Prvi deo

1.

$\cdot$	A	B	C	D
A	D	A	B	C
B	A	B	C	D
C	B	C	D	A
D	C	D	A	B

$$A\cdot A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = D$$

$$A\cdot B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\cdot\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = A$$

$$A\cdot C = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = B$$

$$A\cdot D = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\cdot\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = C$$

$$C\cdot A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\cdot\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = B$$

$$C\cdot B = C$$

$$C\cdot C = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = D$$

$$\vdots$$

Zatvorenost važi, jer su svi elementi u tablici iz skupa $\{A, B, C, D\}$.

Asocijativnost važi, jer je množenje matrica asocijativna operacija.

Neutralni element je B

Inverzni elementi: $A^{-1} = C$, $B^{-1} = B$, $C^{-1} = A$, $D^{-1} = D$.

Komutativnost važi, jer je tablica simetrična u odnosu na glavnu dijagonalu.

Dakle, $(\{A, B, C, D\}, \cdot)$ je Abelova grupa.

2.

$$\frac{2z+1}{i-\bar{z}} = 2z \quad / \cdot (i-\bar{z})$$

$$2z+1 = 2iz - 2\cdot z\bar{z}$$

$$2z+1 = 2iz - 2|z|^2$$

Neka je $z = x+iy$, gde su $x, y \in \mathbb{R}$

$$2\cdot(x+yi)+1 = 2i(x+iy) - 2\cdot(x^2+y^2)$$

$$2x+1+2yi = 2xi - 2y - 2(x^2+y^2)$$

$$2x+1 = -2y - 2(x^2+y^2) \quad \wedge \quad 2y = 2x$$

$$\boxed{y = x}$$

$$2x+1 = -2x - 2(x^2+x^2)$$

$$2x+1 = -2x - 4x^2$$

$$4x^2+4x+1=0$$

$$x_{1/2} = \frac{-4 \pm \sqrt{16-16}}{8}$$

$$x = -\frac{1}{2}$$

$$y = -\frac{1}{2}$$

$$\boxed{\text{Rešenje: } z = -\frac{1}{2} - \frac{1}{2}i}$$

3. $p(x) = x^5 + 3x^4 + 9x^3 + 27x^2 + 81x + 243$

Polinom p možemo posmatrati kao zbir prvih 6 članova geometrijskog niza sa prvim članom $a = 243$ i koeficijentom progresije $q = \frac{1}{3}x$.

$$p(x) = 0 \iff 243 \cdot \frac{1 - \left(\frac{x}{3}\right)^6}{1 - \frac{x}{3}} = 0 \iff \frac{243 - \frac{1}{3}x^6}{1 - \frac{x}{3}} = 0$$

$$\iff 243 - \frac{x^6}{3} = 0 \quad \wedge \quad 1 - \frac{x}{3} \neq 0$$

$$\iff x^6 = 729 \quad \wedge \quad x \neq 3$$

$$\iff x = \sqrt[6]{729 \cdot e^{i \cdot 0}} \in \left\{ 3 \cdot e^{\frac{0 + 2k\pi}{6} i} \,\middle|\, k = -2, -1, 0, 1, 2, 3 \right\} \wedge x \neq 3$$

$$\iff x \in \left\{ 3 \cdot e^{\pm \frac{\pi}{3} i}, 3 \cdot e^{\pm \frac{2\pi}{3} i}, -3 \right\}$$

$$p(x) = (x+3) \cdot \left(x - 3 \cdot e^{\frac{\pi}{3} i}\right)\left(x - 3 \cdot e^{-\frac{\pi}{3} i}\right)\left(x - 3 \cdot e^{\frac{2\pi}{3} i}\right)\left(x - 3 \cdot e^{-\frac{2\pi}{3} i}\right)$$

FAKTORIZACIJA NAD $\mathbb{C}$

$$= (x+3)\left(x^2 - 2x \cdot 3 \cos\frac{\pi}{3} + 9\right)\left(x^2 - 2x \cdot 3 \cdot \cos\frac{2\pi}{3} + 9\right)$$

$$= (x+3)(x^2 - 3x + 9)(x^2 + 3x + 9)$$

FAKTORIZACIJA NAD $\mathbb{R}$ i $\mathbb{Q}$

Drugi deo

$\boxed{1.}$

$$
\begin{aligned}
ax + ay + (a+2)z &= a \quad /\cdot(-1) \\
ax + (a+2)y + (a-1)z &= a \\
ax + (a+2)y + (2a+3)z &= 1
\end{aligned}
$$

$$
\begin{aligned}
ax + ay + (a+2)z &= a \\
2y - 3z &= 0 \quad /\cdot(-1) \\
2y + (a+1)z &= 1-a
\end{aligned}
$$

$$
\begin{aligned}
ax + ay + (a+2)z &= a \\
2y - 3z &= 0 \\
(a+4)z &= 1-a
\end{aligned}
$$

Za $a \notin \{-4, 0\}$ sistem je određen.

Za $a = 0$ je

$$
\begin{aligned}
2z &= 0 \quad /\cdot(-2) \\
2y - 3z &= 0 \\
4z &= 1
\end{aligned}
$$

$$
\begin{aligned}
2z &= 0 \\
2y - 3z &= 0 \\
0 &= 1 \quad ↯
\end{aligned}
$$

KONTRADIKCIJA

Sistem je kontradiktoran.

Za $a = -4$ je:

$$
\begin{aligned}
-4x - 4y - 2z &= 0 \\
2y - 3z &= 0 \\
0 \cdot z &= 5 \quad ↯
\end{aligned}
$$

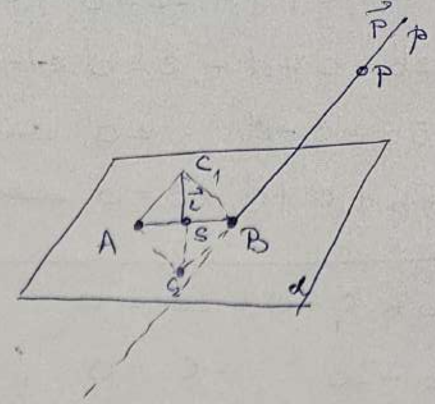
sistem je kontradiktoran!

(2) $p: \vec{r} = \vec{r}_P + t \cdot \vec{p}$

$P \notin \alpha$

$\vec{a} \perp \alpha$

$A \in \alpha$



ΔABC jednakostranični u ravni α

i $B \in p$

$B \in \alpha \ \wedge \ B \in p \Rightarrow B$ – prodor prave p kroz ravan α

$$\vec{r}_B = \vec{r}_P + \frac{(\vec{r}_A - \vec{r}_P) \cdot \vec{a}}{\vec{a} \cdot \vec{p}} \cdot \vec{p}$$

$\vec{r}_S = \dfrac{\vec{r}_A + \vec{r}_B}{2}$ (sredina $\overrightarrow{AB}$) $\quad |\overrightarrow{SC}| = \dfrac{\sqrt{3}}{2} |\overrightarrow{AB}|$

$\vec{r}_C = \vec{r}_S \pm \dfrac{\sqrt{3}}{2} |\overrightarrow{AB}| \cdot \dfrac{\vec{\ell}}{|\vec{\ell}|}$, gde je $\vec{\ell} = \overrightarrow{AB} \times \vec{a}$, jer $\vec{\ell} \perp \overrightarrow{AB}$, $\vec{\ell} \perp \vec{a}$

3.

$$a - b + c + d + 2e = 0 \quad / \cdot (-1) \quad / \cdot 1$$
$$a + 2b - c + d + e = 0$$
$$-a - 5b + c - d = 0$$
$$a - 4b + c + d + e = 0$$

$$a - b + c + d + 2e = 0$$
$$3b - 2c - e = 0 \quad / \cdot 2 \quad / \cdot 1$$
$$-6b + 2c + 2e = 0$$
$$-3b - e = 0$$

$$a - b + c + d + 2e = 0$$
$$3b - 2c - e = 0$$
$$-2c = 0$$
$$-2c - 2e = 0$$

$$c = 0$$
$$e = 0$$
$$b = \frac{2c + e}{3} = 0$$
$$a + d = 0$$

$$b = c = e = 0$$
$$a = -d$$

Jedna baza je $\{d\}$. $\dim V = 1$

Jedina druga baza je $\{a\}$.