

1 Dvostruki integrali

1. Izračunati $\iint_D f(x, y) dx dy$ ako je

- (a) $f(x, y) = 2x^3y + xy$, i $D = \{(x, y) \in \mathbb{R}^2 : -2 \leq x \leq 2, -3 \leq y \leq 1\}$,
- (b) $f(\rho, \varphi) = \rho^2 \varphi$ i $D = \{(\rho, \varphi) \in [0, 2] \times [\frac{\pi}{6}, \frac{\pi}{3}]\}$,
- (c) $f(x, y) = 1$ i $D = \{(x, y) \in \mathbb{R}^2 : 1 \leq x \leq 3, x^2 \leq y \leq \frac{1}{x}\}$,
- (d) $f(x, y) = 1$ i $D = \{(x, y) \in \mathbb{R}^2 : 1 \leq x \leq 25 - y^2, -3 \leq y \leq 0\}$,
- (e) $f(x, y) = 1$ i $D = \{(x, y) \in \mathbb{R}^2 : (x - 1)^2 + (y - 3)^2 \leq 1\}$.

2. Odrediti granice integracije integrala $\iint_D f(x, y) dx dy$ za dva moguća redosleda integracije ako je

- (a) $D = \{(x, y) \in \mathbb{R}^2 : y \leq \sqrt{x}, x \leq \sqrt{y}\}$,
- (b) $D = \{(x, y) \in \mathbb{R}^2 : x^2 + (y - 2)^2 \leq 4, y \leq x\}$,
- (c) $D = \{(x, y) \in \mathbb{R}^2 : (x - 1)^2 + (y - 3)^2 \leq 1\}$ i
- (d) $D = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 8y, x^2 + y^2 \leq 8x\}$.

3. Izračunati površinu figure:

- (a) $\{(x, y) \in \mathbb{R}^2 : 9 \leq x^2 + y^2 \leq 16\}$,
- (b) $\{(x, y) \in \mathbb{R}^2 : 4(x - 1)^2 + 9y^2 \leq 36\}$,
- (c) $\{(x, y) \in \mathbb{R}^2 : 1 \leq x + y \leq 4, 1 \leq \frac{y}{x} \leq 2\}$ i
- (d) $\{(x, y) \in \mathbb{R}^2 : \sqrt[4]{x + y} \leq 1, x \geq 0, y \geq 0\}$.

4. Izračunati masu i težište homogene ploče (npr. $\mu = 1$)

- (a) $\{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq a, 0 \leq y \leq b\}$ i
- (b) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 9, y \geq 0\}$.

5. Izračunati masu i težište ploče

$$D = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 25, 0 \leq y \leq \sqrt{3}x\}$$

gustine $\mu(x, y) = x^2 + y^2$.

6. Izračunati zapreminu oblasti

- (a) $V = \{(x, y, z) \in \mathbb{R}^3 : -2 \leq x \leq 2, -3 \leq y \leq 1, 0 \leq z \leq 5\}$,
- (b) $V = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 \leq 16, z \leq 9 - x^2 - y^2\}$
- (c) $V = \{(x, y, z) \in \mathbb{R}^3 : z - 5 \leq -\sqrt{x^2 + y^2}, z \geq 1\}$ i
- (d) $V = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 \leq 1, x^2 + y^2 - 2 \leq z \leq 2 - x^2 - y^2\}$.

2 Trostruki integrali

1. Izračunati $\iiint_V (x^2y + z) dx dy dz$ ako je

$$V = \{(x, y, z) \in \mathbb{R}^3 : -2 \leq x \leq 2, -3 \leq y \leq 1, 1 \leq z \leq 5\}.$$

2. Izračunati $\iiint_V (x^2 + y^2) dx dy dz$ ako je

$$V = \{(x, y, z) \in \mathbb{R}^3 : 2 \leq z \leq 4 - x^2 - y^2\}.$$

3. Izračunati Jakobijan transformacije

- a) $x = r \cos \varphi \sin \theta, y = r \sin \varphi \sin \theta, z = r \cos \theta,$
 $(r, \varphi, \theta) \in [0, \infty) \times [0, 2\pi] \times [0, \pi].$
b) $x = ar \cos \varphi \sin \theta, y = br \sin \varphi \sin \theta, z = cr \cos \theta,$
 $a > 0, b > 0, c > 0 \quad (r, \varphi, \theta) \in [0, \infty) \times [0, 2\pi] \times [0, \pi].$

4. Izračunati zapreminu oblasti

- (a) $V = \{(x, y, z) \in \mathbb{R}^3 : 2x + 3y + 4z \leq 12, x \geq 0, y \geq 0, z \geq 0\},$
(b) $V = \{(x, y, z) \in \mathbb{R}^3 : 1 \leq x^2 + y^2 + z^2 \leq 9\},$
(c) $V = \{(\rho, \varphi, \theta) \in [0, 3] \times [0, \pi] \times [0, \frac{\pi}{3}]\},$
(d) $V = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 9, 0 \leq \sqrt{3}z \leq \sqrt{x^2 + y^2}, y \geq 0\}$ i
(e) $V = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 \leq z^2, x^2 + y^2 + z^2 \leq 9, z \leq 0\}.$

5. Odrediti težište homogenog tela (gustine $\mu = 1$)

- (a) $V = \{(x, y, z) \in \mathbb{R}^3 : 0 \leq x \leq 2, 0 \leq y \leq 3, 0 \leq z \leq 4\}$ i
(b) $V = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 9, z \geq 0\}.$

6. Izračunati $I = \iiint_V (x^2 + y^2) dx dy dz$, gde je

$$V = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 \geq 2, x^2 + y^2 \leq 7 - z, z \geq 3\}.$$