=0 \Rightarrow y=-x\\ z&=0\end{aligned}$$

Sistem je 1x neodređen

$R=\{(x,-x,0)\mid x\in\mathbb{R}\}$

• $\boxed{a=1}$

$D_x = 2 \neq 0 \Rightarrow$ Sistem je nemoguć

⑦
$$\begin{aligned} 2x+(a-2)y-3az&=0\\ -x\qquad\qquad +az&=b\\ 3x+a^2y\quad-2az&=b^2 \end{aligned}$$

$$D=\begin{vmatrix} 2 & a-2 & -3a\\ -1 & 0 & a\\ 3 & a^2 & -2a\end{vmatrix} = a\begin{vmatrix} 2 & a-2 & -3\\ -1 & 0 & 1\\ 3 & a^2 & -2\end{vmatrix} \begin{matrix} \\ \cdot 2\ /\cdot 3 \\ \\ \end{matrix} = a\begin{vmatrix} 0 & a-2 & -1\\ -1 & 0 & 1\\ 0 & a^2 & 1\end{vmatrix} = a\begin{vmatrix} a-2 & -1\\ a^2 & 1\end{vmatrix}$$

$= a(a-2+a^2) = a(a-1)(a+2)$

• $D\neq 0 \Leftrightarrow a\neq 0 \wedge a\neq 1 \wedge a\neq -2 \Rightarrow$ sistem je određen

• $\boxed{a=0}$

$$\begin{aligned} 2x-2y&=0 \ /:2\\ -x\quad&=b\ /\cdot 3\\ 3x\quad&=b^2 \end{aligned}$$

$$\begin{aligned} x-y&=0\\ -x\quad&=b\\ 0&=b(b+3) \end{aligned}$$

- $b\neq 0 \wedge b\neq -3 \Rightarrow$ sistem je nemoguć
- $b=0 \vee b=-3$
  sistem je 1x neodređen
  $R=\{(-b,-b,z)\mid z\in\mathbb{R}\}$

• $\boxed{a=1}$

$$\begin{aligned} 2x-y-3z&=0\\ -x\quad+z&=b\\ 3x+y-2z&=b^2 \end{aligned}$$

$$\begin{aligned} 2x-y-3z&=0\\ -x\quad+z&=b\ /\cdot 5\\ 5x\quad-5z&=b^2 \end{aligned}$$

$$\begin{aligned} 2x-y-3z&=0\\ -x\quad+z&=b\\ 0&=b(b+5) \end{aligned}$$

- $b\neq 0 \wedge b=-5 \Rightarrow$ sistem je nemoguć
- $b=0 \vee b=-5$
  sistem je 1x neodređen
  $R=\{(x,-x-3b,x+b)\mid x\in\mathbb{R}\}$

• $\boxed{a=-2}$

$$\begin{aligned} 2x-4y+6z&=0\\ -x\quad-2z&=b\\ 3x+4y+4z&=b^2 \end{aligned}$$

$$\begin{aligned} 2x-4y+6z&=0\ /:2\\ -x\quad-2z&=b\ /\cdot 5\\ 5x\quad+10z&=b^2 \end{aligned}$$

$$\begin{aligned} x-2y+3z&=0\\ -x\quad-2z&=b\\ 0&=b(b+5) \end{aligned}$$

- $b\neq 0 \wedge b\neq -5 \Rightarrow$ sistem je nemoguć
- $b=0 \vee b=-5$
  sistem je 1x neodređen
  $R=\{(-2z-b,\frac{z-b}{2},z)\mid z\in\mathbb{R}\}$

⑧
$$A=\begin{bmatrix} 1 & 0 & -1\\ 2 & 1 & 3\\ -1 & -1 & 1\end{bmatrix} \rightsquigarrow |A| = \begin{vmatrix} 1 & 0 & -1\\ 2 & 1 & 3\\ -1 & -1 & 1\end{vmatrix} = \begin{vmatrix} 1 & 0 & 0\\ 2 & 1 & 5\\ -1 & -1 & 0\end{vmatrix} = \begin{vmatrix} 1 & 5\\ -1 & 0\end{vmatrix} = 5 \neq 0$$

$A^{-1} = \frac{1}{|A|}\cdot A^*$

→

$$A^T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ -1 & 3 & 1 \end{bmatrix} \rightsquigarrow$$

$$A^* = \begin{bmatrix} \begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix} & -\begin{vmatrix} 0 & -1 \\ -1 & 1 \end{vmatrix} & \begin{vmatrix} 0 & 1 \\ -1 & 3 \end{vmatrix} \\ -\begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} & \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix} \\ \begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & -1 \\ 0 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 4 & 1 & 1 \\ -5 & 0 & -5 \\ -1 & 1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{5} \begin{bmatrix} 4 & 1 & 1 \\ -5 & 0 & -5 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 4/5 & 1/5 & 1/5 \\ -1 & 0 & -1 \\ -1/5 & 1/5 & 1/5 \end{bmatrix}$$

(9)

(a)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{matrix} \cdot(-4) \,/\, \cdot(-7) \\ + \\ + \end{matrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{bmatrix} \begin{matrix} \\ \cdot(-2) \\ + \end{matrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \text{rang}(A) = 2$$

(b)

$$B = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 4 & 1 & 3 & 2 \\ -2 & 0 & -2 & -4 \\ 33 & 11 & 22 & 0 \end{bmatrix} \begin{matrix} \cdot 2 \\ \\ + \\ \cdot \frac{1}{11} \end{matrix} \sim \begin{bmatrix} 1 & 0 & 1 & 2 \\ 4 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \\ 3 & 1 & 2 & 0 \end{bmatrix} \begin{matrix} \cdot(-1) \\ + \\ \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & 0 & 1 & 2 \\ 3 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 3 & 1 & 2 & 0 \end{bmatrix} \begin{matrix} \\ \cdot(-1) \\ \\ + \end{matrix} \sim \begin{bmatrix} 1 & 0 & 1 & 2 \\ 3 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \text{rang}(B) = 2$$

(10)

$$A = \begin{bmatrix} 3+a & 0 & 3+a \\ a & 1+a & 6+3a \\ 3 & 2 & -3-2a \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3+a & 0 & 3+a \\ a & 1+a & 6+3a \\ 3 & 2 & -3-2a \end{vmatrix} = (3+a) \begin{vmatrix} 1 & 0 & 1 \\ a & 1+a & 6+3a \\ 3 & 2 & -3-2a \end{vmatrix} = (3+a) \begin{vmatrix} 1 & 0 & 0 \\ a & 1+a & 6+2a \\ 3 & 2 & -6-2a \end{vmatrix}$$

(first column $\cdot(-1)$, $+$ to third column)

$$= (3+a)(6+2a) \begin{vmatrix} 1 & 0 & 0 \\ a & 1+a & 1 \\ 3 & 2 & -1 \end{vmatrix} = 2(3+a)^2 \begin{vmatrix} 1+a & 1 \\ 2 & -1 \end{vmatrix} = 2(3+a)^2(-1-a-2) = -2(3+a)^3$$

$$\text{rang}(A) = 3 \Leftrightarrow |A| \neq 0 \Leftrightarrow a \neq -3$$

- $a = -3$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ -3 & -2 & -3 \\ 3 & 2 & 3 \end{bmatrix} \begin{matrix} \\ + \\ \\ \end{matrix} \sim \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 2 & 3 \end{bmatrix} \Rightarrow \text{rang}(A) = 1$$

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$$A=\begin{bmatrix}1 & \lambda & -1 & 2\\ 2 & -1 & \lambda & 5\\ 1 & 10 & -6 & 1\end{bmatrix}\begin{matrix}\cdot(-2)\,/\cdot(-1)\\ \swarrow +\\ \swarrow +\end{matrix} \sim \begin{bmatrix}1 & \lambda & -1 & 2\\ 0 & -1-2\lambda & \lambda+2 & 1\\ 0 & 10-\lambda & -5 & -1\end{bmatrix} \sim \begin{bmatrix}1 & 2 & -1 & \lambda\\ 0 & 1 & \lambda+2 & -1-2\lambda\\ 0 & -1 & -5 & 10-\lambda\end{bmatrix}\Big)+$$

$$\sim\begin{bmatrix}1 & 2 & -1 & \lambda\\ 0 & 1 & \lambda+2 & -1-2\lambda\\ 0 & 0 & \lambda-3 & 9-3\lambda\end{bmatrix}$$

rang(A) = 2 akko su svi elementi u trećoj vrsti poslednje matrice jednaki 0,

tj. $\lambda-3=9-3\lambda=0 \iff \boxed{\lambda=3}$

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$$\begin{aligned} ax - a^2y + 9z &= a\\ ax + 3ay - 3az &= -3\\ a^2x - 9ay + a^3z &= a^2\end{aligned}$$

$$\left[\begin{array}{ccc|c} a & -a^2 & 9 & a\\ a & 3a & -3a & -3\\ a^2 & -9a & a^3 & a^2\end{array}\right]\updownarrow \sim \left[\begin{array}{ccc|c} a & 3a & -3a & -3\\ a & -a^2 & 9 & a\\ a^2 & -9a & a^3 & a^2\end{array}\right]\begin{matrix}\cdot(-1)\,/\cdot(-a)\\ \swarrow +\\ \swarrow +\end{matrix}$$

$$\sim\left[\begin{array}{ccc|c} a & 3a & -3a & -3\\ 0 & -a(a+3) & 3(a+3) & a+3\\ 0 & -3a(a+3) & a^2(a+3) & a(a+3)\end{array}\right]\begin{matrix}\\ \cdot(-3)\\ \swarrow +\end{matrix} \sim \left[\begin{array}{ccc|c} a & 3a & -3a & -3\\ 0 & -a(a+3) & 3(a+3) & a+3\\ 0 & 0 & (a+3)^2(a-3) & (a+3)(a-3)\end{array}\right]$$

- $a\neq 0 \wedge a\neq 3 \wedge a\neq -3 \Rightarrow$ rang(M_s) = rang$(M_p) = 3$
  $\Rightarrow$ sistem je određen

- $a=0$

$$\left[\begin{array}{ccc|c} 0 & 0 & 0 & -3\\ 0 & 0 & 9 & 3\\ 0 & 0 & -27 & -9\end{array}\right]$$

rang$(M_s)=1$, rang$(M_p)=2$ $\Rightarrow$ sistem je nemoguć

- $a=3$

$$\left[\begin{array}{ccc|c} 3 & 9 & -9 & -3\\ 0 & -18 & 18 & 6\\ 0 & 0 & 0 & 0\end{array}\right]$$

rang$(M_s)=2$, rang$(M_p)=2$ $\Rightarrow$ sistem je 1x neodređen

- $a=-3$

$$\left[\begin{array}{ccc|c} -3 & -9 & 9 & -3\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{array}\right]$$

rang$(M_s)=1$, rang$(M_p)=1$ $\Rightarrow$ sistem je 2x neodređen