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3D TOMOGRAPHY RECONSTRUCTION: FROM THEORY TO APPLICATION IN MEDICAL, INDUSTRIAL AND SCIENTIFIC RESEARCH FIELDS

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Abstract. Three-dimensional tomographic reconstruction enables volumetric analysis of internal structures across medical, biological, industrial, and geophysical applications. Building on the Radon transform, modern 3D tomography incorporates analytical inversion, iterative optimization, and deep-learning methods that address noise, limited-angle data, and low-dose constraints. This paper outlines the mathematical foundations of 3D reconstruction and presents representative applications ranging from micro-CT vasculature imaging and Brownian nanoscale tomography to muon-based inspection of shielded infrastructures. These examples demonstrate how 3D tomography increasingly connects experimental imaging with computational modeling, guiding future techniques toward more accurate and interpretable volumetric reconstruction.

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1. Introduction

Tomography refers to the reconstruction of an object's internal structure from its external projections. Its mathematical foundation originates from Radon's 1917 transform, which expresses each projection as a line integral of the unknown object [3]. Early computerized tomography inverted this relation using filtered backprojection (FBP) [3, 5], enabling practical 2D slice reconstruction.

Increasing demands for higher resolution, reduced radiation dose, and improved noise robustness motivated the development of iterative and model-based reconstruction (MBIR), as well as recent deep-learning and diffusion-based approaches [5]. These advances naturally extended to three dimensions, where volumetric imaging integrates multiple projections acquired from a range of orientations.

In this context, 3D tomography provides the mathematical and computational framework for reconstructing full volumetric structure under realistic

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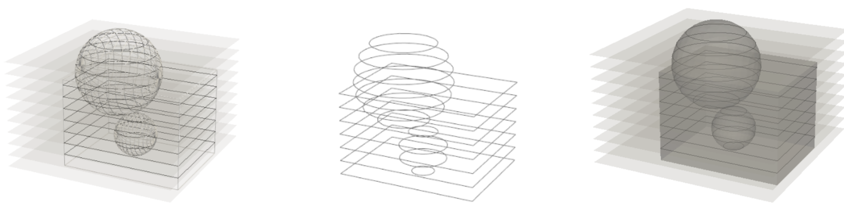
acquisition constraints, including limited-angle sampling and noisy measurements.

2. Mathematical Framework for 3D Tomography

Three-dimensional reconstruction methods generalize 2D principles to recover full volumetric geometry. While 2D tomography reconstructs planar slices, 3D approaches integrate multiple 2D projections acquired across different orientations to recover the object's entire internal structure.



(a) (b) (c)

Figure 1: Illustration of the 3D tomographic reconstruction process.(a) Volumetric object sampled by parallel slice planes.(b) Contour information extracted from each slice. (c) Reconstructed 3D volume obtained by combining all slice measurements.

2.1. From the 2D to the 3D Radon Transform

In 3D tomography, the object is represented by a scalar field $f(x, y, z)$ describing attenuation, density, or another imaging parameter. The 3D Radon transform integrates f over planes orthogonal to a direction $\mathbf{n} = (n_x, n_y, n_z)$ [3]:

$$\mathcal{R}_3 f = \iiint f(x, y, z)\delta(\rho - xn_x - yn_y - zn_z)dxdydz. \tag{2.1}$$

Each projection image recorded on a detector corresponds to sampled values of $\mathcal{R}_3[f]$ after appropriate geometric reparameterization . The inverse 3D Radon transform reconstructs the volume via filtered angular backprojection [3]:

$$f(x, y, z) = -\frac{1}{8\pi^2}\int_{S^2} \frac{\partial^2}{\partial \rho^2}\mathcal{R}_3[f](\rho, \mathbf{n})\Big|_{\rho = xn_x + yn_y + zn_z} d\mathbf{n}, \tag{2.2}$$

In practice, this inversion is approximated using 3D filtered backprojection, cone-beam variants, or numerical projectors implementing discrete ray-driven or voxel-driven integration [3].

2.2. Discrete Model and Inverse Problem

While the continuous 3D Radon transform describes the forward physics of tomography, practical reconstruction requires a discrete numerical model. The object is sampled into a 3D voxel grid and each detector measurement corresponds to a discretized ray–voxel interaction. This yields the linear forward system

$$b = Ax + \varepsilon, \tag{2.3}$$

where x represents the unknown 3D volume, b the stack of 2D projection images, and A the system matrix that maps rays in 3D space to detector pixels [2, 3]. In three-dimensional tomography, A becomes extremely large and cannot be inverted explicitly, motivating the use of numerical reconstruction techniques.

Iterative algorithms such as ART, SIRT, OSEM, and model-based iterative reconstruction (MBIR) solve the discrete inverse problem by minimizing

$$\min_{x} \|Ax - b\|^2 + \lambda R(x), \tag{2.4}$$

where $R(x)$ imposes smoothness, sparsity, or structural constraints [2, 3]. These methods are particularly important in 3D, where limited-angle sampling, noise, and low-dose acquisition lead to significant instability in the reconstruction.

Beyond classical smoothness or sparsity priors, geometric constraints play a key role in stabilizing highly underdetermined 3D inverse problems. In limited-view tomography, where large regions of Fourier space remain unsampled, shape-based regularizers help enforce global structural consistency. A notable example is the shape–centroid prior of Lukić and Balázs [7], which restricts feasible binary reconstructions to those preserving a global centroid estimate. This demonstrates how even minimal geometric information can significantly improve stability under extreme projection sparsity.

3. Challenges in Tomographic 3D Reconstruction

Although the mathematical foundations of tomographic imaging are well established, practical 3D reconstruction remains limited by several persistent issues. Sparse or limited-angle acquisition often restricts angular coverage, producing streaking artifacts and loss of detail [2, 3, 5]. Low-dose requirements further elevate noise, necessitating strong regularization or learned priors to preserve image fidelity [2].

Reconstruction accuracy is also hindered by model mismatch in heterogeneous media, where bone, concrete shielding, or fluidic nanoscale environments distort measured signals [4, 6]. In dynamic imaging scenarios, motion introduces additional inconsistency across projections, forcing algorithms to address temporal undersampling through 4D or motion-compensated reconstruction strategies [4, 5]. Finally, the computational demands of large-scale 3D inverse problems remain high, as both iterative and deep-learning-based methods depend on substantial memory and GPU acceleration [2, 3]. Recent research has begun to address these difficulties through hybrid approaches that combine physics-based modeling with data-driven priors, particularly in applications such as LC-TEM nanoscale tomography, low-dose CT, and muon imaging [4, 6].

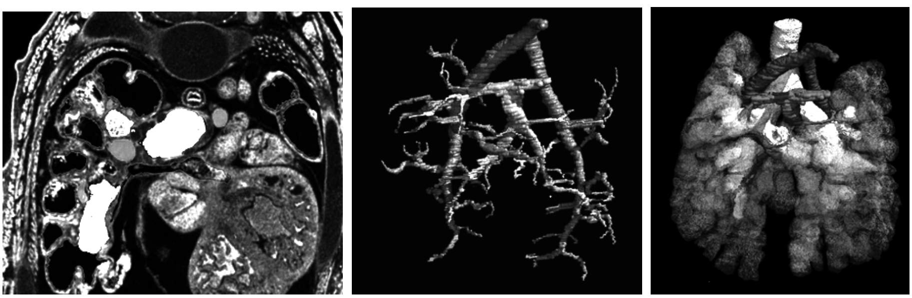
4. Representative Applications

3D tomography enables structural analysis across scales from nanometers to meters. This section summarizes three compact examples illustrating the diversity of imaging scenarios.

4.1. Pulmonary vascular imaging with µCT

High-resolution micro-computed tomography (μCT) enables non-destructive, isotropic imaging of soft-tissue vasculature. In the developmental study of *Monodelphis domestica*, Ferner [1] reconstructed the 3D pulmonary vascular network by scanning postnatal lung samples in 2D slices and using X-ray absorption contrast to visualize fine vessels without physical sectioning.

From a methodological perspective, μCT provides a full volumetric map of the vascular tree, which can then be segmented to extract branching patterns, vessel diameters, and spatial organization (see Figure 2). The purpose of using tomography in this context is capture intact vascular geometry and quantify developmental remodeling, capabilities that conventional histology cannot provide. Such imaging is essential for studying developmental biology, organ growth, and pathological alterations in microvascular networks.

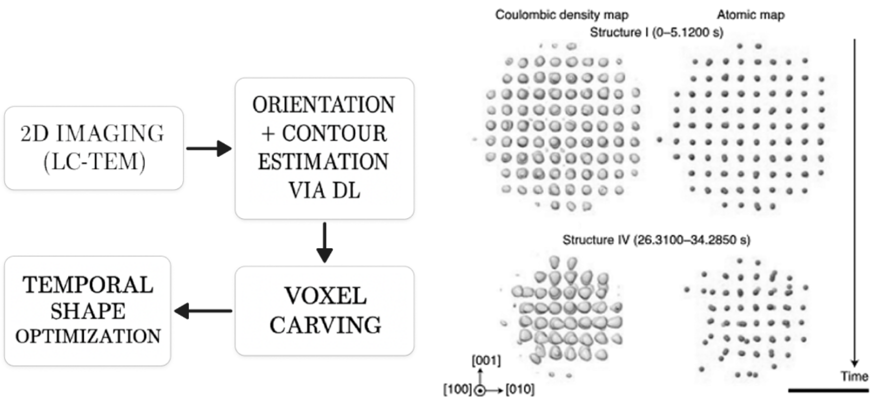


(a) (b) (c)

Figure 2: (a) µCT axial slice. (b) Segmented 3D vasculature. (c) Unsegmented µCT volume. Images adapted from [1].

4.2. Brownian Tomography at the Nanoscale

Kang et al. [4] introduced Brownian tomography, a 4D tomographic framework for reconstructing the shape and motion of single platinum nanocrystals suspended in liquid. In this setting, graphene liquid-cell TEM produces extremely noisy, anisotropic 2D projections of rapidly moving particles, conditions under which classical electron tomography fails because motion and low signal violate the assumption of consistent projection geometry. To address this, the workflow combines fast LC-TEM imaging with a deep neural network that estimates each particle's orientation and 2D contour. These contours act as constraints for voxel carving, yielding coarse 3D volumes that are subsequently refined through temporal shape optimization to ensure smooth frame-to-frame evolution. This enables reconstruction of nanoscale morphological changes (such as rounding, etching, and facet restructuring) during chemical reactions The tomographic approach is crucial because it integrates multiple projections with estimated poses to recover true 3D geometry. without it, only 2D silhouettes would be available, making volumetric and atomic-scale structural evolution impossible to quantify.



(a) (b)

Figure 3: (a) Brownian tomography workflow: LC-TEM imaging, DL-based pose/contour estimation, voxel carving, and temporal shape optimization. (b) Time-resolved 3D density representations showing nanoscale morphological evolution of a single nanocrystal [4]).

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4.3. Muon-Based Tomographic Reconstruction of Shielded Infrastructures

Muon tomography exploits naturally occurring cosmic-ray muons, whose high penetration makes them suitable for imaging large, densely shielded structures. In the study by Lefevre et al. [6], trajectories recorded around the G3 reactor at Marcoule were used to reconstruct a 3D density distribution of the reactor core. The technique operates on a principle analogous to X-ray tomography: as muons pass through the reactor, their energy loss reflects the density of the materials along their paths. A SART-based iterative reconstruction method, refined with machine-learning corrections, processes these recorded trajectories to recover the internal density field and reveal structural anomalies within the heavily shielded environment.

This study shows that 3D tomographic principles remain applicable even under extreme physical constraints, large volumes, sparse and anisotropic ray coverage, and strong material attenuation, thereby expanding the domain of tomography to nuclear infrastructure inspection.

Taken together, these examples highlight the growing role of 3D tomographic research as a link between experimental imaging and computational modeling. Continued advances are essential for improving reconstruction reliability, quantitative accuracy, and interpretability across different domains, and for enabling integrative, simulation-driven approaches capable of revealing complex internal structures with high precision.

5. Conclusion

Three-dimensional tomographic reconstruction continues to evolve from classical analytical formulations toward iterative, model-based, and learning-driven methods capable of operating under limited data, noise, and complex physical conditions. While Radon-based operators remain the mathematical foundation, advances in regularization, statistical modeling, and data-driven priors have substantially improved reconstruction fidelity across diverse imaging settings. The applications demonstrate the versatility and expanding reach of 3D tomography across spatial scales and scientific domains. Future progress will depend on integrating physics-based models with simulation-guided and machine-learning approaches, enabling accurate, interpretable, and computationally efficient reconstructions of increasingly complex internal structures.

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