The 10th Conference on Mathematics in Engineering: Theory and Applications
Novi Sad, December 12-14th 2025

SHAPE CONVEXITY DESCRIPTORS: A COMPARATIVE STUDY

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https://doi.org/10.24867/META.2025.08 Original research article

Abstract. This paper presents a comparative study of shape convexity descriptors with a particular focus on a newly proposed measure based on the first Hu moment invariant. While conventional convexity measures such as area- and perimeter-based ratios offer intuitive interpretations, they lack full invariance under geometric transformations. We propose a novel convexity measure which leverages the relation between the first Hu moment invariant and the integral of squared distances over shape regions. This measure is shown to be invariant under translation, rotation, and scaling, and satisfies all essential properties expected from a convexity descriptor. Experimental results on a variety of planar shapes confirm its discriminative power and robustness, particularly in scenarios involving non-convex and multi-component shapes.

AMS Mathematics Subject Classification (2020): 68U10

Key words and phrases: Shape analysis, convexity measure, Hu moment invariant, moment-based descriptors

1. Introduction

Convexity is one of the fundamental geometric properties of planar shapes, closely related to structural integrity, boundary regularity, and spatial compactness. Quantitative convexity descriptors aim to numerically express how much a shape deviates from being convex. These descriptors play a central role in digital shape analysis, with wide applications across multiple scientific and engineering disciplines.

In engineering, convexity measures are used in automated quality control and defect detection on manufactured parts, where deviation from convexity often signals damage or misalignment. In precision agriculture, shape convexity helps in plant species recognition, leaf disease detection, and crop phenotyping using drone or satellite imagery. In medical imaging, convexity descriptors assist in segmentation and classification tasks—particularly for detecting tumors, lesions, or anatomical irregularities where shape deformities provide diagnostic cues.

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While many convexity measures have been proposed—most notably area- and perimeter-based ones — these are often sensitive to non-similarity transformations or limited in their ability to capture subtle shape deviations. Moment-based descriptors, on the other hand, offer strong mathematical foundations and allow for invariance under translation, rotation, and scaling. In this work, we build upon a recent interpretation of the first Hu moment invariant and derive a new convexity descriptor denoted as $\mathcal{C}^{Hu}(S)$. This measure is theoretically sound and practically robust, especially when dealing with non-convex and multi-component shapes.

The paper is structured as follows: in Section 2, we recall the necessary mathematical preliminaries. Section 3 introduces the $\mathcal{C}^{Hu}(S)$ measure and analyzes its theoretical properties. Section 4 presents comparative experimental results with classical convexity descriptors.

2. Preliminaries

We begin by listing basic definitions to support understanding of the main results and to establish assumptions ensuring theoretical correctness.

A shape, as a fundamental object property similar to color or texture, is typically represented as a bounded (not necessarily connected) region in the plane, defined by foreground pixels in a binary image. For a planar shape S, the *moment of order* $p+q$, denoted as $m_{p,q}(S)$, is defined as:

$$m_{p,q}(S) = \iint_S x^p y^q \, dx dy. \tag{2.1}$$

The zero-order moment $m_{0,0}(S)$ gives the area of the shape, while $\left(\frac{m_{1,0}(S)}{m_{0,0}(S)}, \frac{m_{0,1}(S)}{m_{0,0}(S)}\right)$ determines its centroid. Since translation does not affect the shape, *central moments*, denoted by $\overline{m}_{p,q}(S)$, are introduced:

$$\overline{m}_{p,q}(S) = \iint_{(x,y)\in S} (x - x_c(S))^p \, (y - y_c(S))^q \, dx \, dy. \tag{2.2}$$

However, central moments are not invariant to scaling. To achieve scale invariance, we define the *normalized central moments* as:

$$\mu_{p,q}(S) = \frac{1}{m_{0,0}(S)^{(p+q+2)/2}} \cdot \overline{m}_{p,q}(S). \tag{2.3}$$

Achieving rotational invariance is more challenging. In his seminal work [3], Hu introduced a set of seven algebraic moment invariants with respect to rotation. This paper focuses on the first Hu invariant, which involves only second-order shape moments:

$$\mathcal{H}u(S) = \mu_{2,0}(S) + \mu_{0,2}(S) = \frac{1}{m_{0,0}(S)^2} \cdot \left(\overline{m}_{2,0}(S) + \overline{m}_{0,2}(S)\right). \tag{2.4}$$

Note that all Hu invariants are also geometric moment invariants, as they can be derived from geometric primitives [4]. This enables their use as shape descriptors in image processing and computer vision.

We will evaluate the behavior of our proposed convexity measure against two common alternatives: $\mathcal{C}^{Area}(S)$ and $\mathcal{C}^{Per}(S)$ [1], defined as:

$$\mathcal{C}^{Area}(S) = \frac{Area(S)}{Area(S^{ch})}, \qquad \mathcal{C}^{Per}(S) = \frac{Perimeter(S^{ch})}{Perimeter(S)}, \tag{2.5}$$

where S^{ch} denotes the convex hull of the shape S.

3. Convexity measure based on the first Hu moment invariant

In this section, we present the new measure of shape convexity originally defined in [5] and which is based on a new interpretation of the first Hu moment invariant, introduced in [2], which serves as the base for deriving a new shape measure. Following by this result, we started from the quantity $\mathcal{I}_{SD}(S)$, representing the integral of the square of the distance between all pairs of points of the shape S, defined as follows (for the full derivation, see [5]):

$$\begin{aligned}\mathcal{I}_{SD}(S) &= \frac{1}{2}\cdot \iint\limits_{(x,y)\in S} \iint\limits_{(z,t)\in S} \left((x-z)^2+(y-t)^2\right) dx\,dy\,dz\,dt \\ &= m_{0,0}(S)\cdot\left(\overline{m}_{2,0}(S)+\overline{m}_{0,2}(S)\right). \end{aligned} \tag{3.1}$$

Now, based on (2.4) and (3.1) it follows that

$$\frac{1}{m_{0,0}(S)^3}\cdot\mathcal{I}_{SD}(S) \;=\; \frac{1}{m_{0,0}(S)^2}\cdot\left(\overline{m}_{2,0}(S)+\overline{m}_{0,2}(S)\right) \;=\; \mathcal{H}u(S). \tag{3.2}$$

This result, originally introduced in [2], implies that the first Hu moment invariant can be expressed by means of the integral of the squared distance between all pairs of points belonging to the shape normalized by the the area of the shape itself. Given the quantity $\mathcal{I}_{SD}(S)$ can be expressed in terms of the area and the first Hu moment invariant related to the shape considered, it is invariant to translation and rotation. Based on this, if the same reasoning is applied to the convex hull of S, denoted with S^{ch}, the following holds:

$$\begin{aligned}\mathcal{I}_{SD}(S^{ch}) &= \frac{1}{2}\cdot \iint\limits_{(x,y)\in S^{ch}} \iint\limits_{(z,t)\in S^{ch}} \left((x-z)^2+(y-t)^2\right) dx\,dy\,dz\,dt \\ &\geq \frac{1}{2}\cdot \iint\limits_{(x,y)\in S} \iint\limits_{(z,t)\in S} \left((x-z)^2+(y-t)^2\right) dx\,dy\,dz\,dt \;=\; \mathcal{I}_{SD}(S). \end{aligned} \tag{3.3}$$

The latter represents a key result that provides the foundation for deriving in [5] a new shape measure evaluating the degree to which a given shape is convex. In this context, we present the following lemma, whose results are a direct consequence of the last inequality in (3.3).

Lemma 3.1. *For an arbitrary shape S, the following statements hold:*

(a) $0 < \dfrac{\mathcal{I}_{SD}(S)}{\mathcal{I}_{SD}(S^{ch})} \leq 1$;

(b) $\mathcal{I}_{SD}(S) = \mathcal{I}_{SD}(S^{ch}) \Leftrightarrow S \equiv S^{ch}$.

Based on the results in Lemma 3.1, we can conclude that the observed ratio can be used in a certain way to characterize how much the shape S deviates from its convex hull, i.e., how convex it is. Accordingly, we have the following definition.

Definition 3.2. Given a shape S, the *shape convexity measure* $\mathcal{C}^{Hu}(S)$ is defined as

$$\mathcal{C}^{Hu}(S) = \frac{\mathcal{I}_{SD}(S)}{\mathcal{I}_{SD}(S^{ch})} = \frac{m_{0,0}(S)^3 \cdot \mathcal{H}u(S)}{m_{0,0}(S^{ch})^3 \cdot \mathcal{H}u(S^{ch})}. \tag{3.4}$$

Such a defined convexity measure $\mathcal{C}^{Hu}(S)$ also satisfies all the basic properties that any well-defined convexity measure should satisfy. Some of these properties are given in the following theorem, which we state here without proof. For more details about $\mathcal{C}^{Hu}(S)$ and its properties, the reader is referred to the paper [5] where the measure $\mathcal{C}^{Hu}(S)$ itself was originally designed.

Theorem 3.3. *For the convexity measure $\mathcal{C}^{Hu}(S)$ the following holds:*

(a) $\mathcal{C}^{Hu}(S) \in (0, 1]$, for all shapes S;

(b) $\mathcal{C}^{Hu}(S) = 1 \Leftrightarrow S$ is a convex shape;

(c) $\mathcal{C}^{Hu}(S)$ is invariant under similarity transformations (i.e., translation, rotation and scaling);

(d) the lower bound 0 for $\mathcal{C}^{Hu}(S)$ is the best possible, i.e., for each $\epsilon > 0$ there exists a shape S such that $0 < \mathcal{C}^{Hu}(S) < \epsilon$.

4. Experimental illustration of the convexity measures

In this section, we present several illustrative experiments to compare the behavior of the shape convexity measure $\mathcal{C}^{Hu}(S)$ with two commonly used convexity measures: $\mathcal{C}^{Area}(S)$ and $\mathcal{C}^{Per}(S)$ [1]. The performance of these measures is illustrated on synthetically generated shapes, with modifications including the size and relative position of the holes within the shape.

The first experiment illustrates the behavior of the three convexity measures with respect to the relative position of the indentation within the shape, without changing its size (shape examples are shown in Figure 1, (a)–(c)). As observed from the first plot in Figure 2, the area and perimeter of both the shape and its convex hull remain unchanged, causing $\mathcal{C}^{Area}(S)$ and $\mathcal{C}^{Per}(S)$ to be insensitive to the indentation's position. This can be considered a weakness compared to $\mathcal{C}^{Hu}(S)$, which is able to distinguish between the given shapes. Notably, the measured $\mathcal{C}^{Hu}(S)$ value is invariant with respect to the symmetrical placement of the indentation. This behavior aligns with the theoretical results established in the previous section, confirming that the proposed measure is invariant under shape similarity transformations (the examined shape pairs are rotation-invariant around the vertical axis of symmetry).

In the second experiment, we examine how the convexity measures respond when the width of the indentation increases (Figure 1, (d)–(f)). As expected, the convexity decreases, which is reflected in both $\mathcal{C}^{Hu}(S)$ and $\mathcal{C}^{Area}(S)$ (see the central plot in Figure 2). In contrast, the perimeter-based measure $\mathcal{C}^{Per}(S)$ remains unchanged, indicating its insensitivity to such deformation. This is consistent with its definition: increasing the indentation width does not alter the perimeters of the shape or its convex hull, while the area of the shape decreases relative to the unchanged convex hull area.

In the final experiment, we consider the case where the side length of the square hole within the shape increases (Figure 1, (g)–(i)). As anticipated, the convexity decreases, which is captured by all three measures (see the third plot in Figure 2). This behavior is in full accordance with their definitions: a larger internal hole leads to a greater deviation between the shape and its convex hull.

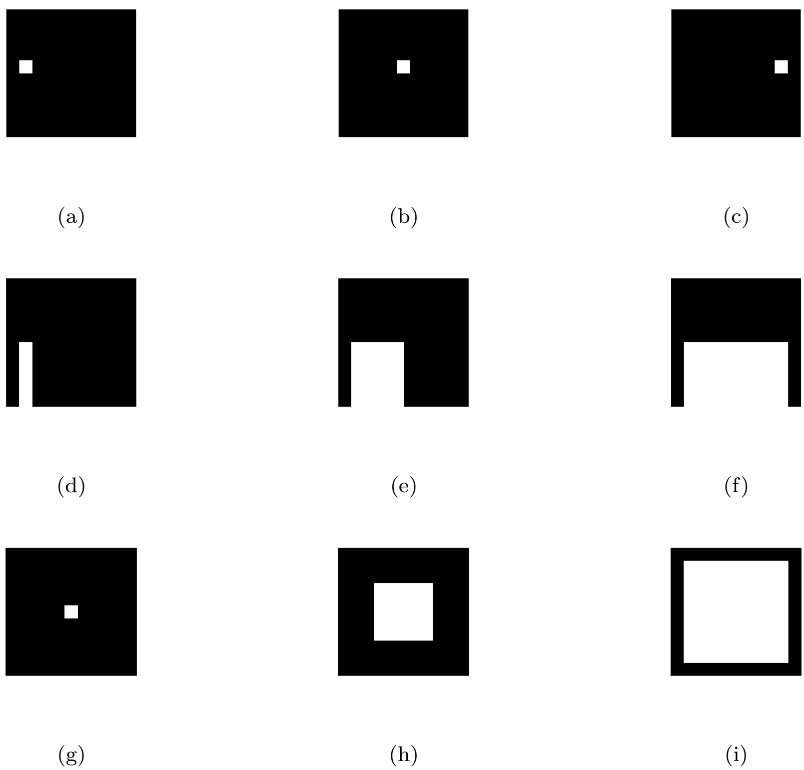

Figure 1: Shapes obtained from the squares for different hole positions (first row), indentation width (second row) and size of the holes (third row).

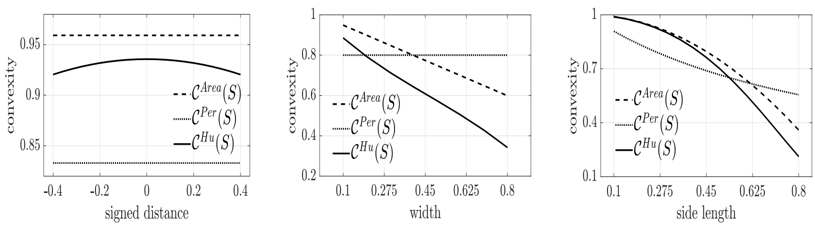

Figure 2: Plots of computed $\mathcal{C}^{Hu}(S), \mathcal{C}^{Area}(S)$ and $\mathcal{C}^{Per}(S)$ values with respect to hole position, indentation width, and hole size within the shape.

Acknowledgement

This research was supported by the Department of Fundamental Disciplines in Engineering through the project entitled "Socio-technological aspects of improving the teaching process in the English language in fundamental disciplines".

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