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FUZZY MODEL FOR NOISE REMOVAL IN MONTE CARLO RENDERING BASED ON AGGREGATION FUNCTION[1]

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Abstract. Monte Carlo rendering represents a modern and highly precise technique for simulating light behavior in three-dimensional scenes, enabling a high level of photorealism. However, a limited number of samples per pixel often leads to the occurrence of pronounced statistical noise, which significantly affects the quality and usability of rendered images. To address this issue, this paper proposes a fuzzy model for noise removal in Monte Carlo rendering, based on aggregation functions (and an adaptive window size during the filtering process). The model applies fuzzy principles to determine the degree of similarity between neighboring pixels and dynamically adjusts the filter parameters according to the local image structure. This ensures efficient noise reduction while preserving important visual details and avoiding artifacts. Experimental results indicate that the proposed approach provides a better trade-off between image quality and computational complexity compared to standard filtering methods.

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1. Introduction

Denoising digital images is one of the basic problems in image processing and computer visualization. The quality of the image affects the reliability of the analysis and the visual experience, so the development of effective filtering methods is of great importance. In computer graphics, Monte Carlo rendering generates stochastically noise due to random sampling of light paths, which appears as low-intensity variational noise ("grain noise") and occasional extreme outliers ("fireflies"). These phenomena degrade visual quality and increase rendering time by requiring a larger number of samples per pixel. This work builds on a previously developed fuzzy model for removing impulse noise

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using the $AWAM$ (Adaptive Weighted Aggregation Mean) filter [3], which combines noise detection and adaptive filtering with dynamic window expansion. In this research the model is extended to Monte Carlo noise through introduction of statistical detection based on local mean and standard deviation, as well as special processing of extremely bright pixels ("fireflies"). The proposed fuzzy-$AWAM$ approach introduces a combined spatial photometric weighted aggregation mechanism that enables efficient noise removal while preserving image structure and realistic light transitions. Experiments show that the model gives more stable and natural results in comparison with standard filtering methods [1] and provides a basis for further development of advanced fuzzy algorithms in rendering.

2. Monte Carlo rendering

Monte Carlo rendering is a method for creating realistic images by simulating the movement of light through a scene using random sampling. It is employed because an analytical solution to the integrals that describe light material interactions is often infeasible, especially in complex scenes with reflections, refractions, and volumetric effects [2]. The goal is to calculate the average contribution of a large number of light paths for each pixel.

The main challenge of Monte Carlo rendering is the high estimation variance, which manifests as random ("grain") noise pixels may appear slightly brighter or darker than the true value. Noise is particularly pronounced when the number of samples per pixel is low, which is common in practice due to limited computational resources and rendering time.

To reduce variance, techniques such as importance sampling, stratified sampling, and adaptive sampling are applied, but these methods do not eliminate noise completely. Therefore, post-filtering (denoising) is used to remove the remaining variations by exploiting information from neighboring pixels or additional geometric and lighting data.

Approaches to noise reduction are generally divided into methods that improve sampling during rendering and those applied as post-processing on the final image. In recent years, deep-learning-based models have become dominant; however, they require large training data sets, substantial computational resources, and often exhibit limited generalization and detail preservation.

For these reasons, this paper considers a statistical and fuzzy approach to Monte Carlo denoising, similar to the method presented in [5], where the distribution of samples for each pixel is analyzed and relevant neighbors are selected for filtering. Fuzzy logic, unlike classical filters with sharp thresholds, uses degrees of membership that enable adaptive decision-making regarding the contribution of each pixel. This allows better preservation of edges and fine details, does not require pretrained models, and can be directly applied to rendered images resulting in flexible and efficient noise removal suitable for production-level rendering.

3. Image filtering - algorithm

The proposed model is based on the principles of fuzzy aggregation and adaptive weighted filtering in the spatial-photometric domain. The algorithm consists of two basic phases: noise detection and a filtering phase incorporating a dynamic window expansion mechanism that enables stable estimation even in areas of high lighting variance. Unlike the impulse noise treated in the previous work, this study considers variational Monte Carlo noise and extreme light values ("fireflies"), which require a statistical detection approach.

3.1. Noise detection phase

For each image point $A(i,j)$, a local window $W_{i,j}$ of size $m \times m$ is defined. Within this window, the local arithmetic mean and standard deviation are computed:

$$\mu_{i,j} = \frac{1}{|W_{i,j}|} \sum_{(p,q)\in W_{i,j}} A(p,q), \quad \sigma_{i,j} = \sqrt{\frac{1}{|W_{i,j}|} \sum_{(p,q)\in W_{i,j}} (A(p,q) - \mu_{i,j})^2}.$$

Based on these values, the deviation of the pixel from the local average is computed:

$$d(i,j) = |A(i,j) - \mu_{i,j}|.$$

A pixel is marked as "noisy" if the deviation exceeds the threshold proportional to the local variance:

$$b(i,j) = \begin{cases} 255, & d(i,j) > T \cdot \sigma_{i,j} \\ 0, & \text{otherwise} \end{cases}.$$

The parameter T represents the detection threshold, which is experimentally determined sets in the range $T \in [1.5, 3.0]$. In this way, a binary noise mask $b(i,j)$ is formed, where a value of 255 denotes noise and 0 denotes a "clean" pixel. To identify rare extreme values ("fireflies"), a more robust criterion based on the local median $M_{i,j}$ is used:

$$b_f(i,j) = \begin{cases} 255, & A(i,j) > M_{i,j} + K\sigma_{i,j} \\ 0, & \text{otherwise} \end{cases},$$

where K is a constant that determines the threshold for identification outliers (most often $K \in [4, 8]$). Pixels detected as fireflies are corrected in a preprocessing step by replacing them with the local median, preventing extreme values from negatively affecting the filtering phase.

3.2. Filtering phase

For each pixel marked as noisy, a set of clean neighboring pixels is determined as $S_{i,j} = \{A(p,q) | b(p,q) = 0\}$ (b is the binary noise mask) within the window $W_{i,j}$. If the number of clean pixels in the current window is insufficient ($|S_{i,j}| < n$), the window is dynamically expanded $W_{i,j}^{(m)} \to W_{i,j}^{(m+2)}$ (index m

- indicates the size of the window in pixels, is changing) until the condition $|S_{i,j}| \geq n$) is met or until the maximum size m_{max} is reached.

Let the pixel values in the set $S_{i,j}$ be denoted by p_k, and their distances from the center by d_k. Spatial weights and photometric weights are defined as follows:

$$\alpha_k = \frac{1}{\beta^{d_k}}, \quad s_k = \exp(-\frac{|p_k - A(i,j)|}{\sigma_I}),$$

where $\beta > 1$ is a parameter that controls the weight decay with distance, and σ_I is the coefficient controlling sensitivity to intensity differences.

The combined weight for each neighbor is:

$$w_k = \frac{\alpha_k \cdot s_k}{\sum\limits_r \alpha_r s_r}.$$

The reconstruction of the damaged pixel is performed using the weighted power-root mean (WAM):

$$\hat{A}(i,j) = (\sum_k w_k p_k^{\kappa})^{1/\kappa}.$$

where the parameter κ enables interpolation between different types of means:

$\kappa = 1$: arithmetic mean,

$\kappa \to 0$: approximately the geometric mean,

$\kappa > 1$ more weight to larger values.

This defines the $AWAM$ fuzzy aggregation operator adapted to Monte Carlo noise.

3.3. Final reconstruction

After filtering all detected pixels, the resulting image $\hat{A}$ represents the denoised estimate of the original image. To evaluate the performance of the model, the following metric functions are used:

$$MAE = \frac{1}{N} \sum |A - \hat{A}|, \quad PSNR = 10 \log_{10} \frac{255^2}{MSE},$$

where $MSE = \frac{1}{N} \sum (A - \hat{A})^2$.

4. Experimental setup and evaluation

A custom $3D$ scene was used to evaluate the proposed fuzzy model. The scene was rendered in the Autodesk 3ds Max environment using the V-Ray renderer. The image was generated in multiple variants with different numbers of samples per pixel (spp): 2, 4, 8, 16, and 32 spp. The reference image was obtained by rendering at 128 spp, which provides a visually and numerically stable ground truth.

The developed algorithm was then applied to the rendered images. The results obtained using the proposed method were compared with standard filters commonly used in practice: Median, Gaussian, Bilateral, and Non-Local

Means (NLM). The comparison was performed using two error metrics MAE and $PSNR$ computed relative to the reference render.

The obtained results show that the proposed $AWAM$ model significantly outperforms all benchmark methods across all noise levels. $AWAM$ achieves the lowest MAE values (often 36 times lower than those of the other filters) and the highest $PSNR$ values, with improvements of up to $+7$ dB in favor of the proposed method. These findings indicate that the fuzzy $AWAM$ approach better corresponds to the statistical nature of Monte Carlo noise and enables more stable and precise image reconstruction compared to classical filtering techniques.

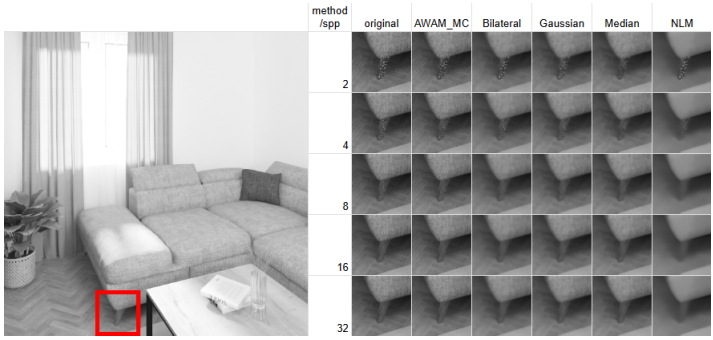

Figure 1: Comparative visualization of Monte Carlo noise filtering results on a local image segment. Rendered versions with different spp values and the filtering outputs of the $AWAM_MC$ method and standard filters are shown. The $AWAM_MC$ method produces the most visually faithful results.

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