ALGEBRA

① $f_1(x,y) = (-y,-x)$

$f_2(x,y) = (y,x)$

$f_3(x,y) = (x,y)$

$f_4(x,y) = (-x,-y)$

$\circ$	f_1	f_2	f_3	f_4
f_1	f_3	f_4	f_1	f_2
f_2	f_4	f_3	f_2	f_1
f_3	f_1	f_2	f_3	f_4
f_4	f_2	f_1	f_4	f_3

$(f_2 \circ f_1)(x,y) = f_2(f_1(x,y)) = f_2(-y,-x) = (-x,-y) = f_4(x,y)$ ...

1. zatvorenost: u tablici vidimo da su svi rezultati iz skupa $\{f_1, f_2, f_3, f_4\}$
2. asocijativnost: Kompozicija fja je asocijativna
3. neutralni element: f_3 (identična fja) (vrsta koja odgovara f_3 u tablici jednaka je graničnoj vrsti; kolona f_3 jednaka je graničnoj koloni)
4. inverzni elementi: $f_1^{-1} = f_1$, $f_2^{-1} = f_2$, $f_3^{-1} = f_3$, $f_4^{-1} = f_4$.
5. komutativnost: Tablica je simetrična u odnosu na glavnu dijagonalu → grupa je komutativna

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② $p(x) = ax^5 + 5x^4 - 2x^3 - 2x^2 + 16x + b$

$p(1+i) = a(1+i)^5 + 5\cdot(1+i)^4 - 2\cdot(1+i)^3 - 2\cdot(1+i)^2 + 16(1+i) + b =$

$= a\cdot(-4-4i) + 5\cdot(-4) - 2\cdot(2i-2) - 2\cdot 2i + 16(1+i) + b =$

$= (\underbrace{-4a - 20 + 4 + 16 + b}_{=0}) + (\underbrace{-4a - 4 - 4 + 16}_{=0})i = 0$

$-4a + b = 0$

$-4a + 8 = 0 \rightarrow -4a = -8 \rightarrow \boxed{a=2}$ $\qquad -4\cdot 2 + b = 0 \rightarrow \boxed{b=8}$

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③ $z_1 = 2+2i \qquad z_2 = 4+i$

$\boxed{1^\circ}$ $z_3 = S_{z_2, -\frac{\pi}{2}}(z_1) = z_2 + (z_1 - z_2)\cdot e^{-i\frac{\pi}{2}} = 4+i + (-2+i)\cdot(-i) =$

$= 4+i+2i+1 = \boxed{5+3i}$

$z_1 + z_3 = z_2 + z_4 \qquad z_4 = z_1 + z_3 - z_2 = 2+2i+5+3i-4-i = \boxed{3+4i}$

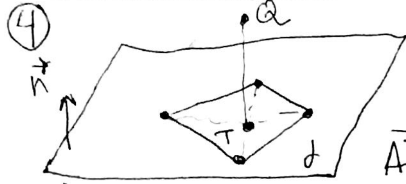
$\boxed{2^\circ}$ $z_1 = \frac{z_4 + w_4}{2} \qquad w_4 = 2z_1 - z_4 = 4+4i-3-4i = \boxed{1}$

$z_2 = \frac{z_3 + w_3}{2} \qquad w_3 = 2z_2 - z_3 = 8+2i-5-3i = \boxed{3-i}$

$\boxed{3^\circ}$ $S = \frac{z_1 + z_3}{2} = \frac{2+2i+5+3i}{2} = \boxed{\frac{7}{2} + \frac{5}{2}i}$

$T = \frac{z_1 + w_3}{2} = \frac{2+2i+3-i}{2} = \boxed{\frac{5}{2} + \frac{1}{2}i}$

(4)



T - пресек дијагонала $\quad T = proj_{d}\,Q$

$$\vec{r}_T = \vec{r}_Q + \boxed{\frac{(\vec{r}_A - \vec{r}_Q)\vec{n}}{\vec{n}\cdot\vec{n}}\cdot\vec{n}}$$

$$\vec{AT} = \vec{TC} \Rightarrow \boxed{\vec{r}_C = 2\vec{r}_T - \vec{r}_A}$$

$$\vec{BT} \perp \vec{n} \wedge \vec{BT} \perp \vec{AT}$$

$$\vec{BT} \parallel \vec{m} = \vec{n} \times \vec{AT}$$

$$\boxed{\vec{r}_{B,C} = \vec{r}_T \pm |\vec{AT}|\,\frac{\vec{m}}{|\vec{m}|}}$$

(5)

$$\begin{aligned} ax + y + z &= 1 \\ ax + a^2 y + (1-2a)z &= 4 \\ ax + y + az &= b+3 \end{aligned} \quad\longrightarrow\quad \begin{aligned} ax + y + z &= 1 \\ (a^2-1)y - 2az &= 3 \\ (a-1)z &= b+2 \end{aligned}$$

$1^\circ)$ $a \notin \{-1, 0, 1\}$ одређен систем

$$z = \frac{b+2}{a-1} \qquad y = \frac{1}{a^2-1}\left(\frac{2a(b+2)}{a-1} + 3\right) \qquad x = \frac{1}{a}\left(1 - \frac{1}{a^2-1}\left(\frac{2a(b+2)}{a-1}+3\right) - \frac{b+2}{a-1}\right)$$

$2^\circ)$ $a = 0$

$$\begin{aligned} y + z &= 1 \\ &= 3 \\ -y \quad - z &= b+2 \end{aligned} \longrightarrow \begin{aligned} y+z &= 1 \\ z &= 4 \\ -z &= b+2 \end{aligned} \longrightarrow \begin{aligned} y+z &= 1 \\ z &= 4 \\ 0 &= b+6 \end{aligned}$$

$b \neq -6$ немогућ

$b = -6$ 1x неодређен

$x = d \in \mathbb{R}$

$z = 4 \quad y = -3$

$3^\circ)$ $a = -1$

$$\begin{aligned} -x + y + z &= 1 \\ 2z &= 3/1 \\ -2z &= b+5 \end{aligned} \longrightarrow \begin{aligned} -x + z + y &= 1 \\ z &= \tfrac{3}{2} \\ 0 &= b+4 \end{aligned}$$

$b \neq -5$ немогућ

$b = -5$ 1x неодређен

$y = d \in \mathbb{R}$

$z = \frac{3}{2} \quad x = d$

$4^\circ)$ $a = 1$

$$\begin{aligned} x + y + z &= 1 \\ -2z &= 3 \\ 0 &= b+2 \end{aligned}$$

$b \neq -2$ немогућ

$b = -2$ 1x неодређен

$y = d \in \mathbb{R} \quad z = -\frac{3}{2} \quad x = \frac{5}{2} - d$

(6) $r = (x, y, z) \quad a = (1, 2, -1) \quad b = (-1, 0, 3)$

$a \times r = (y + 2z, -x - z, -2x + y)$

$f(r) = f(x, y, z) = (a \times r - r) \times b =$

$= (-x + y + 2z, -x - y - z, -2x + y - z) \times (-1, 0, 3) =$

$= (-3x - 3y - 3z, 5x - 4y - 5z, -x - y - z) \longleftarrow$ лин. тр.

$f(e_1) = (-3, 5, -1) = b_1$

$f(e_2) = (-3, -4, -1) = b_2$

$f(e_3) = (-3, -5, -1) = b_3$

$$\begin{vmatrix} -3 & 5 & -1 \\ -3 & -4 & -1 \\ -3 & -5 & -1 \end{vmatrix} = 0 \Rightarrow \dim\left(f(\mathbb{R}^3)\right) < 3$$

b_1, b_2 - независни јер су непропорционални

$\Rightarrow \{b_1, b_2\}$ је база $f(\mathbb{R}^3)$